Diskriminant polynomu třetího stupně

```
> restart;
> p:=a*x^3+b*x^2+c*x+d;
```

$$p := a\,x^3 + b\,x^2 + c\,x + d$$

```
> p_n:=expand(p/a);
```

$$p\_n := x^3 + \frac{b\,x^2}{a} + \frac{c\,x}{a} + \frac{d}{a}$$

```
> p_s:=(x-x1)*(x-x2)*(x-x3);
  p_s:=collect(p_s,x,factor);
```

$$p\_s := (x - x1)\,(x - x2)\,(x - x3)$$

$$p\_s := x^3 + (-x1 - x2 - x3)\,x^2 + (x1\,x2 + x3\,x1 + x3\,x2)\,x - x1\,x2\,x3$$

Viétovy vztahy - vyjádření koeficientů polynomu pomocí elementárních symetrických polynomů

```
> s1:=coeff(p_s,x^2)=coeff(p_n,x^2);
  s2:=coeff(p_s,x)=coeff(p_n,x);
  s3:=coeff(p_s,x,0)=coeff(p_n,x,0);
```

$$s1 := -x1 - x2 - x3 = \frac{b}{a}$$

$$s2 := x1\,x2 + x3\,x1 + x3\,x2 = \frac{c}{a}$$

$$s3 := -x1\,x2\,x3 = \frac{d}{a}$$

Výpočet diskriminantu programem:

```
> Diskriminant(a,b,c,d):=discrim(p,x);
```

$$\text{Diskriminant}(a, b, c, d) := -27\, a^2\, d^2 + 18\, a\, d\, b\, c + b^2\, c^2 - 4\, b^3\, d - 4\, a\, c^3$$

Z Viétových vztahů vyjádříme keoeficienty *b*, *c*, *d*

```
> Reseni_bcd:=solve([s1,s2,s3],{b,c,d});
```

$$Reseni\_bcd := \{ d = -x1\, x2\, x3\, a,\ b = -x1\, a - x2\, a - x3\, a,\ c = x1\, x2\, a + x3\, x1\, a + x3\, x2\, a \}$$

a dosadíme do diskriminantu (viz proměnná Diskriminant):

```
> Diskriminant2:=eval(Diskriminant(a,b,c,d),
  Reseni_bcd);
```

$$\begin{aligned} Diskriminant2 := & -27\, a^4\, x1^2\, x2^2\, x3^2 - 18\, a^2\, x1\, x2\, x3 \\ & (-x1\, a - x2\, a - x3\, a)\, (x1\, x2\, a + x3\, x1\, a + x3\, x2\, a) \\ & + (-x1\, a - x2\, a - x3\, a)^2\, (x1\, x2\, a + x3\, x1\, a + x3\, x2\, a)^2 \\ & + 4\, (-x1\, a - x2\, a - x3\, a)^3\, x1\, x2\, x3\, a \\ & - 4\, a\, (x1\, x2\, a + x3\, x1\, a + x3\, x2\, a)^3 \end{aligned}$$

Po zjednodušení dostaneme diskriminant vyjádřený pomocí kořenů *x1*, *x2*, *x3* polynomu *p*:

```
> Diskriminant(x1,x2,x3):=factor(Diskriminan
  t2);
```

$$\text{Diskriminant}(x1, x2, x3) := a^4\, (x2 - x3)^2\, (x1 - x3)^2\, (x1 - x2)^2$$

Souvislost diskriminantu a Vandermondova determinantu (determinantu Vandermondovy matice)

```
> with(LinearAlgebra):
> V:=VandermondeMatrix(<x1,x2,x3>);
```

$$V := \begin{bmatrix} 1 & x1 & x1^2 \\ 1 & x2 & x2^2 \\ 1 & x3 & x3^2 \end{bmatrix}$$

```
> D3:=Transpose(V).V;
```

$$D3 := \begin{bmatrix} 3 & x1+x2+x3 & x1^2+x2^2+x3^2 \\ x1+x2+x3 & x1^2+x2^2+x3^2 & x1^3+x2^3+x3^3 \\ x1^2+x2^2+x3^2 & x1^3+x2^3+x3^3 & x1^4+x2^4+x3^4 \end{bmatrix}$$

```
> factor(Determinant(D3));
```

$$(x2-x3)^2\,(x1-x3)^2\,(x1-x2)^2$$

Diskriminanty polynomů 2., 3. a 4. stupně:

```
> discrim(a*x^2+b*x+c,x);
```

$$-4\,a\,c+b^2$$

```
> discrim(a*x^3+b*x^2+c*x+d,x);
```

$$-27\,a^2\,d^2+18\,a\,d\,b\,c+b^2\,c^2-4\,b^3\,d-4\,a\,c^3$$

```
> discrim(a*x^4+b*x^3+c*x^2+d*x+e,x);
```

$$\begin{aligned} &-128\,a^2\,e^2\,c^2-4\,b^3\,d^3-27\,b^4\,e^2-4\,a\,c^3\,d^2+16\,a\,c^4\,e \\ &\quad-6\,a\,e\,b^2\,d^2+144\,a\,e^2\,c\,b^2+144\,a^2\,e\,c\,d^2+18\,a\,b\,d^3\,c \\ &\quad+c^2\,b^2\,d^2-4\,c^3\,b^2\,e+256\,a^3\,e^3-192\,a^2\,e^2\,b\,d-80\,a\,b\,d\,c^2\,e \\ &\quad+18\,b^3\,d\,c\,e-27\,a^2\,d^4 \end{aligned}$$

```
>
```