

1.13 Klasifikace kvadrik - řešené příklady

Příklad 3: Vyšetřete kvadriku

$$3x^2 + 3y^2 + 3z^2 + 4\sqrt{2}xy + 2yz + 6x + 2y(2\sqrt{2} - 1) - 6z - 9 = 0$$

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[ > restart;
[ > with(LinearAlgebra):
[ > with(linalg):
[ > with(plots):
[ > X:=Vector[row]([x,y,z,1]);
                                X := [x, y, z, 1]
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                K :=
                                [ a(1,1)  a(1,2)  a(1,3)  a(1,4)
                                [ a(1,2)  a(2,2)  a(2,3)  a(2,4)
                                [ a(1,3)  a(2,3)  a(3,3)  a(3,4)
                                [ a(1,4)  a(2,4)  a(3,4)  a(4,4) ]
[ > Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
Kv := a(1,1)x^2 + 2a(1,2)xy + 2a(1,3)xz + a(2,2)y^2 + 2a(2,3)yz + a(3,3)z^2
      + 2a(1,4)x + 2a(2,4)y + 2a(3,4)z + a(4,4) = 0
[ > Tecna := [x=m+t*u, y=n+t*v, z=p+t*w];
                                Tecna := [x = m + t u, y = n + t v, z = p + t w]
[ > Kv0:=simplify(eval(Kv,[x=m,y=n,z=p]));
Kv0 := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2
      + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) = 0
[ > Kv1:=simplify(eval(Kv,Tecna));
Kv1 := 2a(2,3)np + 2a(1,1)m t u + 2a(1,2)m t v + 2a(1,2)t u n + 2a(1,2)t^2 u v
      + 2a(1,3)m t w + 2a(1,3)t u p + 2a(1,3)t^2 u w + 2a(2,2)n t v + 2a(2,3)n t w
      + 2a(2,3)t v p + 2a(2,3)t^2 v w + 2a(3,3)p t w + a(1,1)t^2 u^2 + a(2,2)t^2 v^2
      + a(3,3)t^2 w^2 + 2a(1,4)t u + 2a(2,4)t v + 2a(3,4)t w + 2a(1,2)m n + 2a(1,3)m p
      + a(1,1)m^2 + a(2,2)n^2 + a(3,3)p^2 + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4) = 0
[ > A:=coeff(lhs(Kv1),t^2);
      A := 2a(1,2)u v + 2a(1,3)u w + 2a(2,3)v w + a(1,1)u^2 + a(2,2)v^2 + a(3,3)w^2
[ > B:=1/2*coeff(lhs(Kv1),t);
      B := a(1,1)m u + a(1,2)m v + a(1,2)u n + a(1,3)m w + a(1,3)u p + a(2,2)n v
      + a(2,3)n w + a(2,3)v p + a(3,3)p w + a(1,4)u + a(2,4)v + a(3,4)w
[ > C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
      C := a(1,1)m^2 + 2a(1,2)mn + 2a(1,3)mp + a(2,2)n^2 + 2a(2,3)np + a(3,3)p^2
      + 2a(1,4)m + 2a(2,4)n + 2a(3,4)p + a(4,4)
[ > B1:=collect(B,[u,v,w]);
      B1 := (a(1,1)m + a(1,3)p + a(1,4) + a(1,2)n)u
      + (a(1,2)m + a(2,2)n + a(2,4) + a(2,3)p)v
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+ (a(3, 3) p + a(2, 3) n + a(1, 3) m + a(3, 4)) w
> a1:=sort(coeff(B1,u),[m,n,p]); b1:=sort(coeff(B1,v),[m,n,p]);
c1:=sort(coeff(B1,w),[m,n,p]);
d1:=sort(coeff(coeff(coeff(B1,u,0),v,0),w,0),[m,n,p]);
      a1 := a(1, 1) m + a(1, 2) n + a(1, 3) p + a(1, 4)
      b1 := a(1, 2) m + a(2, 2) n + a(2, 3) p + a(2, 4)
      c1 := a(1, 3) m + a(2, 3) n + a(3, 3) p + a(3, 4)
      d1 := 0
> B2:=collect(B,[m,n,p]);
B2 := (a(1, 1) u + a(1, 2) v + a(1, 3) w) m + (a(1, 2) u + a(2, 2) v + a(2, 3) w) n
      + (a(2, 3) v + a(3, 3) w + a(1, 3) u) p + a(3, 4) w + a(1, 4) u + a(2, 4) v
> ra:=sort(coeff(B2,m),[u,v,w]); rb:=sort(coeff(B2,n),[u,v,w]);
rc:=sort(coeff(B2,p),[u,v,w]);
rd:=sort(coeff(coeff(coeff(B2,m,0),n,0),p,0),[u,v,w]);
      ra := a(1, 1) u + a(1, 2) v + a(1, 3) w
      rb := a(1, 2) u + a(2, 2) v + a(2, 3) w
      rc := a(1, 3) u + a(2, 3) v + a(3, 3) w
      rd := a(1, 4) u + a(2, 4) v + a(3, 4) w
> U:=Vector[row]([u,v,w,0]);
      U := [u, v, w, 0]
> PrumerR:=collect(expand(evalm(U*K*Transpose(X))),[x,y,z])=0;
PrumerR := (a(1, 1) u + a(1, 2) v + a(1, 3) w) x + (a(1, 2) u + a(2, 2) v + a(2, 3) w) y
      + (a(1, 3) u + a(2, 3) v + a(3, 3) w) z + a(1, 4) u + a(2, 4) v + a(3, 4) w = 0
> n:=[coeff(lhs(PrumerR),x),coeff(lhs(PrumerR),y),coeff(lhs(Prumer
R),z)];
n := [a(1, 1) u + a(1, 2) v + a(1, 3) w, a(1, 2) u + a(2, 2) v + a(2, 3) w,
      a(1, 3) u + a(2, 3) v + a(3, 3) w]

```

Dosadíme hodnoty parametrů dle zadání:

```

> kv:=3*x^2+3*y^2+3*z^2+4*sqrt(2)*x*y+2*y*z+6*x+2*y*(2*sqrt(2)-1)-
6*z-9=0;
      kv := 3 x2 + 3 y2 + 3 z2 + 4 √2 x y + 2 y z + 6 x + 2 y (2 √2 - 1) - 6 z - 9 = 0
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
a(3,3):=coeff(lhs(kv),z^2);
a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
      a(1, 1) := 3

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a(2, 2) := 3
a(3, 3) := 3
a(1, 2) := 2*sqrt(2)
a(1, 3) := 0
a(1, 4) := 3
a(2, 3) := 1
a(2, 4) := 2*sqrt(2) - 1
a(3, 4) := -3
a(4, 4) := -9

```

```
> K;
```

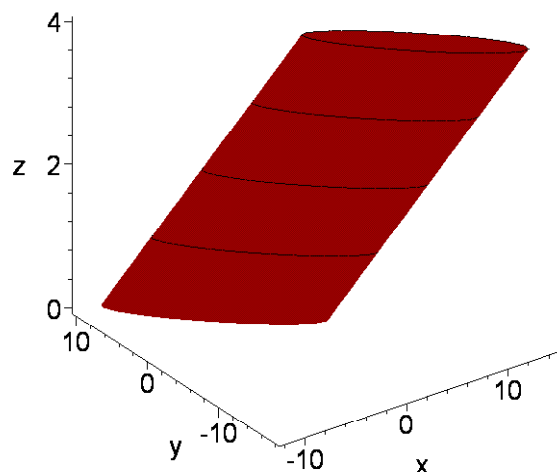
$$\begin{bmatrix} 3 & 2\sqrt{2} & 0 & 3 \\ 2\sqrt{2} & 3 & 1 & 2\sqrt{2}-1 \\ 0 & 1 & 3 & -3 \\ 3 & 2\sqrt{2}-1 & -3 & -9 \end{bmatrix}$$

```
> kv;
```

$$3x^2 + 3y^2 + 3z^2 + 4\sqrt{2}xy + 2yz + 6x + 2y(2\sqrt{2}-1) - 6z - 9 = 0$$

```
> kvg:=plots[implicitplot3d](kv,x=-12..15,y=-18..10,z=0..4,axes=frame,color=red,style=patchcontour,grid=[40,40,10],lightmodel=light1,tickmarks=[3,3,3],orientation=[52,63],scaling=constrained,contours=5):
```

```
> display(kvg,axes=frame,scaling=unconstrained,orientation=[-126,60]);
```



```
> n;
```

$$[3u + 2\sqrt{2}v, 2\sqrt{2}u + 3v + w, v + 3w]$$

```
> uv:=[u,v,w];
```

$$uv := [u, v, w]$$

```

> r:=evalm(n-lambda*uv);
      r := [3 u + 2√2 v - λ u, 2√2 u + 3 v + w - λ v, v + 3 w - λ w]
> ChM:=linalg[genmatrix]([r[1],r[2],r[3]], [u,v,w]);
      ChM := ⎡ 3 - λ   2√2   0
              2√2   3 - λ   1
              0     1     3 - λ ⎤
> ChR:=linalg[det](ChM)=0;
      ChR := -18 λ + 9 λ2 - λ3 = 0
> collect(ChR, [lambda^3, lambda^2, lambda]);
      -18 λ + 9 λ2 - λ3 = 0
> I1:=coeff(lhs(ChR), lambda, 2);
      I1 := 9
> I2:=coeff(lhs(ChR), lambda, 1);
      I2 := -18
> A44:=coeff(lhs(ChR), lambda, 0);
      A44 := 0
> Delta:=linalg[det](K);
      Δ := 0
> lambda_sol:=solve(ChR, {lambda});
      lambda_sol := {λ = 0}, {λ = 6}, {λ = 3}
> r1:=eval(r, lambda_sol[1]); r2:=eval(r, lambda_sol[2]);
r3:=eval(r, lambda_sol[3]);
      r1 := [3 u + 2√2 v, 2√2 u + 3 v + w, v + 3 w]
      r2 := [-3 u + 2√2 v, 2√2 u - 3 v + w, v - 3 w]
      r3 := [2√2 v, 2√2 u + w, v]
> hs1:=eval(uv, solve({r1[1], r1[2], r1[3]}, {u,v,w}));
hs2:=eval(uv, solve({r2[1], r2[2], r2[3]}, {u,v,w}));
hs3:=eval(uv, solve({r3[1], r3[2], r3[3]}, {u,v,w}));
      hs1 := [2√2 w, -3 w, w]
      hs2 := [2√2 w, 3 w, w]
      hs3 := [u, 0, -2√2 u]
> PrumerR;
      (3 u + 2√2 v) x + (2√2 u + 3 v + w) y + (v + 3 w) z + 3 u + (2√2 - 1) v - 3 w = 0
> Smer1:=eval(hs1, {u=1, v=1, w=1});
      Smer1 := [2√2, -3, 1]
> PrumerR1:=eval(PrumerR, [u=Smer1[1], v=Smer1[2], w=Smer1[3]]);
      PrumerR1 := 0 = 0
> Smer2:=eval(hs2, {u=1, v=1, w=1});
      Smer2 := [2√2, 3, 1]
> PrumerR2:=eval(PrumerR, [u=Smer2[1], v=Smer2[2], w=Smer2[3]]);

```

$$\text{PrumerR2} := 12\sqrt{2}x + 18y + 6z + 12\sqrt{2} - 6 = 0$$

```
> Smer3:=eval(hs3,{u=1,v=1,w=1});
```

$$\text{Smer3} := [1, 0, -2\sqrt{2}]$$

```
> PrumerR3:=eval(PrumerR,[u=Smer3[1],v=Smer3[2],w=Smer3[3]]);
```

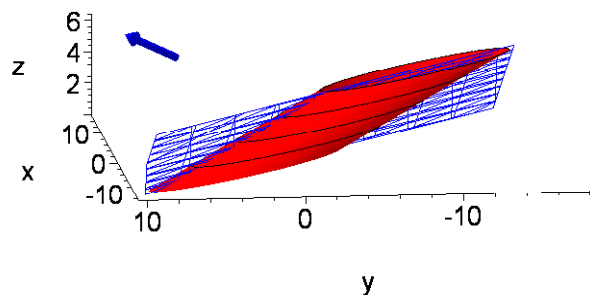
$$\text{PrumerR3} := 3x + 3 - 6\sqrt{2}z + 6\sqrt{2} = 0$$

```
> col2:=blue:
```

```
> PrumerR2g:=implicitplot3d(PrumerR2,x=-12..15,y=-18..10,z=0..4,grid=[10,10,10],style=wireframe,color=col2):
```

```
> Smer2g:=plottools[arrow]([8, 6, 5], vector(Smer2), 0.5, 1, 0.2, cylindrical_arrow,color=col2):
```

```
> display(kvg,PrumerR2g,Smer2g,axes=frame,scaling=constrained,orientation=[173,78],light=[50,45,1,1,1]);
```

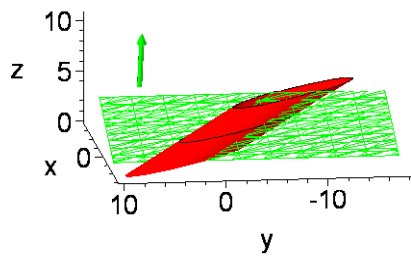


```
> col3:=green:
```

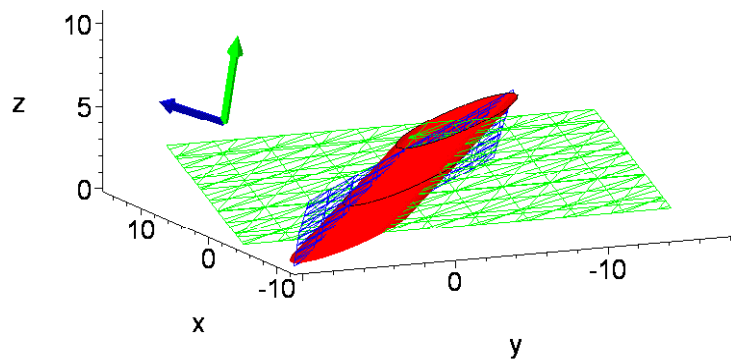
```
> PrumerR3g:=implicitplot3d(PrumerR3,x=-12..15,y=-18..10,z=0..4,grid=[10,10,10],style=wireframe,color=col3):
```

```
> Smer3g:=plottools[arrow]([8,6,5], vector(-2*Smer3), 0.5, 1, 0.2, cylindrical_arrow,color=col3):
```

```
> display(kvg,PrumerR3g,Smer3g,axes=frame,scaling=constrained,orientation=[173,78],light=[50,45,1,1,1]);
```



```
> display(kvg,PrumerR2g,Smer2g,PrumerR3g,Smer3g,axes=frame,scaling
=constrained,orientation=[153,78],light=[50,45,1,1,1]);
```



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[ >
[ >
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