Binomické rovnice. Výpočet n-tých odmocnin z komplexního čísla.

1. $x^3 - 8 = 0$

```
> restart;
> Rovnice:=x^3-8=0;
```

$$Rovnice := x^3 - 8 = 0$$

```
> Reseni:=solve(Rovnice,{x});
```

$$Reseni := \{x = 2\}, \{x = -1 + \sqrt{3}\, I\}, \{x = -1 - \sqrt{3}\, I\}$$

Následuje posloupnost příkazů vedoucí ke grafickému znázornění řešení binomické rovnice.
Nejprve převedeme jednotlivá řešení z algebraického tvaru do formy uspořádaných dvojic. Začneme tím, že zjistíme stupeň rovnice n. Třeba pomocí počtu jejích řešení:

```
> n:=nops([Reseni]);
```

$$n := 3$$

Potom definujeme n-prvkové pole Z, do něhož jednotlivé dvojice uložíme:

```
> Z:=array(1..n):
> for i from 1 to n do Z[i]:=eval([Re(x),Im(x)],Reseni[i]) od;
```

$$Z_1 := [2, 0]$$

$$Z_2 := [-1, \sqrt{3}]$$

$$Z_3 := [-1, -\sqrt{3}]$$

Obrazy těchto bodů uložíme do proměnné Body:

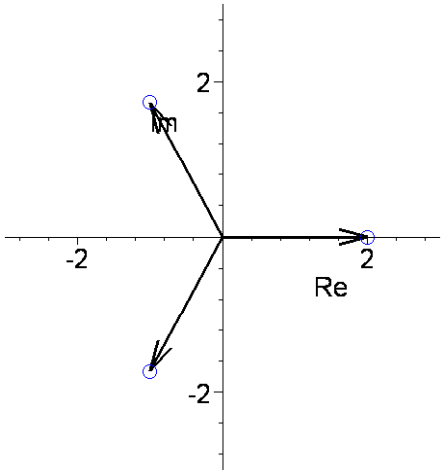
```
> Body:=plots[pointplot](Z,symbol=circle,color=blue,thickness=3,symb
  olsize=30,tickmarks=[3,3],labels=[Re,Im],view=[-3..3,-3..3]):
```

Podobně vytvoříme grafickou reprezentaci šipek - průvodičů jednotlivých odmocnin. Budou uloženy v n-prvkovém poli R:

```
> R:=array(1..n):
> for i from 1 to n do
  R[i]:=plots[arrow](Z[i],thickness=3,shape=arrow) od:
```

Nakonec body i vektory zobrazíme:

```
> plots[display](Body,seq(R[i],i=1..n));
```



2. $x^{13} - 8 = 0$

```
> restart;
> Rovnice:=x^13-8=0;
```

$$Rovnice := x^{13} - 8 = 0$$

```
> Reseni:=solve(Rovnice,{x});
```

$$Reseni := \{x = 2^{(3/13)}\}, \{x = \left(\cos\left(\frac{2\pi}{13}\right) + \cos\left(\frac{9\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(\cos\left(\frac{4\pi}{13}\right) + \cos\left(\frac{5\pi}{26}\right)I\right)2^{(3/13)}\}, \{x = \left(\cos\left(\frac{6\pi}{13}\right) + \cos\left(\frac{\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(-\cos\left(\frac{5\pi}{13}\right) + \cos\left(\frac{3\pi}{26}\right)I\right)2^{(3/13)}\}, \{x = \left(-\cos\left(\frac{3\pi}{13}\right) + \cos\left(\frac{7\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(-\cos\left(\frac{\pi}{13}\right) + \cos\left(\frac{11\pi}{26}\right)I\right)2^{(3/13)}\}, \{x = \left(-\cos\left(\frac{\pi}{13}\right) - \cos\left(\frac{11\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(-\cos\left(\frac{3\pi}{13}\right) - \cos\left(\frac{7\pi}{26}\right)I\right)2^{(3/13)}\}, \{x = \left(-\cos\left(\frac{5\pi}{13}\right) - \cos\left(\frac{3\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(\cos\left(\frac{6\pi}{13}\right) - \cos\left(\frac{\pi}{26}\right)I\right)2^{(3/13)}\}, \{x = \left(\cos\left(\frac{4\pi}{13}\right) - \cos\left(\frac{5\pi}{26}\right)I\right)2^{(3/13)}\},$$
$$\{x = \left(\cos\left(\frac{2\pi}{13}\right) - \cos\left(\frac{9\pi}{26}\right)I\right)2^{(3/13)}\}$$

```
> n:=nops([Reseni]);
```

$$n := 13$$

```
> Z:=array(1..n):
> for i from 1 to n do Z[i]:=eval([Re(x),Im(x)],Reseni[i]) od;
```

$$Z_1 := [2^{(3/13)}, 0]$$
$$Z_2 := \left[2^{(3/13)}\cos\left(\frac{2\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{9\pi}{26}\right)\right]$$
$$Z_3 := \left[2^{(3/13)}\cos\left(\frac{4\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{5\pi}{26}\right)\right]$$
$$Z_4 := \left[2^{(3/13)}\cos\left(\frac{6\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{\pi}{26}\right)\right]$$
$$Z_5 := \left[-2^{(3/13)}\cos\left(\frac{5\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{3\pi}{26}\right)\right]$$
$$Z_6 := \left[-2^{(3/13)}\cos\left(\frac{3\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{7\pi}{26}\right)\right]$$
$$Z_7 := \left[-2^{(3/13)}\cos\left(\frac{\pi}{13}\right), 2^{(3/13)}\cos\left(\frac{11\pi}{26}\right)\right]$$

$$Z_8 := \left[-2^{(3/13)} \cos\left(\frac{\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{11\,\pi}{26}\right) \right]$$

$$Z_9 := \left[-2^{(3/13)} \cos\left(\frac{3\,\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{7\,\pi}{26}\right) \right]$$

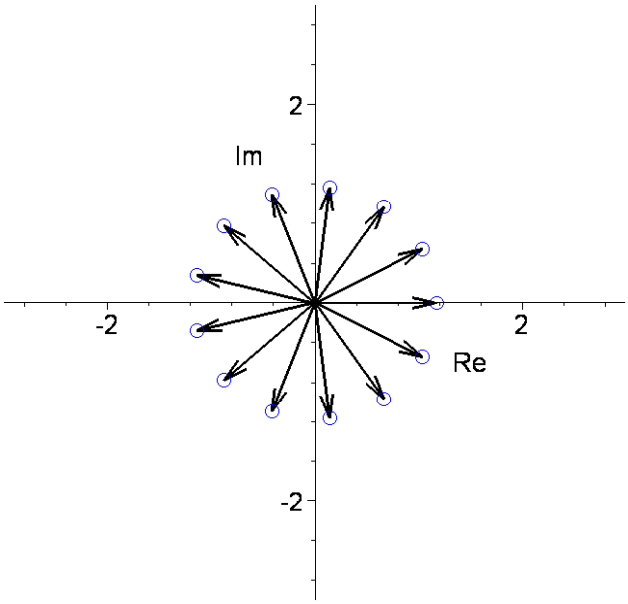
$$Z_{10} := \left[-2^{(3/13)} \cos\left(\frac{5\,\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{3\,\pi}{26}\right) \right]$$

$$Z_{11} := \left[2^{(3/13)} \cos\left(\frac{6\,\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{\pi}{26}\right) \right]$$

$$Z_{12} := \left[2^{(3/13)} \cos\left(\frac{4\,\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{5\,\pi}{26}\right) \right]$$

$$Z_{13} := \left[2^{(3/13)} \cos\left(\frac{2\,\pi}{13}\right), -2^{(3/13)} \cos\left(\frac{9\,\pi}{26}\right) \right]$$

```
> Body:=plots[pointplot](Z,symbol=circle,color=blue,thickness=3,symb
  olsize=30,tickmarks=[3,3],labels=[Re,Im],view=[-3..3,-3..3]):
> R:=array(1..n):
> for i from 1 to n do
  R[i]:=plots[arrow](Z[i],thickness=3,shape=arrow) od:
> plots[display](Body,seq(R[i],i=1..n));
```



3. Určete všechny třetí odmocniny z čísla -2+2i. (Tj. řešte rovnici $x^3 + 2 - 2\,I = 0$)

```
> restart;
> Rovnice:=x^3+2-2*I=0;
```

$$Rovnice := x^3 + 2 - 2\,I = 0$$

```
> Reseni:=solve(Rovnice,{x});
```

$$Reseni := \{x = -\frac{1}{2} - \frac{1}{2}I + \frac{\sqrt{3}}{2} - \frac{1}{2}I\sqrt{3}\}, \{x = -\frac{1}{2} - \frac{1}{2}I - \frac{\sqrt{3}}{2} + \frac{1}{2}I\sqrt{3}\}, \{x = 1 + I\}$$

```
> n:=nops([Reseni]);
```

$$n := 3$$

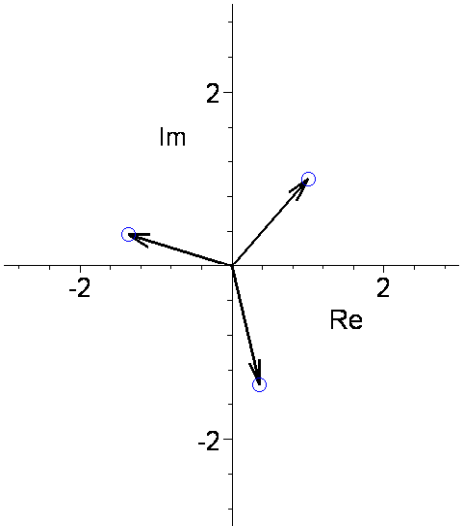
```
> Z:=array(1..n):
> for i from 1 to n do Z[i]:=eval([Re(x),Im(x)],Reseni[i]) od;
```

$$Z_1 := \left[-\frac{1}{2} + \frac{\sqrt{3}}{2}, -\frac{1}{2} - \frac{\sqrt{3}}{2}\right]$$

$$Z_2 := \left[-\frac{1}{2} - \frac{\sqrt{3}}{2}, -\frac{1}{2} + \frac{\sqrt{3}}{2}\right]$$

$$Z_3 := [1, 1]$$

```
> Body:=plots[pointplot](Z,symbol=circle,color=blue,thickness=3,symb
  olsize=30,tickmarks=[3,3],labels=[Re,Im],view=[-3..3,-3..3]):
> R:=array(1..n):
> for i from 1 to n do
  R[i]:=plots[arrow](Z[i],thickness=3,shape=arrow) od:
> plots[display](Body,seq(R[i],i=1..n));
```



4. Určete všechny třetí odmocniny z čísla i. (Tj. řešte rovnici $x^3 - I = 0$)

```
> restart;
> Rovnice:=x^3-I=0;
```

$$Rovnice := x^3 - I = 0$$

```
> Reseni:=solve(Rovnice,{x});
```

$$Reseni := \{x = \frac{1}{2}I + \frac{\sqrt{3}}{2}\}, \{x = \frac{1}{2}I - \frac{\sqrt{3}}{2}\}, \{x = -I\}$$

```
> n:=nops([Reseni]);
```

$$n := 3$$

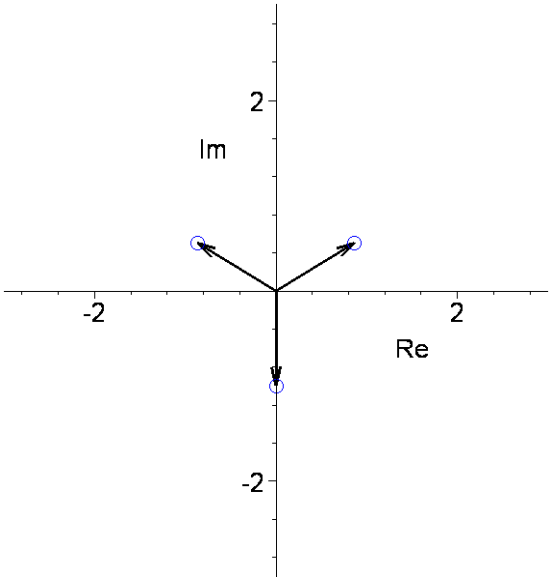
```
> Z:=array(1..n):
> for i from 1 to n do Z[i]:=eval([Re(x),Im(x)],Reseni[i]) od;
```

$$Z_1 := \left[\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$$

$$Z_2 := \left[-\frac{\sqrt{3}}{2}, \frac{1}{2}\right]$$

$$Z_3 := [0, -1]$$

```
> Body:=plots[pointplot](Z,symbol=circle,color=blue,thickness=3,symb
  olsize=30,tickmarks=[3,3],labels=[Re,Im],view=[-3..3,-3..3]):
> R:=array(1..n):
> for i from 1 to n do
  R[i]:=plots[arrow](Z[i],thickness=3,shape=arrow) od:
> plots[display](Body,seq(R[i],i=1..n));
```



5. Určete všechny třetí odmocniny z čísla -4. (Tj. řešte rovnici $x^4 + 4 = 0$)

```
> restart;
> Rovnice:=x^4+4=0;
```

$$Rovnice := x^4 + 4 = 0$$

```
> Reseni:=solve(Rovnice,{x});
```

$$Reseni := \{x = -1 + I\}, \{x = -1 - I\}, \{x = 1 + I\}, \{x = 1 - I\}$$

```
> n:=nops([Reseni]);
```

$$n := 4$$

```
> Z:=array(1..n):
> for i from 1 to n do Z[i]:=eval([Re(x),Im(x)],Reseni[i]) od;
```

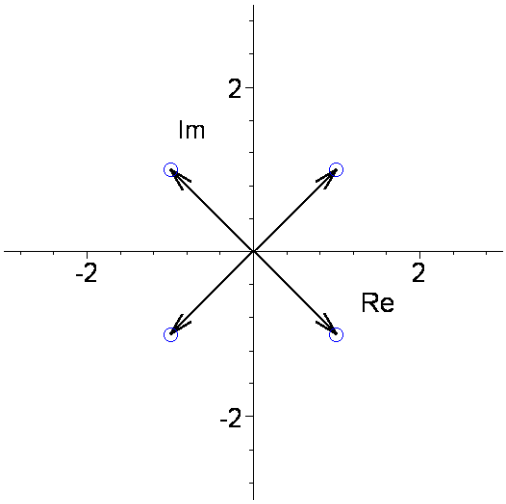
$$Z_1 := [-1, 1]$$

$$Z_2 := [-1, -1]$$

$$Z_3 := [1, 1]$$

$$Z_4 := [1, -1]$$

```
> Body:=plots[pointplot](Z,symbol=circle,color=blue,thickness=3,symb
  olsize=30,tickmarks=[3,3],labels=[Re,Im],view=[-3..3,-3..3]):
> R:=array(1..n):
> for i from 1 to n do
  R[i]:=plots[arrow](Z[i],thickness=3,shape=arrow) od:
> plots[display](Body,seq(R[i],i=1..n));
```




```
>
```