1.13 Klasifikace kvadrik - řešené příklady

Příklad 2: Vyšetřete kvadriku $5x^2 - y^2 + z^2 + 4xy + 6xz + 2x + 4y + 6z - 8 = 0$.

```
> restart;
> with(LinearAlgebra):
> with(linalg):
> with(plots):
> X:=Vector[row]([x,y,z,1]);
```

$$X := [x, y, z, 1]$$

```
> K:=Matrix(a,1..4,1..4,shape=symmetric);
```

$$K := \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$$

```
> Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
```

$$Kv := a(1,1)x^2 + 2a(1,2)xy + 2a(1,3)xz + a(2,2)y^2 + 2a(2,3)yz + a(3,3)z^2 + 2a(1,4)x + 2a(2,4)y + 2a(3,4)z + a(4,4) = 0$$

```
> Tecna:=[x=m+t*u,y=n+t*v,z=p+t*w];
```

$$Tecna := [x = m + tu, y = n + tv, z = p + tw]$$

```
> Kv0:=simplify(eval(Kv,[x=m,y=n,z=p]));
```

$$Kv0 := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) = 0$$

```
> Kv1:=simplify(eval(Kv,Tecna));
```

$$Kv1 := 2a(1,2)mn + 2a(1,3)mp + 2a(2,3)np + a(1,1)m^2 + a(2,2)n^2 + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) + a(1,1)t^2u^2 + a(2,2)t^2v^2 + a(3,3)t^2w^2 + 2a(1,4)tu + 2a(2,4)tv + 2a(3,4)tw + 2a(1,1)mtu + 2a(1,2)mtv + 2a(1,2)tun + 2a(1,2)t^2uv + 2a(1,3)mtw + 2a(1,3)tup + 2a(1,3)t^2uw + 2a(2,2)ntv + 2a(2,3)ntw + 2a(2,3)tvp + 2a(2,3)t^2vw + 2a(3,3)ptw = 0$$

```
> A:=coeff(lhs(Kv1),t^2);
```

$$A := a(1,1)u^2 + a(2,2)v^2 + a(3,3)w^2 + 2a(1,2)uv + 2a(1,3)uw + 2a(2,3)vw$$

```
> B:=1/2*coeff(lhs(Kv1),t);
```

$$B := a(1,4)u + a(2,4)v + a(3,4)w + a(1,1)mu + a(1,2)mv + a(1,2)un + a(1,3)mw + a(1,3)up + a(2,2)nv + a(2,3)nw + a(2,3)vp + a(3,3)pw$$

```
> C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
```

$$C := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4)$$

```
> B1:=collect(B,[u,v,w]);
```

$$B1 := (a(1,1)m + a(1,4) + a(1,2)n + a(1,3)p)u + (a(1,2)m + a(2,2)n + a(2,4) + a(2,3)p)v + (a(2,3)n + a(3,4) + a(3,3)p + a(1,3)m)w$$

```
> a1:=sort(coeff(B1,u),[m,n,p]); b1:=sort(coeff(B1,v),[m,n,p]);
  c1:=sort(coeff(B1,w),[m,n,p]);
  d1:=sort(coeff(coeff(coeff(B1,u,0),v,0),w,0),[m,n,p]);
```

$$a1 := a(1,1)\,m + a(1,2)\,n + a(1,3)\,p + a(1,4)$$

$$b1 := a(1,2)\,m + a(2,2)\,n + a(2,3)\,p + a(2,4)$$

$$c1 := a(1,3)\,m + a(2,3)\,n + a(3,3)\,p + a(3,4)$$

$$d1 := 0$$

```
> B2:=collect(B,[m,n,p]);
```

$$B2 := (a(1,2)\,v + a(1,3)\,w + a(1,1)\,u)\,m + (a(2,2)\,v + a(1,2)\,u + a(2,3)\,w)\,n + (a(1,3)\,u + a(2,3)\,v + a(3,3)\,w)\,p + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> ra:=sort(coeff(B2,m),[u,v,w]); rb:=sort(coeff(B2,n),[u,v,w]);
  rc:=sort(coeff(B2,p),[u,v,w]);
  rd:=sort(coeff(coeff(coeff(B2,m,0),n,0),p,0),[u,v,w]);
```

$$ra := a(1,1)\,u + a(1,2)\,v + a(1,3)\,w$$

$$rb := a(1,2)\,u + a(2,2)\,v + a(2,3)\,w$$

$$rc := a(1,3)\,u + a(2,3)\,v + a(3,3)\,w$$

$$rd := a(1,4)\,u + a(2,4)\,v + a(3,4)\,w$$

```
> U:=Vector[row]([u,v,w,0]);
```

$$U := [u, v, w, 0]$$

```
> PrumerR:=collect(expand(evalm(U&*K&*Transpose(X))),[x,y,z])=0;
```

$$PrumerR := (a(1,1)\,u + a(1,2)\,v + a(1,3)\,w)\,x + (a(1,2)\,u + a(2,2)\,v + a(2,3)\,w)\,y + (a(1,3)\,u + a(2,3)\,v + a(3,3)\,w)\,z + a(1,4)\,u + a(2,4)\,v + a(3,4)\,w = 0$$

```
> n:=[coeff(lhs(PrumerR),x),coeff(lhs(PrumerR),y),coeff(lhs(Prumer
  R),z)];
```

$$n := [a(1,1)\,u + a(1,2)\,v + a(1,3)\,w,\ a(1,2)\,u + a(2,2)\,v + a(2,3)\,w,\ a(1,3)\,u + a(2,3)\,v + a(3,3)\,w]$$

Definujeme hodnoty parametrů dle zadání:

```
> kv:=5*x^2-y^2+z^2+4*x*y+6*x*z+2*x+4*y+6*z-8=0;
```

$$kv := 5x^2 - y^2 + z^2 + 4xy + 6xz + 2x + 4y + 6z - 8 = 0$$

```
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
  a(3,3):=coeff(lhs(kv),z^2);
  a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
  a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
  a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
  a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
  a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
  a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
  a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
```

$$a(1,1) := 5$$

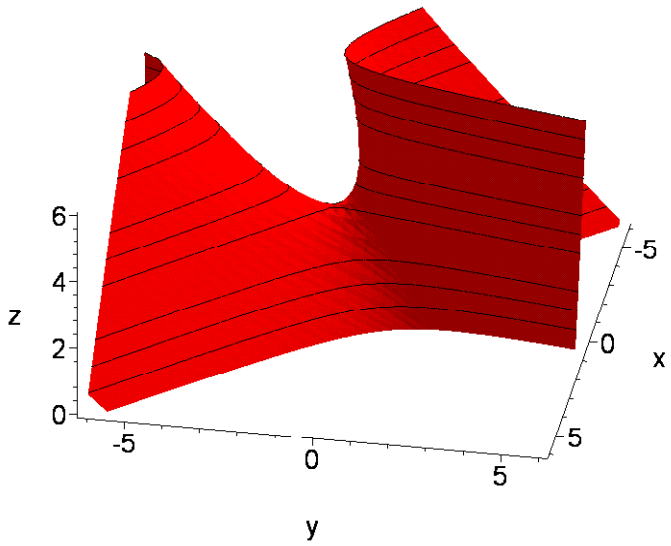
$$a(2,2) := -1$$

$$a(3,3) := 1$$
$$a(1,2) := 2$$
$$a(1,3) := 3$$
$$a(1,4) := 1$$
$$a(2,3) := 0$$
$$a(2,4) := 2$$
$$a(3,4) := 3$$
$$a(4,4) := -8$$

```
> K;
```

$$\begin{bmatrix} 5 & 2 & 3 & 1 \\ 2 & -1 & 0 & 2 \\ 3 & 0 & 1 & 3 \\ 1 & 2 & 3 & -8 \end{bmatrix}$$

```
> kvg:=plots[implicitplot3d](kv,x=-6..6,y=-6..6,z=0..6,axes=frame,
  color=red,style=patchcontour,grid=[40,40,40],lightmodel=light1,t
  ickmarks=[3,3,3],orientation=[52,63],scaling=constrained):
> display(kvg,axes=frame,scaling=constrained,orientation=[10,60],c
  ontours=11);
```




```
> n;
```

$$[5u + 2v + 3w, 2u - v, 3u + w]$$

```
> uv:=[u,v,w];
```

$$uv := [u, v, w]$$

```
> r:=evalm(n-lambda*uv);
```

$$r := [5\,u + 2\,v + 3\,w - \lambda\,u,\ 2\,u - v - \lambda\,v,\ 3\,u + w - \lambda\,w]$$

```
> ChM:=linalg[genmatrix]([r[1],r[2],r[3]],[u,v,w]);
```

$$ChM := \begin{bmatrix} 5-\lambda & 2 & 3 \\ 2 & -1-\lambda & 0 \\ 3 & 0 & 1-\lambda \end{bmatrix}$$

```
> ChR:=linalg[det](ChM)=0;
```

$$ChR := 14\,\lambda + 5\,\lambda^2 - \lambda^3 = 0$$

```
> collect(ChR,[lambda^3,lambda^2,lambda]);
```

$$14\,\lambda + 5\,\lambda^2 - \lambda^3 = 0$$

```
> I1:=coeff(lhs(ChR),lambda,2);
```

$$I1 := 5$$

```
> I2:=coeff(lhs(ChR),lambda,1);
```

$$I2 := 14$$

```
> A44:=coeff(lhs(ChR),lambda,0);
```

$$A44 := 0$$

```
> Delta:=linalg[det](K);
```

$$\Delta := 16$$

```
> lambda_sol:=solve(ChR,{lambda});
```

$$lambda\_sol := \{\lambda = 0\}, \{\lambda = 7\}, \{\lambda = -2\}$$

```
> r1:=eval(r,lambda_sol[1]); r2:=eval(r,lambda_sol[2]);
  r3:=eval(r,lambda_sol[3]);
```

$$r1 := [5\,u + 2\,v + 3\,w,\ 2\,u - v,\ 3\,u + w]$$

$$r2 := [-2\,u + 2\,v + 3\,w,\ 2\,u - 8\,v,\ 3\,u - 6\,w]$$

$$r3 := [7\,u + 2\,v + 3\,w,\ 2\,u + v,\ 3\,u + 3\,w]$$

```
> hs1:=eval(uv,solve({r1[1],r1[2],r1[3]},{u,v,w}));
  hs2:=eval(uv,solve({r2[1],r2[2],r2[3]},{u,v,w}));
  hs3:=eval(uv,solve({r3[1],r3[2],r3[3]},{u,v,w}));
```

$$hs1 := [u,\ 2\,u,\ -3\,u]$$

$$hs2 := [4\,v,\ v,\ 2\,v]$$

$$hs3 := [-w,\ 2\,w,\ w]$$

```
> PrumerR;
```

$$(5\,u + 2\,v + 3\,w)\,x + (2\,u - v)\,y + (3\,u + w)\,z + u + 2\,v + 3\,w = 0$$

```
> Smer1:=eval(hs1,{u=1,v=1,w=1});
```

$$Smer1 := [1, 2, -3]$$

```
> PrumerR1:=eval(PrumerR,[u=Smer1[1],v=Smer1[2],w=Smer1[3]]);
```

$$PrumerR1 := -4 = 0$$

```
> kv;
```

$$5\,x^2 - y^2 + z^2 + 4\,x\,y + 6\,x\,z + 2\,x + 4\,y + 6\,z - 8 = 0$$

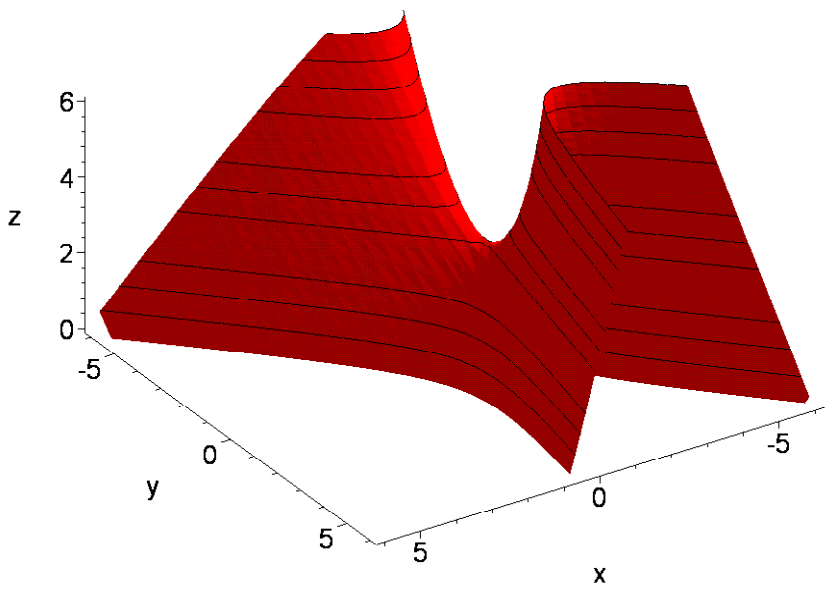
```
> {kv,PrumerR1};
```

$$\{-4 = 0,\ 5\,x^2 - y^2 + z^2 + 4\,x\,y + 6\,x\,z + 2\,x + 4\,y + 6\,z - 8 = 0\}$$

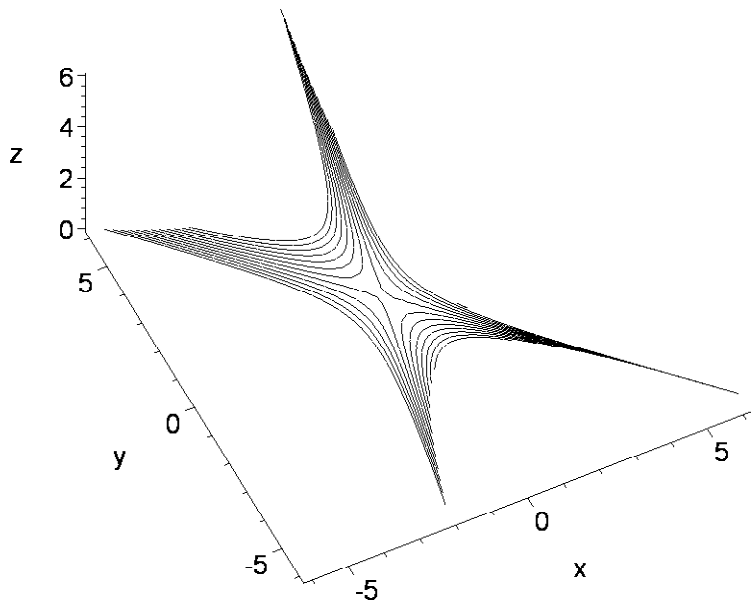
```
> res1:=solve({kv,PrumerR1},{x,y,z});
```

$$res1 :=$$

```
> display(kvg,axes=frame,scaling=constrained,orientation=[53,62],c
  ontours=11);
```




```
> kvg:=plots[implicitplot3d](kv,x=-6..6,y=-6..6,z=0..6,axes=frame,
  color=black,style=contour,grid=[40,40,40],light=[90,-5,1,1,1],ti
  ckmarks=[3,3,3],orientation=[52,63],scaling=constrained,numpoint
  s=100000):
> display(kvg,axes=frame,scaling=constrained,orientation=[-115,39]
  ,light=[230,200,0,0,0]);
```


z
6
4
2
0
5
0
y
-5
-5
0
5
x