```
> restart;
> with(LinearAlgebra):
> X:=Vector[row]([x,y,z,1]);
```

$$X := [x, y, z, 1]$$

```
> K:=Matrix(a,1..4,1..4,shape=symmetric);
```

$$K := \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$$

```
> Kv:=expand(X.K.Transpose(X))=0;
```

$$Kv := x^2\, a(1,1) + 2\, x\, y\, a(1,2) + 2\, x\, z\, a(1,3) + 2\, x\, a(1,4) + y^2\, a(2,2) + 2\, y\, z\, a(2,3) + 2\, y\, a(2,4) + z^2\, a(3,3) + 2\, z\, a(3,4) + a(4,4) = 0$$

2) $h(K) = 2$

```
> alpha:=a1*x+b1*y+c1*z+d1=0;  beta:=a2*x+b2*y+c2*z+d2=0;
```

$$\alpha := a1\, x + b1\, y + c1\, z + d1 = 0$$

$$\beta := a2\, x + b2\, y + c2\, z + d2 = 0$$

```
> Kv2:=lhs(alpha)*lhs(beta)=0;
```

$$Kv2 := (a1\, x + b1\, y + c1\, z + d1)\,(a2\, x + b2\, y + c2\, z + d2) = 0$$

```
> Kv2:=expand(Kv2);
```

$$Kv2 := a1\, x^2\, a2 + a1\, x\, b2\, y + a1\, x\, c2\, z + a1\, x\, d2 + b1\, y\, a2\, x + b1\, y^2\, b2 + b1\, y\, c2\, z + b1\, y\, d2 + c1\, z\, a2\, x + c1\, z\, b2\, y + c1\, z^2\, c2 + c1\, z\, d2 + d1\, a2\, x + d1\, b2\, y + d1\, c2\, z + d1\, d2 = 0$$

```
> r1:=coeff(lhs(Kv-Kv2),x,2); r2:=coeff(lhs(Kv-Kv2),y,2);
  r3:=coeff(lhs(Kv-Kv2),z,2); r4:=coeff(coeff(lhs(Kv-Kv2),x),y);
  r5:=coeff(coeff(lhs(Kv-Kv2),x),z);
  r6:=coeff(coeff(lhs(Kv-Kv2),y),z);
  r7:=coeff(coeff(coeff(lhs(Kv-Kv2),x),y,0),z,0);
  r8:=coeff(coeff(coeff(lhs(Kv-Kv2),y),x,0),z,0);
  r9:=coeff(coeff(coeff(lhs(Kv-Kv2),z),x,0),y,0);
  r10:=coeff(coeff(coeff(lhs(Kv-Kv2),x,0),y,0),z,0);
```

$$r1 := -a1\, a2 + a(1,1)$$

$$r2 := -b1\, b2 + a(2,2)$$

$$r3 := -c1\, c2 + a(3,3)$$

$$r4 := -a1\, b2 - b1\, a2 + 2\, a(1,2)$$

$$r5 := -a1\, c2 - c1\, a2 + 2\, a(1,3)$$

$$r6 := 2\, a(2,3) - b1\, c2 - c1\, b2$$

$$r7 := -a1\, d2 - d1\, a2 + 2\, a(1,4)$$

$$r8 := -b1\, d2 - d1\, b2 + 2\, a(2,4)$$

$$r9 := -c1\, d2 - d1\, c2 + 2\, a(3,4)$$

$$r10 := a(4,4) - d1\, d2$$

```
> res:=solve({r1,r2,r3,r4,r5,r6,r7,r8,r9,r10},{a(1,1),a(1,2),a(1,3
```

```
),a(1,4),a(2,2),a(2,3),a(2,4),a(3,3),a(3,4),a(4,4)});
```

$$res := \{a(1,1) = a1\ a2,\ a(1,2) = \frac{a1\ b2}{2} + \frac{b1\ a2}{2},\ a(1,3) = \frac{a1\ c2}{2} + \frac{c1\ a2}{2},$$

$$a(1,4) = \frac{a1\ d2}{2} + \frac{d1\ a2}{2},\ a(2,2) = b1\ b2,\ a(2,3) = \frac{b1\ c2}{2} + \frac{c1\ b2}{2},\ a(2,4) = \frac{b1\ d2}{2} + \frac{d1\ b2}{2},$$

$$a(3,3) = c1\ c2,\ a(3,4) = \frac{c1\ d2}{2} + \frac{d1\ c2}{2},\ a(4,4) = d1\ d2\}$$

```
> K2:=2*map(x->eval(x,res),K);
```

$$K2 := \begin{bmatrix} 2\ a1\ a2 & a1\ b2 + b1\ a2 & a1\ c2 + c1\ a2 & a1\ d2 + d1\ a2 \\ a1\ b2 + b1\ a2 & 2\ b1\ b2 & b1\ c2 + c1\ b2 & b1\ d2 + d1\ b2 \\ a1\ c2 + c1\ a2 & b1\ c2 + c1\ b2 & 2\ c1\ c2 & c1\ d2 + d1\ c2 \\ a1\ d2 + d1\ a2 & b1\ d2 + d1\ b2 & c1\ d2 + d1\ c2 & 2\ d1\ d2 \end{bmatrix}$$

Příklad: Dvě různoběžné roviny

```
> alpha:=x+2*y-z+1=0;  beta:=2*x+y+3*z-2=0;
```

$$\alpha := x + 2\ y - z + 1 = 0$$

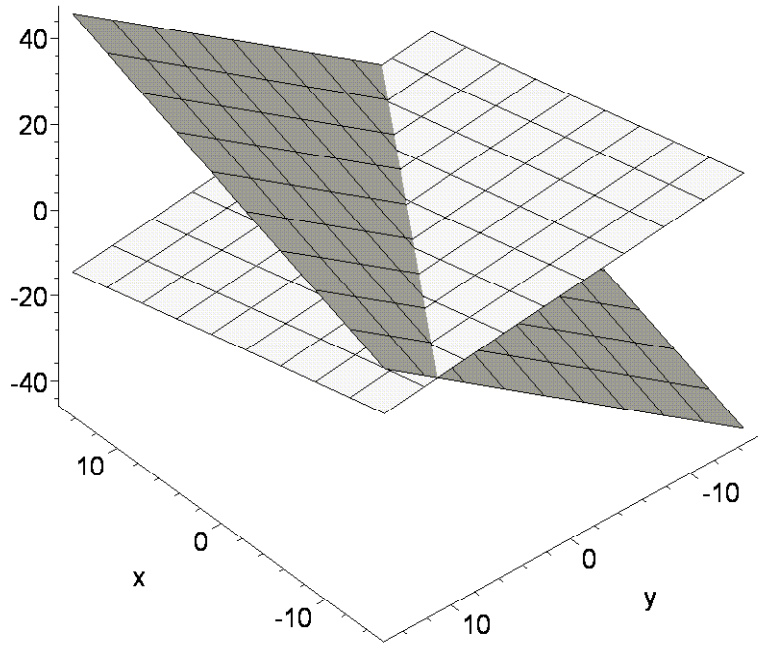
$$\beta := 2\ x + y + 3\ z - 2 = 0$$

```
> plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);
> r1:=solve(alpha,z); r2:=solve(beta,z);
```

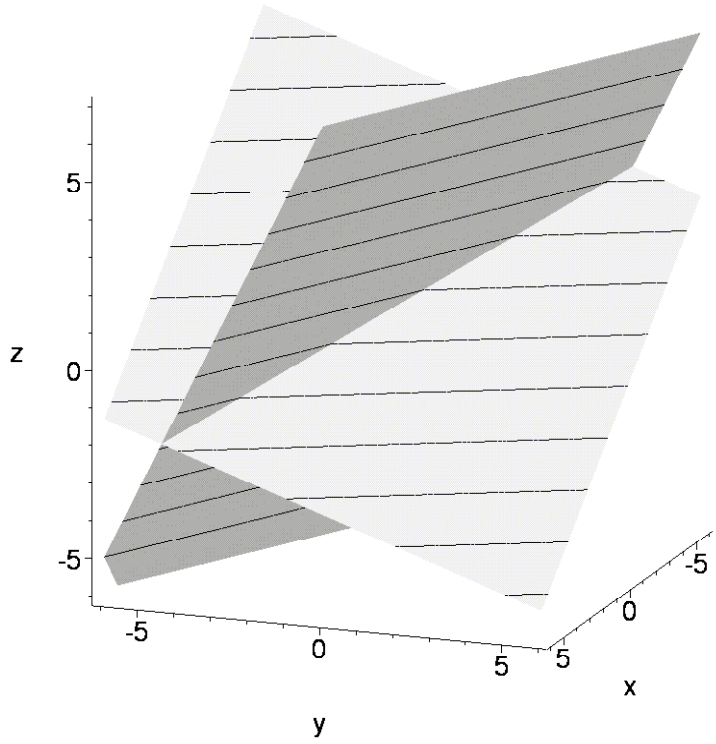
$$r1 := x + 2\ y + 1$$

$$r2 := -\frac{2\ x}{3} - \frac{y}{3} + \frac{2}{3}$$

```
> r1:=plot3d(r1,x=-15..15,y=-15..15,axes=frame,orientation=[139,52
  ],color=grey,grid=[10,10],color=COLOR(RGB,140/255,140/255,140/25
  5),tickmarks=[3,3,3],numpoints=100000):
> r2:=plot3d(r2,x=-15..15,y=-15..15,axes=frame,orientation=[139,52
  ],color=grey,grid=[10,10],color=COLOR(RGB,243/255,243/255,243/25
  5),tickmarks=[3,3,3],numpoints=100000):
> plots[display](r1,r2);
```




```
> plots[implicitplot3d]([alpha,beta],x=-6..6,y=-6..6,z=-6..7,orien
  tation=[20,75],gridstyle=rectangular,axes=frame,light=[90,-5,1,1
  ,1],grid=[30,30,30],style=patchcontour,color=[COLOR(RGB,240/255,
  240/255,240/255),COLOR(RGB,243/255,243/255,243/255)],scaling=con
  strained,tickmarks=[3,3,3]);
```



3) $h(K) = 1$

```
> Kv3:=(a*x+b*y+c*z+d)^2=0;
```

$$Kv3 := (a\,x + b\,y + c\,z + d)^2 = 0$$

```
> Kv3:=expand(Kv3);
```

$Kv3 :=$

$$a^2\,x^2 + 2\,a\,x\,b\,y + 2\,a\,x\,c\,z + 2\,a\,x\,d + b^2\,y^2 + 2\,b\,y\,c\,z + 2\,b\,y\,d + c^2\,z^2 + 2\,c\,z\,d + d^2 = 0$$

```
> r1:=coeff(lhs(Kv-Kv3),x,2); r2:=coeff(lhs(Kv-Kv3),y,2);
  r3:=coeff(lhs(Kv-Kv3),z,2); r4:=coeff(coeff(lhs(Kv-Kv3),x),y);
  r5:=coeff(coeff(lhs(Kv-Kv3),x),z);
  r6:=coeff(coeff(lhs(Kv-Kv3),y),z);
  r7:=coeff(coeff(coeff(lhs(Kv-Kv3),x),y,0),z,0);
  r8:=coeff(coeff(coeff(lhs(Kv-Kv3),y),x,0),z,0);
  r9:=coeff(coeff(coeff(lhs(Kv-Kv3),z),x,0),y,0);
  r10:=coeff(coeff(coeff(lhs(Kv-Kv3),x,0),y,0),z,0);
```

$$r1 := \mathrm{a}(1, 1) - a^2$$

$$r2 := \mathrm{a}(2, 2) - b^2$$

$$r3 := \mathrm{a}(3, 3) - c^2$$

$$r4 := 2\,a(1,2) - 2\,a\,b$$
$$r5 := 2\,a(1,3) - 2\,a\,c$$
$$r6 := 2\,a(2,3) - 2\,b\,c$$
$$r7 := 2\,a(1,4) - 2\,a\,d$$
$$r8 := 2\,a(2,4) - 2\,b\,d$$
$$r9 := 2\,a(3,4) - 2\,c\,d$$
$$r10 := a(4,4) - d^2$$

```
> res:=solve({r1,r2,r3,r4,r5,r6,r7,r8,r9,r10},{a(1,1),a(1,2),a(1,3
  ),a(1,4),a(2,2),a(2,3),a(2,4),a(3,3),a(3,4),a(4,4)});
```

$$res := \{a(1,1) = a^2,\ a(1,2) = a\,b,\ a(1,3) = a\,c,\ a(1,4) = a\,d,\ a(2,2) = b^2,\ a(2,3) = b\,c,$$
$$a(2,4) = b\,d,\ a(3,3) = c^2,\ a(3,4) = c\,d,\ a(4,4) = d^2\}$$

```
> K3:=map(x->eval(x,res),K);
```

$$K3 := \begin{bmatrix} a^2 & a\,b & a\,c & a\,d \\ a\,b & b^2 & b\,c & b\,d \\ a\,c & b\,c & c^2 & c\,d \\ a\,d & b\,d & c\,d & d^2 \end{bmatrix}$$

Příklad: Dvojnásobná rovina

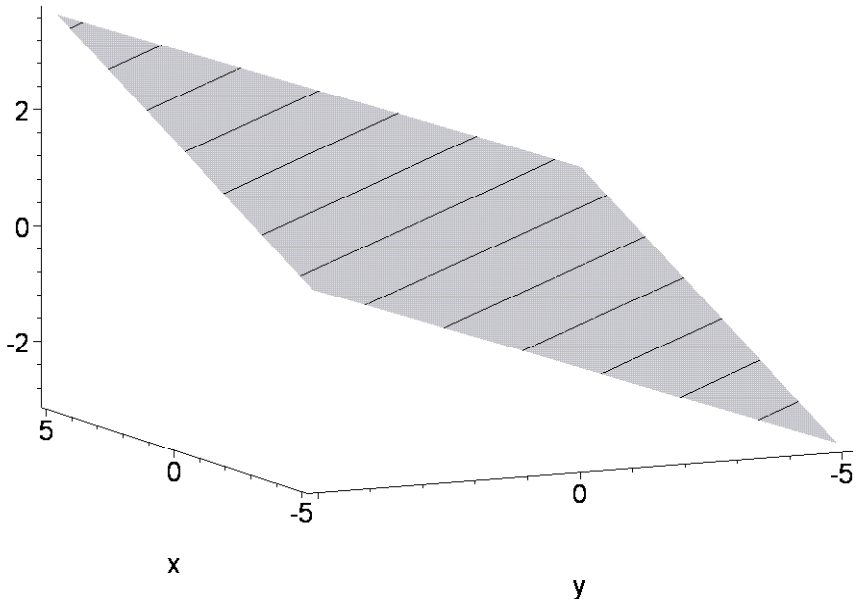
```
> Kv3:=(x+y-3*z+1)^2=0;
```

$$Kv3 := (x + y - 3\,z + 1)^2 = 0$$

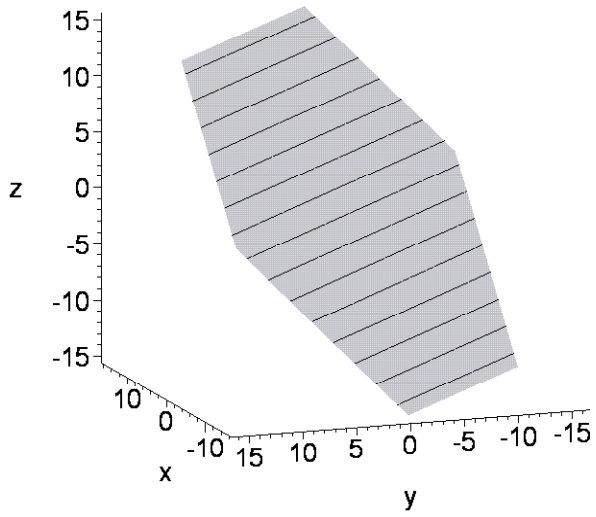
```
> op(lhs(Kv3))[1];
```

$$x + y - 3\,z + 1$$

```
> plot3d(solve(op(lhs(Kv3))[1],z),x=-5..5,y=-5..5,axes=frame,orien
  tation=[154,81],style=surfacecontour,color=grey,grid=[10,10],sca
  ling=constrained,tickmarks=[3,3,3]);
```




```
> plots[implicitplot3d](op(lhs(Kv3))[1],x=-16..16,y=-16..16,z=-15.
  .15,orientation=[170,60],gridstyle=rectangular,axes=frame,grid=[
  10,10,10],style=patchcontour,color=grey,scaling=constrained);
```


15
10
5
z
0
-5
-10
-15
10
0
-10
x
15
10
5
0
-5
-10
-15
y