1.13 Klasifikace kvadrik - řešené příklady

Příklad 1: Vyšetřete kvadriku $7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 22x + 24y + 2z + 30 = 0$.

```
> restart;
> with(LinearAlgebra):
> with(linalg):
> with(plots):
> X:=Vector[row]([x,y,z,1]);
```

$$X := [x, y, z, 1]$$

```
> K:=Matrix(a,1..4,1..4,shape=symmetric);
```

$$K := \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$$

```
> Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
```

$$Kv := a(1,1)x^2 + 2a(1,2)xy + 2a(1,3)xz + a(2,2)y^2 + 2a(2,3)yz + a(3,3)z^2 + 2a(1,4)x + 2a(2,4)y + 2a(3,4)z + a(4,4) = 0$$

```
> Tecna:=[x=m+t*u,y=n+t*v,z=p+t*w];
```

$$Tecna := [x = m + tu, y = n + tv, z = p + tw]$$

```
> Kv0:=simplify(eval(Kv,[x=m,y=n,z=p]));
```

$$Kv0 := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) = 0$$

```
> Kv1:=simplify(eval(Kv,Tecna));
```

$$Kv1 := 2a(1,2)mn + 2a(1,3)mp + 2a(2,3)np + a(1,1)m^2 + a(2,2)n^2 + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) + a(1,1)t^2u^2 + a(2,2)t^2v^2 + a(3,3)t^2w^2 + 2a(1,4)tu + 2a(2,4)tv + 2a(3,4)tw + 2a(1,1)mtu + 2a(1,2)mtv + 2a(1,2)tun + 2a(1,2)t^2uv + 2a(1,3)mtw + 2a(1,3)tup + 2a(1,3)t^2uw + 2a(2,2)ntv + 2a(2,3)ntw + 2a(2,3)tvp + 2a(2,3)t^2vw + 2a(3,3)ptw = 0$$

```
> A:=coeff(lhs(Kv1),t^2);
```

$$A := a(1,1)u^2 + a(2,2)v^2 + a(3,3)w^2 + 2a(1,2)uv + 2a(1,3)uw + 2a(2,3)vw$$

```
> B:=1/2*coeff(lhs(Kv1),t);
```

$$B := a(1,4)u + a(2,4)v + a(3,4)w + a(1,1)mu + a(1,2)mv + a(1,2)un + a(1,3)mw + a(1,3)up + a(2,2)nv + a(2,3)nw + a(2,3)vp + a(3,3)pw$$

```
> C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
```

$$C := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4)$$

```
> B1:=collect(B,[u,v,w]);
```

$$B1 := (a(1,1)m + a(1,4) + a(1,2)n + a(1,3)p)u + (a(1,2)m + a(2,2)n + a(2,4) + a(2,3)p)v + (a(2,3)n + a(3,4) + a(3,3)p + a(1,3)m)w$$

```
> a1:=sort(coeff(B1,u),[m,n,p]); b1:=sort(coeff(B1,v),[m,n,p]);
  c1:=sort(coeff(B1,w),[m,n,p]);
  d1:=sort(coeff(coeff(coeff(B1,u,0),v,0),w,0),[m,n,p]);
```

$$a1 := a(1,1)\,m + a(1,2)\,n + a(1,3)\,p + a(1,4)$$

$$b1 := a(1,2)\,m + a(2,2)\,n + a(2,3)\,p + a(2,4)$$

$$c1 := a(1,3)\,m + a(2,3)\,n + a(3,3)\,p + a(3,4)$$

$$d1 := 0$$

```
> B2:=collect(B,[m,n,p]);
```

$$B2 := (a(1,2)\,v + a(1,3)\,w + a(1,1)\,u)\,m + (a(2,2)\,v + a(1,2)\,u + a(2,3)\,w)\,n + (a(1,3)\,u + a(2,3)\,v + a(3,3)\,w)\,p + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> ra:=sort(coeff(B2,m),[u,v,w]); rb:=sort(coeff(B2,n),[u,v,w]);
  rc:=sort(coeff(B2,p),[u,v,w]);
  rd:=sort(coeff(coeff(coeff(B2,m,0),n,0),p,0),[u,v,w]);
```

$$ra := a(1,1)\,u + a(1,2)\,v + a(1,3)\,w$$

$$rb := a(1,2)\,u + a(2,2)\,v + a(2,3)\,w$$

$$rc := a(1,3)\,u + a(2,3)\,v + a(3,3)\,w$$

$$rd := a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> U:=Vector[row]([u,v,w,0]);
```

$$U := [u, v, w, 0]$$

```
> PrumerR:=collect(expand(evalm(U&*K&*Transpose(X))),[x,y,z])=0;
```

$$PrumerR := (a(1,1)\,u + a(1,2)\,v + a(1,3)\,w)\,x + (a(1,2)\,u + a(2,2)\,v + a(2,3)\,w)\,y + (a(1,3)\,u + a(2,3)\,v + a(3,3)\,w)\,z + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w = 0$$

```
> n:=[coeff(lhs(PrumerR),x),coeff(lhs(PrumerR),y),coeff(lhs(Prumer
  R),z)];
```

$$n := [a(1,1)\,u + a(1,2)\,v + a(1,3)\,w,\ a(1,2)\,u + a(2,2)\,v + a(2,3)\,w,\ a(1,3)\,u + a(2,3)\,v + a(3,3)\,w]$$

Definujeme hodnoty parametrů dle zadání:

```
> kv:=7*x^2+6*y^2+5*z^2-4*x*y-4*y*z-22*x+24*y+2*z+30=0;
```

$$kv := 7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 22x + 24y + 2z + 30 = 0$$

```
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
  a(3,3):=coeff(lhs(kv),z^2);
  a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
  a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
  a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
  a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
  a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
  a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
  a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
```

$$a(1,1) := 7$$

$$a(2,2) := 6$$

$$a(3,3) := 5$$

$$a(1,2) := -2$$

$$a(1,3) := 0$$

$$a(1,4) := -11$$

$$a(2,3) := -2$$

$$a(2,4) := 12$$

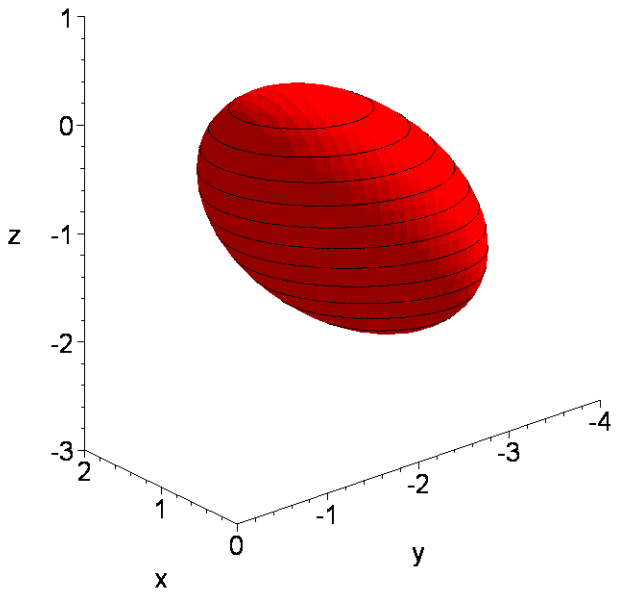
$$a(3,4) := 1$$

$$a(4,4) := 30$$

```
> K;
```

$$\begin{bmatrix} 7 & -2 & 0 & -11 \\ -2 & 6 & -2 & 12 \\ 0 & -2 & 5 & 1 \\ -11 & 12 & 1 & 30 \end{bmatrix}$$

```
> kvg:=plots[implicitplot3d](kv,x=-2..3,y=-4..0,z=-3..2,axes=frame
  ,color=red,style=patchcontour,grid=[40,40,40],tickmarks=[3,3,3],
  lightmodel=light1,orientation=[52,63],scaling=constrained):
> display(kvg,axes=frame,scaling=constrained,orientation=[140,66],
  view=[0..2,-4..0,-3..1]);
```




```
> n;
```

$$[7u - 2v, -2u + 6v - 2w, -2v + 5w]$$

```
> uv:=[u,v,w];
```

$$uv := [u, v, w]$$

```
> r:=evalm(n-lambda*uv);
```

$$r := [7\,u - 2\,v - \lambda\,u, -2\,u + 6\,v - 2\,w - \lambda\,v, -2\,v + 5\,w - \lambda\,w]$$

```
> ChM:=linalg[genmatrix]([r[1],r[2],r[3]],[u,v,w]);
```

$$ChM := \begin{bmatrix} 7-\lambda & -2 & 0 \\ -2 & 6-\lambda & -2 \\ 0 & -2 & 5-\lambda \end{bmatrix}$$

```
> ChR:=linalg[det](ChM)=0;
```

$$ChR := 162 - 99\,\lambda + 18\,\lambda^2 - \lambda^3 = 0$$

```
> collect(ChR,[lambda^3,lambda^2,lambda]);
```

$$162 - 99\,\lambda + 18\,\lambda^2 - \lambda^3 = 0$$

```
> I1:=coeff(lhs(ChR),lambda,2);
```

$$I1 := 18$$

```
> I2:=coeff(lhs(ChR),lambda,1);
```

$$I2 := -99$$

```
> A44:=coeff(lhs(ChR),lambda,0);
```

$$A44 := 162$$

```
> Delta:=linalg[det](K);
```

$$\Delta := -972$$

```
> lambda_sol:=solve(ChR,{lambda});
```

$$lambda\_sol := \{\lambda = 6\}, \{\lambda = 3\}, \{\lambda = 9\}$$

```
> r1:=eval(r,lambda_sol[1]); r2:=eval(r,lambda_sol[2]);
  r3:=eval(r,lambda_sol[3]);
```

$$r1 := [u - 2\,v, -2\,u - 2\,w, -2\,v - w]$$

$$r2 := [4\,u - 2\,v, -2\,u + 3\,v - 2\,w, -2\,v + 2\,w]$$

$$r3 := [-2\,u - 2\,v, -2\,u - 3\,v - 2\,w, -2\,v - 4\,w]$$

```
> hs1:=eval(uv,solve({r1[1],r1[2],r1[3]},{u,v,w}));
  hs2:=eval(uv,solve({r2[1],r2[2],r2[3]},{u,v,w}));
  hs3:=eval(uv,solve({r3[1],r3[2],r3[3]},{u,v,w}));
```

$$hs1 := [2\,v, v, -2\,v]$$

$$hs2 := [u, 2\,u, 2\,u]$$

$$hs3 := [2\,w, -2\,w, w]$$

```
> PrumerR;
```

$$(7\,u - 2\,v)\,x + (-2\,u + 6\,v - 2\,w)\,y + (-2\,v + 5\,w)\,z - 11\,u + 12\,v + w = 0$$

```
> Smer1:=eval(hs1,{u=1,v=1,w=1});
```

$$Smer1 := [2, 1, -2]$$

```
> PrumerR1:=eval(PrumerR,[u=Smer1[1],v=Smer1[2],w=Smer1[3]]);
```

$$PrumerR1 := 12\,x + 6\,y - 12\,z - 12 = 0$$

```
> kv;
```

$$7\,x^2 + 6\,y^2 + 5\,z^2 - 4\,x\,y - 4\,y\,z - 22\,x + 24\,y + 2\,z + 30 = 0$$

```
> {kv,PrumerR1};
```

$$\{12x+6y-12z-12=0,\ 7x^2+6y^2+5z^2-4xy-4yz-22x+24y+2z+30=0\}$$

```
> res1:=solve({kv,PrumerR1},{x,y,z});
```

$$res1 := \{x=-\frac{1}{2}\,\mathrm{RootOf}(13\,\_Z^2+(-20z+32)\,\_Z+16z^2-8z+20)+z+1,$$

$$y=\mathrm{RootOf}(13\,\_Z^2+(-20z+32)\,\_Z+16z^2-8z+20),\ z=z\}$$

```
> Kuz:=allvalues(res1);
```

$$Kuz := \{x=\frac{8z}{13}+\frac{21}{13}-\frac{\sqrt{-27z^2-54z-1}}{13},\ y=\frac{10z}{13}-\frac{16}{13}+\frac{2\sqrt{-27z^2-54z-1}}{13},\ z=z\},$$

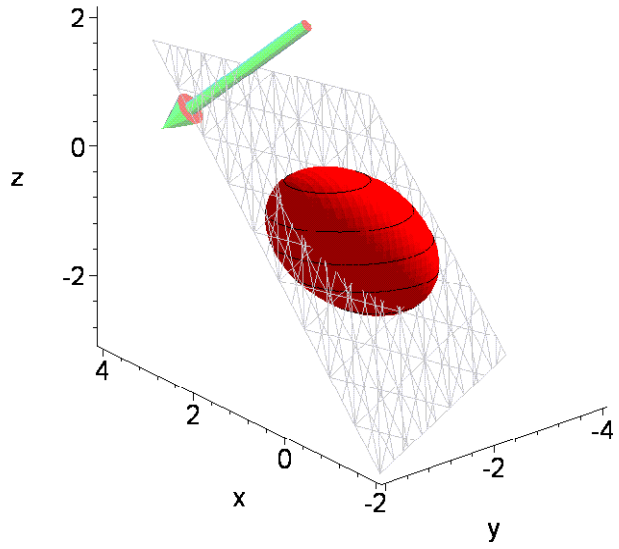
$$\{x=\frac{8z}{13}+\frac{21}{13}+\frac{\sqrt{-27z^2-54z-1}}{13},\ y=\frac{10z}{13}-\frac{16}{13}-\frac{2\sqrt{-27z^2-54z-1}}{13},\ z=z\}$$

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := \left[\frac{8z}{13}+\frac{21}{13}-\frac{\sqrt{-27z^2-54z-1}}{13},\ \frac{10z}{13}-\frac{16}{13}+\frac{2\sqrt{-27z^2-54z-1}}{13},\ z\right]$$

$$Kuz2 := \left[\frac{8z}{13}+\frac{21}{13}+\frac{\sqrt{-27z^2-54z-1}}{13},\ \frac{10z}{13}-\frac{16}{13}-\frac{2\sqrt{-27z^2-54z-1}}{13},\ z\right]$$

```
> col1:=grey:
> PrumerR1g:=implicitplot3d(PrumerR1,x=-2..3,y=-4..0,z=-3..2,grid=
  [10,10,10],style=wireframe,color=col1):
> ug1:=plots[arrow]([-5, -5, 10], vector(Smer1), width=[0.2,
  relative],length=5, head_length=[0.2, relative], color=green):
> Smer1g:=plottools[arrow]([2, -2, 2], vector(Smer1), 0.2, 0.5,
  0.2, cylindrical_arrow,color=col1):
> Kuz1g:=spacecurve(Kuz1,z=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,z=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PrumerR1g,Kuz1g,Kuz2g,Smer1g,axes=frame,scaling=cons
  trained,orientation=[140,66]);
```




```
> Smer2:=eval(hs2,{u=1,v=1,w=1});
```

$$Smer2 := [1, 2, 2]$$

```
> PrumerR2:=eval(PrumerR,[u=Smer2[1],v=Smer2[2],w=Smer2[3]]);
```

$$PrumerR2 := 3x + 6y + 6z + 15 = 0$$

```
> kv;
```

$$7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 22x + 24y + 2z + 30 = 0$$

```
> {kv,PrumerR2};
```

$$\{3x + 6y + 6z + 15 = 0,\ 7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 22x + 24y + 2z + 30 = 0\}$$

```
> res2:=solve({kv,PrumerR2},{x,y,z});
```

$$res2 := \{x = -2\,\mathrm{RootOf}(14\_Z^2 + (20z + 76)\_Z + 11z^2 + 62z + 105) - 2z - 5,$$
$$y = \mathrm{RootOf}(14\_Z^2 + (20z + 76)\_Z + 11z^2 + 62z + 105), z = z\}$$

```
> Kuz:=allvalues(res2);
```

$$Kuz :=$$

$$\{x = -\frac{4z}{7} + \frac{3}{7} - \frac{\sqrt{-54z^2 - 108z - 26}}{7}, y = -\frac{5z}{7} - \frac{19}{7} + \frac{\sqrt{-54z^2 - 108z - 26}}{14}, z = z\},$$

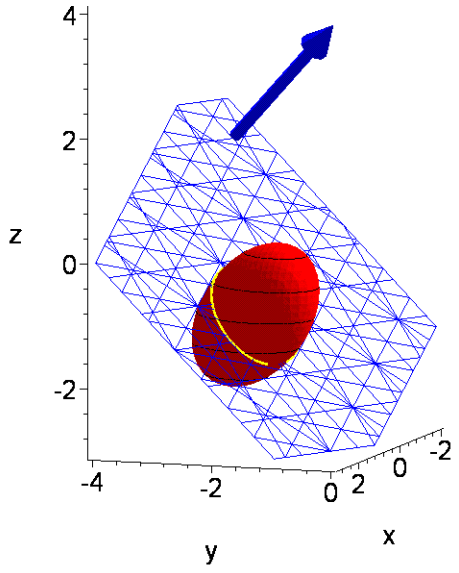
$$\{x = -\frac{4z}{7} + \frac{3}{7} + \frac{\sqrt{-54z^2 - 108z - 26}}{7}, y = -\frac{5z}{7} - \frac{19}{7} - \frac{\sqrt{-54z^2 - 108z - 26}}{14}, z = z\}$$

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := \left[-\frac{4z}{7} + \frac{3}{7} - \frac{\sqrt{-54z^2 - 108z - 26}}{7}, -\frac{5z}{7} - \frac{19}{7} + \frac{\sqrt{-54z^2 - 108z - 26}}{14}, z\right]$$

$$Kuz2 := \left[-\frac{4z}{7} + \frac{3}{7} + \frac{\sqrt{-54z^2 - 108z - 26}}{7}, -\frac{5z}{7} - \frac{19}{7} - \frac{\sqrt{-54z^2 - 108z - 26}}{14}, z \right]$$

```
> col2:=blue:
> PrumerR2g:=implicitplot3d(PrumerR2,x=-2..3,y=-4..0,z=-3..2,grid=
  [10,10,10],style=wireframe,color=col2):
> Smer2g:=plottools[arrow]([2, -2, 2], vector(Smer2), 0.2, 0.5,
  0.2, cylindrical_arrow,color=col2):
> ug2:=plots[arrow]([-5, -5, 10], vector(Smer2), width=[0.2,
  relative],length=5, head_length=[0.2, relative], color=blue):
> Kuz1g:=spacecurve(Kuz1,z=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,z=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PrumerR2g,Kuz1g,Kuz2g,Smer2g,axes=frame,scaling=cons
  trained);
```




```
> Smer3:=eval(hs3,{u=1,v=1,w=1});
```

$$Smer3 := [2, -2, 1]$$

```
> PrumerR3:=eval(PrumerR,[u=Smer3[1],v=Smer3[2],w=Smer3[3]]);
```

$$PrumerR3 := 18x - 18y + 9z - 45 = 0$$

```
> kv;
```

$$7x^2 + 6y^2 + 5z^2 - 4xy - 4yz - 22x + 24y + 2z + 30 = 0$$

```
> {kv,PrumerR3};
```

$$\{18x-18y+9z-45=0,\ 7x^2+6y^2+5z^2-4xy-4yz-22x+24y+2z+30=0\}$$

```
> res3:=solve({kv,PrumerR3},{x,y,z});
```

$$res3 := \{x = \text{RootOf}(9\_Z^2 + (-12y-42)\_Z + 6y^2 + 36y + 55),\ y = y,$$
$$z = -2\,\text{RootOf}(9\_Z^2 + (-12y-42)\_Z + 6y^2 + 36y + 55) + 2y + 5\}$$

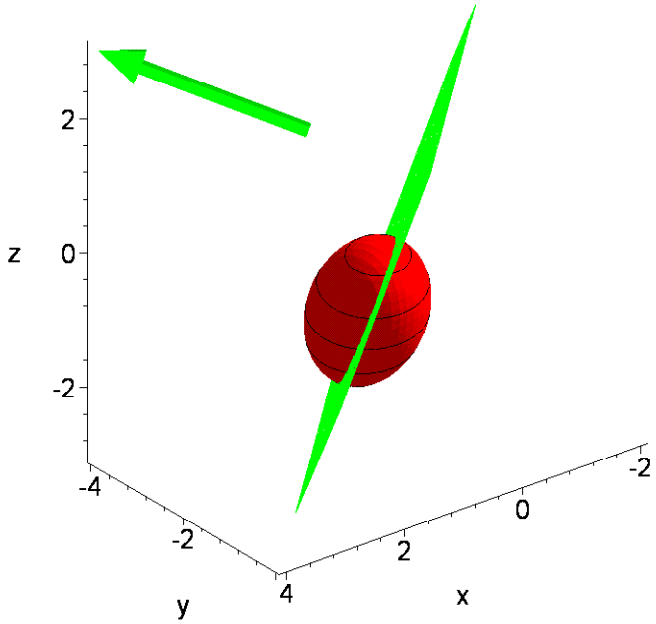
```
> Kuz:=allvalues(res3);
```

$$Kuz := \left\{x = \frac{2y}{3}+\frac{7}{3}+\frac{\sqrt{-2y^2-8y-6}}{3},\ y = y,\ z = \frac{2y}{3}+\frac{1}{3}-\frac{2\sqrt{-2y^2-8y-6}}{3}\right\},$$
$$\left\{x = \frac{2y}{3}+\frac{7}{3}-\frac{\sqrt{-2y^2-8y-6}}{3},\ y = y,\ z = \frac{2y}{3}+\frac{1}{3}+\frac{2\sqrt{-2y^2-8y-6}}{3}\right\}$$

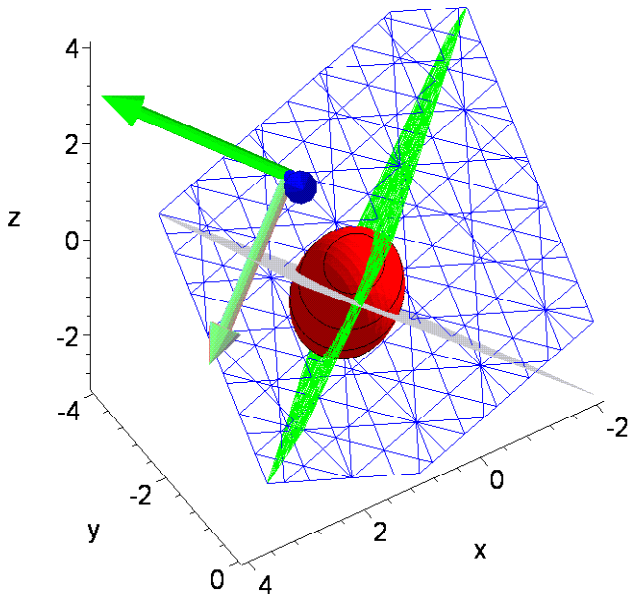
```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := \left[\frac{2y}{3}+\frac{7}{3}+\frac{\sqrt{-2y^2-8y-6}}{3},\ y,\ \frac{2y}{3}+\frac{1}{3}-\frac{2\sqrt{-2y^2-8y-6}}{3}\right]$$
$$Kuz2 := \left[\frac{2y}{3}+\frac{7}{3}-\frac{\sqrt{-2y^2-8y-6}}{3},\ y,\ \frac{2y}{3}+\frac{1}{3}+\frac{2\sqrt{-2y^2-8y-6}}{3}\right]$$

```
> col3:=green:
> PrumerR3g:=implicitplot3d(PrumerR3,x=-2..3,y=-4..0,z=-3..2,grid=
  [10,10,10],style=wireframe,color=col3):
> Smer3g:=plottools[arrow]([2, -2, 2], vector(Smer3), 0.2, 0.5,
  0.2, cylindrical_arrow,color=col3):
> ug3:=plots[arrow]([-5, -5, 10], vector(Smer3), width=[0.2,
  relative],length=5, head_length=[0.2, relative], color=violet):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PrumerR3g,Kuz1g,Kuz2g,Smer3g,axes=frame,scaling=cons
  trained);
```




```
> display(kvg,PrumerR1g,PrumerR2g,PrumerR3g,Kuz1g,Kuz2g,Smer1g,Sme
  r2g,Smer3g,axes=frame,scaling=constrained,orientation=[58,44]);
```


4
2
z
0
-2
-4
-2
y
0
4
2
x
0
-2

>