1.10 Hlavní směry

```
> restart;
> with(LinearAlgebra):
> with(linalg):
> with(plots):
> X:=Vector[row]([x,y,z,1]);
```

$$X := [x, y, z, 1]$$

Matice kvadriky:

```
> K:=Matrix(a,1..4,1..4,shape=symmetric);
```

$$K := \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$$

Rovnice kvadriky:

```
> Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
```

$$Kv := a(1,1)\,x^2 + 2\,a(1,2)\,x\,y + 2\,a(1,3)\,x\,z + a(2,2)\,y^2 + 2\,a(2,3)\,y\,z + a(3,3)\,z^2 + 2\,a(1,4)\,x + 2\,a(2,4)\,y + 2\,a(3,4)\,z + a(4,4) = 0$$

Parametrické rovnice tečny se směrem $sm = [u, v, w]$ a s bodem dotyku $M = [m, n, p]$

```
> Tecna:=[x=m+t*u,y=n+t*v,z=p+t*w];
```

$$Tecna := [x = m + t\,u, y = n + t\,v, z = p + t\,w]$$

```
> Kv0:=simplify(eval(Kv,[x=m,y=n,z=p]));
```

$$Kv0 := a(1,1)\,m^2 + 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p + a(2,2)\,n^2 + 2\,a(2,3)\,n\,p + a(3,3)\,p^2 + 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p + a(4,4) = 0$$

```
> Kv1:=simplify(eval(Kv,Tecna));
```

$$Kv1 := 2\,a(1,1)\,m\,t\,u + 2\,a(1,2)\,m\,t\,v + 2\,a(1,2)\,t\,u\,n + 2\,a(1,2)\,t^2\,u\,v + 2\,a(1,3)\,m\,t\,w + 2\,a(1,3)\,t\,u\,p + 2\,a(1,3)\,t^2\,u\,w + 2\,a(2,2)\,n\,t\,v + 2\,a(2,3)\,n\,t\,w + 2\,a(2,3)\,t\,v\,p + 2\,a(2,3)\,t^2\,v\,w + 2\,a(3,3)\,p\,t\,w + a(1,1)\,m^2 + a(2,2)\,n^2 + a(3,3)\,p^2 + 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p + a(4,4) + a(1,1)\,t^2\,u^2 + a(2,2)\,t^2\,v^2 + a(3,3)\,t^2\,w^2 + 2\,a(1,4)\,t\,u + 2\,a(2,4)\,t\,v + 2\,a(3,4)\,t\,w + 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p + 2\,a(2,3)\,n\,p = 0$$

```
> A:=coeff(lhs(Kv1),t^2);
```

$$A := 2\,a(1,2)\,u\,v + 2\,a(1,3)\,u\,w + 2\,a(2,3)\,v\,w + a(1,1)\,u^2 + a(2,2)\,v^2 + a(3,3)\,w^2$$

```
> B:=1/2*coeff(lhs(Kv1),t);
```

$$B := a(1,1)\,m\,u + a(1,2)\,m\,v + a(1,2)\,u\,n + a(1,3)\,m\,w + a(1,3)\,u\,p + a(2,2)\,n\,v$$

$$+ a(2,3)\,n\,w + a(2,3)\,v\,p + a(3,3)\,p\,w + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
```

$$C := a(1,1)\,m^2 + 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p + a(2,2)\,n^2 + 2\,a(2,3)\,n\,p + a(3,3)\,p^2 + 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p + a(4,4)$$

```
> B1:=collect(B,[u,v,w]);
```

$$B1 := (a(1,1)\,m + a(1,3)\,p + a(1,4) + a(1,2)\,n)\,u + (a(1,2)\,m + a(2,2)\,n + a(2,4) + a(2,3)\,p)\,v + (a(3,3)\,p + a(2,3)\,n + a(1,3)\,m + a(3,4))\,w$$

```
> a1:=sort(coeff(B1,u),[m,n,p]); b1:=sort(coeff(B1,v),[m,n,p]);
  c1:=sort(coeff(B1,w),[m,n,p]);
  d1:=sort(coeff(coeff(coeff(B1,u,0),v,0),w,0),[m,n,p]);
```

$$a1 := a(1,1)\,m + a(1,2)\,n + a(1,3)\,p + a(1,4)$$

$$b1 := a(1,2)\,m + a(2,2)\,n + a(2,3)\,p + a(2,4)$$

$$c1 := a(1,3)\,m + a(2,3)\,n + a(3,3)\,p + a(3,4)$$

$$d1 := 0$$

```
> B2:=collect(B,[m,n,p]);
```

$$B2 := (a(1,1)\,u + a(1,2)\,v + a(1,3)\,w)\,m + (a(1,2)\,u + a(2,2)\,v + a(2,3)\,w)\,n + (a(2,3)\,v + a(3,3)\,w + a(1,3)\,u)\,p + a(3,4)\,w + a(1,4)\,u + a(2,4)\,v$$

```
> ra:=sort(coeff(B2,m),[u,v,w]); rb:=sort(coeff(B2,n),[u,v,w]);
  rc:=sort(coeff(B2,p),[u,v,w]);
  rd:=sort(coeff(coeff(coeff(B2,m,0),n,0),p,0),[u,v,w]);
```

$$ra := a(1,1)\,u + a(1,2)\,v + a(1,3)\,w$$

$$rb := a(1,2)\,u + a(2,2)\,v + a(2,3)\,w$$

$$rc := a(1,3)\,u + a(2,3)\,v + a(3,3)\,w$$

$$rd := a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> U:=Vector[row]([u,v,w,0]);
```

$$U := [u, v, w, 0]$$

```
> PrumerR:=collect(expand(evalm(U&*K&*Transpose(X))),[x,y,z])=0;
```

$$PrumerR := (a(1,1)\,u + a(1,2)\,v + a(1,3)\,w)\,x + (a(1,2)\,u + a(2,2)\,v + a(2,3)\,w)\,y + (a(1,3)\,u + a(2,3)\,v + a(3,3)\,w)\,z + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w = 0$$

```
> n:=[coeff(lhs(PrumerR),x),coeff(lhs(PrumerR),y),coeff(lhs(Prumer
  R),z)];
```

$$n := [a(1,1)\,u + a(1,2)\,v + a(1,3)\,w,\ a(1,2)\,u + a(2,2)\,v + a(2,3)\,w,\ a(1,3)\,u + a(2,3)\,v + a(3,3)\,w]$$

Příklad: Hlavní směry a roviny kvadriky $x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$.

```
> kv:=x^2+4*y^2+16*z^2-144=0;
```

$$kv := x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$$

```
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
  a(3,3):=coeff(lhs(kv),z^2);
  a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
```

```
a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
```

$$a(1,1) := 1$$
$$a(2,2) := 4$$
$$a(3,3) := 16$$
$$a(1,2) := 0$$
$$a(1,3) := 0$$
$$a(1,4) := 0$$
$$a(2,3) := 0$$
$$a(2,4) := 0$$
$$a(3,4) := 0$$
$$a(4,4) := -144$$

```
> K;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -144 \end{bmatrix}$$

```
> n;
```

$$[u, 4v, 16w]$$

```
> uv:=[u,v,w];
```

$$uv := [u, v, w]$$

```
> r:=evalm(n-lambda*uv);
```

$$r := [u - \lambda u, 4v - \lambda v, 16w - \lambda w]$$

```
> ChM:=linalg[genmatrix]([r[1],r[2],r[3]],[u,v,w]);
```

$$ChM := \begin{bmatrix} 1-\lambda & 0 & 0 \\ 0 & 4-\lambda & 0 \\ 0 & 0 & 16-\lambda \end{bmatrix}$$

```
> ChR:=linalg[det](ChM)=0;
```

$$ChR := (1-\lambda)(4-\lambda)(16-\lambda) = 0$$

```
> collect(ChR,[lambda^3,lambda^2,lambda]);
```

$$64 - \lambda^3 + 21\lambda^2 - 84\lambda = 0$$

```
> I1:=coeff(lhs(ChR),lambda,2);
```

$$I1 := 21$$

```
> I2:=coeff(lhs(ChR),lambda,1);
```

$$I2 := -84$$

```
> A44:=coeff(lhs(ChR),lambda,0);
```

$$A44 := 64$$

```
> Delta:=linalg[det](K);
```

$$\Delta := -9216$$

```
> lambda_sol:=solve(ChR,{lambda});
```

$$lambda\_sol := \{\lambda = 1\}, \{\lambda = 4\}, \{\lambda = 16\}$$

```
> r1:=eval(r,lambda_sol[1]); r2:=eval(r,lambda_sol[2]);
  r3:=eval(r,lambda_sol[3]);
```

$$r1 := [0, 3\,v, 15\,w]$$

$$r2 := [-3\,u, 0, 12\,w]$$

$$r3 := [-15\,u, -12\,v, 0]$$

```
> hs1:=eval(uv,solve({r1[1],r1[2],r1[3]},{u,v,w}));
  hs2:=eval(uv,solve({r2[1],r2[2],r2[3]},{u,v,w}));
  hs3:=eval(uv,solve({r3[1],r3[2],r3[3]},{u,v,w}));
```

$$hs1 := [u, 0, 0]$$

$$hs2 := [0, v, 0]$$

$$hs3 := [0, 0, w]$$

```
> PrumerR;
```

$$u\,x + 4\,v\,y + 16\,w\,z = 0$$

```
> Smer1:=eval(hs1,u=1);
```

$$Smer1 := [1, 0, 0]$$

```
> PrumerR1:=eval(PrumerR,[u=Smer1[1],v=Smer1[2],w=Smer1[3]]);
```

$$PrumerR1 := x = 0$$

```
> kv;
```

$$x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$$

```
> {kv,PrumerR1};
```

$$\{x = 0, x^2 + 4\,y^2 + 16\,z^2 - 144 = 0\}$$

```
> res1:=solve({kv,PrumerR1},{x,y,z});
```

$$res1 := \{x = 0, y = 2\,\mathrm{RootOf}(\_Z^2 + z^2 - 9, label = \_L1), z = z\}$$

```
> Kuz:=allvalues(res1);
```

$$Kuz := \{x = 0, y = 2\sqrt{9 - z^2}, z = z\}, \{x = 0, y = -2\sqrt{9 - z^2}, z = z\}$$

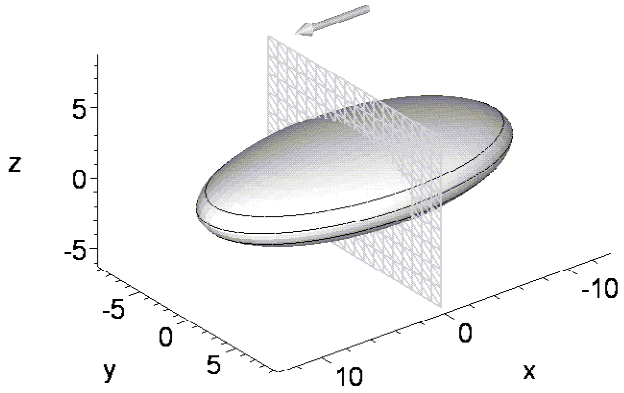
```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := [0, 2\sqrt{9 - z^2}, z]$$

$$Kuz2 := [0, -2\sqrt{9 - z^2}, z]$$

```
> col1:=COLOR(RGB,200/255,200/255,200/255):
> plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);
> kvg0:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,grid=[3
  0,30,30],style=patchcontour,color=red,lightmodel=light1,scaling=
  constrained):
> kvg:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,axes=fra
  me,color=COLOR(RGB,250/255,250/255,250/255),style=patchcontour,g
  rid=[30,30,40],light=[90,-5,1,1,1],tickmarks=[3,3,3],orientation
  =[52,63],scaling=constrained):
> PrumerR1g:=implicitplot3d(PrumerR1,x=-13..13,y=-9..9,z=-6..6,gri
  d=[10,10,10],thickness=2,style=wireframe,color=col1):
```

```
> ug1:=plots[arrow]([-5, -5, 10], vector(Smer1), width=[0.2,
  relative],length=5, head_length=[0.2, relative], color=green):
> Smer1g:=plottools[arrow]([-5, -5, 8], vector(6*Smer1), 0.5, 1,
  0.2, cylindrical_arrow,color=col1):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PrumerR1g,Smer1g,Kuz1g,Kuz2g,axes=frame,scaling=cons
  trained);
```




```
> Smer2:=eval(hs2,v=1);
```

$$Smer2 := [0, 1, 0]$$

```
> PrumerR2:=eval(PrumerR,[u=Smer2[1],v=Smer2[2],w=Smer2[3]]);
```

$$PrumerR2 := 4\,y = 0$$

```
> kv;
```

$$x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$$

```
> {kv,PrumerR2};
```

$$\{4\,y = 0,\ x^2 + 4\,y^2 + 16\,z^2 - 144 = 0\}$$

```
> res2:=solve({kv,PrumerR2},{x,y,z});
```

$$res2 := \{x = 4\ \mathrm{RootOf}(\_Z^2 + z^2 - 9,\ label = \_L3),\ y = 0,\ z = z\}$$

```
> Kuz:=allvalues(res2);
```

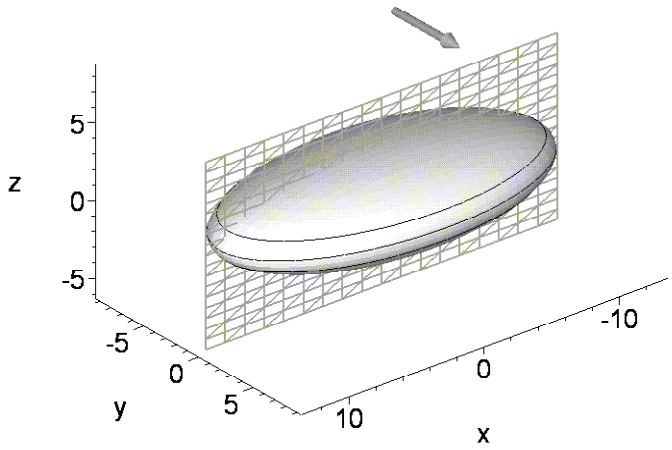
$$Kuz := \{x = 4\sqrt{9 - z^2},\ y = 0,\ z = z\},\ \{x = -4\sqrt{9 - z^2},\ y = 0,\ z = z\}$$

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := [4\sqrt{9 - z^2}, 0, z]$$

$$Kuz2 := [-4\sqrt{9 - z^2}, 0, z]$$

```
> col2:=COLOR(RGB,150/255,150/255,150/255):
> kvg0:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,grid=[3
  0,30,30],style=patchcontour,color=red,lightmodel=light1,scaling=
  constrained):
> kvg:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,axes=fra
  me,color=COLOR(RGB,250/255,250/255,250/255),style=patchcontour,g
  rid=[30,30,40],light=[90,-5,1,1,1],tickmarks=[3,3,3],orientation
  =[52,63],scaling=constrained):
> PrumerR2g:=implicitplot3d(PrumerR2,x=-13..13,y=-9..9,z=-6..6,gri
  d=[10,10,10],thickness=2,style=wireframe,gridstyle=rectangular,c
  olor=col2):
> Smer2g:=plottools[arrow]([-5, -5, 8], vector(6*Smer2), 0.5, 1,
  0.2, cylindrical_arrow,color=col2):
> ug2:=plots[arrow]([-5, -5, 10], vector(Smer2), width=[0.2,
  relative],length=5, head_length=[0.2, relative], color=blue):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PrumerR2g,Smer2g,Kuz1g,Kuz2g,axes=frame,scaling=cons
  trained);
```




```
> Smer3:=eval(hs3,w=1);
```

$$Smer3 := [0, 0, 1]$$

```
> PrumerR3:=eval(PrumerR,[u=Smer3[1],v=Smer3[2],w=Smer3[3]]);
```

$$PrumerR3 := 16\,z = 0$$

```
> kv;
```

$$x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$$

```
> {kv,PrumerR3};
```

$$\{16\,z = 0,\ x^2 + 4\,y^2 + 16\,z^2 - 144 = 0\}$$

```
> res3:=solve({kv,PrumerR3},{x,y,z});
```

$$res3 := \{x = 2\,\mathrm{RootOf}(\_Z^2 + y^2 - 36,\ label = \_L4),\ y = y,\ z = 0\}$$

```
> Kuz:=allvalues(res3);
```

$$Kuz := \{x = 2\sqrt{36 - y^2},\ y = y,\ z = 0\},\ \{x = -2\sqrt{36 - y^2},\ y = y,\ z = 0\}$$

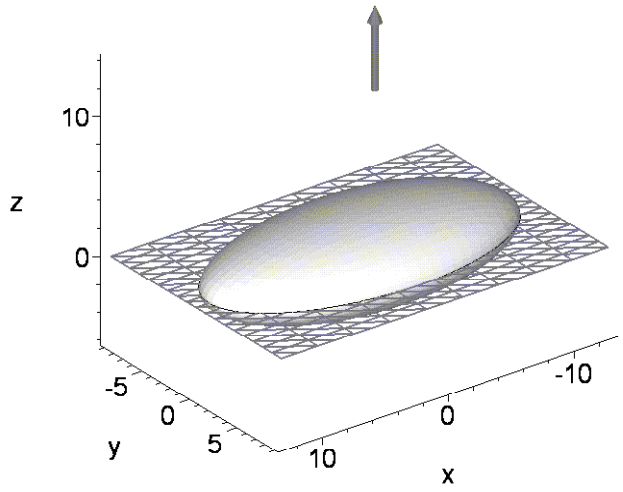
```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := [2\sqrt{36 - y^2},\ y,\ 0]$$

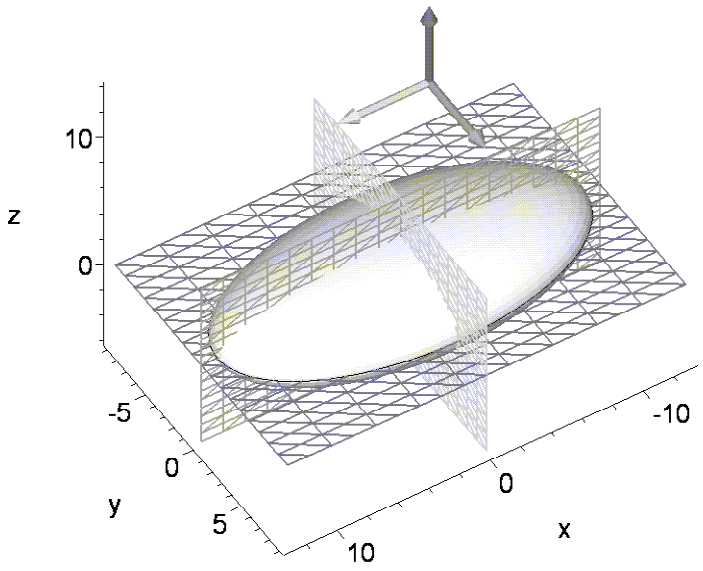
$$Kuz2 := [-2\sqrt{36 - y^2},\ y,\ 0]$$

```
> col3:=COLOR(RGB,100/255,100/255,100/255):
> kvg0:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,grid=[3
  0,30,30],style=patchcontour,color=red,lightmodel=light1,scaling=
  constrained):
> kvg:=plots[implicitplot3d](kv,x=-13..13,y=-9..9,z=-6..6,axes=fra
```

```
   me,color=COLOR(RGB,250/255,250/255,250/255),style=patchcontour,g
   rid=[30,30,40],light=[90,-5,1,1,1],tickmarks=[3,3,3],orientation
   =[52,63],scaling=constrained):
> PrumerR3g:=implicitplot3d(PrumerR3,x=-13..13,y=-9..9,z=-6..6,gri
   d=[10,10,10],thickness=2,gridstyle=rectangular,style=wireframe,c
   olor=col3):
> Smer3g:=plottools[arrow]([-5, -5, 8], vector(6*Smer3), 0.5, 1,
   0.2, cylindrical_arrow,color=col3):
> ug3:=plots[arrow]([-5, -5, 10], vector(Smer3), width=[0.2,
   relative],length=5, head_length=[0.2, relative], color=violet):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,colo
   r=yellow):
   Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,colo
   r=yellow):
> display(kvg,PrumerR3g,Smer3g,Kuz1g,Kuz2g,axes=frame,scaling=cons
   trained);
```




```
> display(kvg,PrumerR1g,Smer1g,PrumerR2g,Smer2g,PrumerR3g,Smer3g,K
   uz1g,Kuz2g,axes=frame,scaling=constrained,orientation=[58,44]);
```


z
10
0
-5
0
y
5
10
-10
0
x