1.8 Polární rovina

```
> restart;
> with(LinearAlgebra):
> with(plots):
> X:=Vector[row]([x,y,z,1]);
```

$$X := [x, y, z, 1]$$

Matice kvadriky:

```
> K:=Matrix(a,1..4,1..4,shape=symmetric);
```

$$K := \begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$$

Rovnice kvadriky:

```
> Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
```

$$Kv := a(1,1)\,x^2 + 2\,a(1,2)\,x\,y + 2\,a(1,3)\,x\,z + a(2,2)\,y^2 + 2\,a(2,3)\,y\,z + a(3,3)\,z^2 + 2\,a(1,4)\,x + 2\,a(2,4)\,y + 2\,a(3,4)\,z + a(4,4) = 0$$

Parametrické rovnice tečny z bodu $R = [r, s, u]$ s bodem dotyku $T = [m, n, p]$

```
> R:=[r,s,u]; T:=[m,n,p];
```

$$R := [r, s, u]$$

$$T := [m, n, p]$$

```
> Tecna:=[x=m+t*w1,y=n+t*w2,z=p+t*w3];
```

$$Tecna := [x = m + t\,w1, y = n + t\,w2, z = p + t\,w3]$$

```
> Kv1:=simplify(eval(Kv,Tecna));
```

$$Kv1 := 2\,a(1,1)\,m\,t\,w1 + 2\,a(1,2)\,m\,t\,w2 + 2\,a(1,2)\,t\,w1\,n + 2\,a(1,2)\,t^2\,w1\,w2 + 2\,a(1,3)\,m\,t\,w3 + 2\,a(1,3)\,t\,w1\,p + 2\,a(1,3)\,t^2\,w1\,w3 + 2\,a(2,2)\,n\,t\,w2 + 2\,a(2,3)\,n\,t\,w3 + 2\,a(2,3)\,t\,w2\,p + 2\,a(2,3)\,t^2\,w2\,w3 + 2\,a(3,3)\,p\,t\,w3 + a(4,4) + a(1,1)\,m^2 + a(2,2)\,n^2 + a(3,3)\,p^2 + 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p + a(1,1)\,t^2\,w1^2 + 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p + a(2,2)\,t^2\,w2^2 + 2\,a(2,3)\,n\,p + a(3,3)\,t^2\,w3^2 + 2\,a(1,4)\,t\,w1 + 2\,a(2,4)\,t\,w2 + 2\,a(3,4)\,t\,w3 = 0$$

```
> Kv1 :=
  a(2,2)*n^2+a(4,4)+a(1,1)*m^2+2*a(2,3)*n*p+2*a(1,3)*m*p+2*a(1,2)*
  m*n+2*a(3,3)*p*t*w3+2*a(1,2)*m*t*w2+2*a(1,2)*t^2*w1*w2+2*a(1,3)*
  m*t*w3+2*a(1,3)*t*w1*p+a(1,1)*t^2*w1^2+a(2,2)*t^2*w2^2+2*a(1,4)*
  t*w1+2*a(1,3)*%?^2*w1*w3+2*a(2,2)*n*t*w2+2*a(1,1)*m*t*w1+2*a(1,2
  )*t*w1*n+2*a(2,3)*n*t*w3+2*a(2,3)*t*w2*p+2*a(2,3)*t^2*w2*w3+a(3,
  3)*t^2*w3^2+2*a(3,4)*t*w3+2*a(2,4)*t*w2+2*a(3,4)*p+2*a(2,4)*n+2*
  a(1,4)*m+a(3,3)*p^2 = 0;
```

$$Kv1 := 2\,a(1,1)\,m\,t\,w1 + 2\,a(1,2)\,m\,t\,w2 + 2\,a(1,2)\,t\,w1\,n + 2\,a(1,2)\,t^2\,w1\,w2$$

$$+ 2\,a(1,3)\,m\,t\,w3 + 2\,a(1,3)\,t\,w1\,p + 2\,a(2,2)\,n\,t\,w2 + 2\,a(2,3)\,n\,t\,w3 + 2\,a(2,3)\,t\,w2\,p$$
$$+ 2\,a(2,3)\,t^2\,w2\,w3 + 2\,a(3,3)\,p\,t\,w3 + a(4,4) + a(1,1)\,m^2 + a(2,2)\,n^2 + a(3,3)\,p^2$$
$$+ 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p + a(1,1)\,t^2\,w1^2 + 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p$$
$$+ a(2,2)\,t^2\,w2^2 + 2\,a(2,3)\,n\,p + a(3,3)\,t^2\,w3^2 + 2\,a(1,4)\,t\,w1 + 2\,a(2,4)\,t\,w2$$
$$+ 2\,a(3,4)\,t\,w3 + 2\,a(1,3)\,?^2\,w1\,w3 = 0$$

```
> A:=coeff(lhs(Kv1),t^2);
```

$$A := 2\,a(1,2)\,w1\,w2 + 2\,a(2,3)\,w2\,w3 + a(1,1)\,w1^2 + a(2,2)\,w2^2 + a(3,3)\,w3^2$$

```
> B:=1/2*coeff(lhs(Kv1),t);
```

$$B := a(1,1)\,m\,w1 + a(1,2)\,m\,w2 + a(1,2)\,w1\,n + a(1,3)\,m\,w3 + a(1,3)\,w1\,p + a(2,2)\,n\,w2$$
$$+ a(2,3)\,n\,w3 + a(2,3)\,w2\,p + a(3,3)\,p\,w3 + a(1,4)\,w1 + a(2,4)\,w2 + a(3,4)\,w3$$

```
> C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
```

$$C := a(4,4) + a(1,1)\,m^2 + a(2,2)\,n^2 + a(3,3)\,p^2 + 2\,a(1,4)\,m + 2\,a(2,4)\,n + 2\,a(3,4)\,p$$
$$+ 2\,a(1,2)\,m\,n + 2\,a(1,3)\,m\,p + 2\,a(2,3)\,n\,p + 2\,a(1,3)\,?^2\,w1\,w3$$

```
> B:=collect(B,[w1,w2,w3]);
```

$$B := (a(1,1)\,m + a(1,3)\,p + a(1,4) + a(1,2)\,n)\,w1$$
$$+ (a(1,2)\,m + a(2,2)\,n + a(2,4) + a(2,3)\,p)\,w2$$
$$+ (a(3,3)\,p + a(2,3)\,n + a(1,3)\,m + a(3,4))\,w3$$

```
> n1:=sort(coeff(B,w1),[m,n,p]); n2:=sort(coeff(B,w2),[m,n,p]);
  n3:=sort(coeff(B,w3),[m,n,p]);
```

$$n1 := a(1,1)\,m + a(1,2)\,n + a(1,3)\,p + a(1,4)$$
$$n2 := a(1,2)\,m + a(2,2)\,n + a(2,3)\,p + a(2,4)$$
$$n3 := a(1,3)\,m + a(2,3)\,n + a(3,3)\,p + a(3,4)$$

```
> BD:=linalg[dotprod](T-R,[n1,n2,n3],orthogonal);
```

$$BD := (-r + m)\,(a(1,1)\,m + a(1,2)\,n + a(1,3)\,p + a(1,4))$$
$$+ (-s + n)\,(a(1,2)\,m + a(2,2)\,n + a(2,3)\,p + a(2,4))$$
$$+ (-u + p)\,(a(1,3)\,m + a(2,3)\,n + a(3,3)\,p + a(3,4))$$

```
> BD:=collect(eval(lhs(Kv),[x=m,y=n,z=p])-expand(BD),[m,n,p]);
```

$$BD := (r\,a(1,1) + u\,a(1,3) + s\,a(1,2) + a(1,4))\,m$$
$$+ (a(2,4) + s\,a(2,2) + u\,a(2,3) + r\,a(1,2))\,n$$
$$+ (u\,a(3,3) + r\,a(1,3) + s\,a(2,3) + a(3,4))\,p + a(4,4) + s\,a(2,4) + u\,a(3,4) + r\,a(1,4)$$

```
> ra:=sort(coeff(BD,m),[r,s,u]); rb:=sort(coeff(BD,n),[r,s,u]);
  rc:=sort(coeff(BD,p),[r,s,u]);
  rd:=sort(coeff(coeff(coeff(BD,m,0),n,0),p,0),[r,s,u]);
```

$$ra := a(1,1)\,r + a(1,2)\,s + a(1,3)\,u + a(1,4)$$
$$rb := a(1,2)\,r + a(2,2)\,s + a(2,3)\,u + a(2,4)$$
$$rc := a(1,3)\,r + a(2,3)\,s + a(3,3)\,u + a(3,4)$$
$$rd := a(1,4)\,r + a(2,4)\,s + a(3,4)\,u + a(4,4)$$

```
> pi:=ra*x+rb*y+rc*z+rd=0;
```

$$\pi := (a(1,1)\,r + a(1,2)\,s + a(1,3)\,u + a(1,4))\,x$$
$$+ (a(1,2)\,r + a(2,2)\,s + a(2,3)\,u + a(2,4))\,y$$

$$+ (a(1,3)\,r + a(2,3)\,s + a(3,3)\,u + a(3,4))\,z + a(1,4)\,r + a(2,4)\,s + a(3,4)\,u + a(4,4) = 0$$

```
> R:=Vector[row]([r,s,u,1]);
```

$$R := [r, s, u, 1]$$

```
> PolR:=collect(expand(R.K.Transpose(X)),[x,y,z])=0;
```

$$PolR := (a(1,1)\,r + a(1,2)\,s + a(1,3)\,u + a(1,4))\,x + (a(1,2)\,r + a(2,2)\,s + a(2,3)\,u + a(2,4))\,y + (a(1,3)\,r + a(2,3)\,s + a(3,3)\,u + a(3,4))\,z + a(1,4)\,r + a(2,4)\,s + a(3,4)\,u + a(4,4) = 0$$

Příklad: Polární rovina bodu R = [15,-4, 5] vzhledem ke kvadrice $x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$

```
> kv:=x^2+4*y^2+16*z^2-144=0;
```

$$kv := x^2 + 4\,y^2 + 16\,z^2 - 144 = 0$$

```
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
  a(3,3):=coeff(lhs(kv),z^2);
  a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
  a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
  a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
  a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
  a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
  a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
  a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
```

$$a(1,1) := 1$$
$$a(2,2) := 4$$
$$a(3,3) := 16$$
$$a(1,2) := 0$$
$$a(1,3) := 0$$
$$a(1,4) := 0$$
$$a(2,3) := 0$$
$$a(2,4) := 0$$
$$a(3,4) := 0$$
$$a(4,4) := -144$$

```
> K;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -144 \end{bmatrix}$$

```
> R:=Vector[row]([15,-4,5,1]);
```

$$R := [15, -4, 5, 1]$$

```
> PolR:=collect(expand(R.K.Transpose(X)),[x,y,z])=0;
```

$$PolR := -144 + 15\,x - 16\,y + 80\,z = 0$$

```
> PolR_ex:=solve(PolR,z);
```

$$PolR\_ex := \frac{9}{5} - \frac{3\,x}{16} + \frac{y}{5}$$

```
> {kv,PolR};
```

$$\{-144 + 15\,x - 16\,y + 80\,z = 0,\ x^2 + 4\,y^2 + 16\,z^2 - 144 = 0\}$$

```
> res1:=solve({kv,PolR},{y,z});
```

$$res1 := \{$$

$$y = -9 + \frac{15\,x}{16} + 5\,\mathrm{RootOf}(7424\,\_Z^2 + (-23040 + 2400\,x)\,\_Z + 289\,x^2 + 11520 - 4320\,x),$$

$$z = \mathrm{RootOf}(7424\,\_Z^2 + (-23040 + 2400\,x)\,\_Z + 289\,x^2 + 11520 - 4320\,x)\}$$

```
> Kuz:=allvalues(res1);
```

$$Kuz := \{$$

$$y = -\frac{36}{29} + \frac{15\,x}{116} + \frac{5\sqrt{46080 + 4320\,x - 689\,x^2}}{232},\ z = \frac{45}{29} - \frac{75\,x}{464} + \frac{\sqrt{46080 + 4320\,x - 689\,x^2}}{232}\},$$

$$\{y = -\frac{36}{29} + \frac{15\,x}{116} - \frac{5\sqrt{46080 + 4320\,x - 689\,x^2}}{232},\ z = \frac{45}{29} - \frac{75\,x}{464} - \frac{\sqrt{46080 + 4320\,x - 689\,x^2}}{232}$$

$$\}$$

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 :=$$

$$\left[x, -\frac{36}{29} + \frac{15\,x}{116} + \frac{5\sqrt{46080 + 4320\,x - 689\,x^2}}{232}, \frac{45}{29} - \frac{75\,x}{464} + \frac{\sqrt{46080 + 4320\,x - 689\,x^2}}{232}\right]$$

$$Kuz2 :=$$

$$\left[x, -\frac{36}{29} + \frac{15\,x}{116} - \frac{5\sqrt{46080 + 4320\,x - 689\,x^2}}{232}, \frac{45}{29} - \frac{75\,x}{464} - \frac{\sqrt{46080 + 4320\,x - 689\,x^2}}{232}\right]$$

```
> T1:=evalf(eval(Kuz1,x=0)); T2:=evalf(eval(Kuz1,x=5));
  T3:=evalf(eval(Kuz2,x=0)); T4:=evalf(eval(Kuz1,x=10));
```

$$T1 := [0.,\ 3.384968229,\ 2.476993646]$$

$$T2 := [5.,\ 4.246161736,\ 1.711732347]$$

$$T3 := [0.,\ -5.867726849,\ 0.6264546302]$$

$$T4 := [10.,\ 3.128416755,\ 0.5506833509]$$

```
> Rk:=[R[1],R[2],R[3]];
```

$$Rk := [15,\ -4,\ 5]$$

```
> t:='t':
```

```
> tec1:=evalm(Rk+t*(T1-Rk)); tec2:=evalm(Rk+t*(T2-Rk));
  tec3:=evalm(Rk+t*(T3-Rk)); tec4:=evalm(Rk+t*(T4-Rk));
```

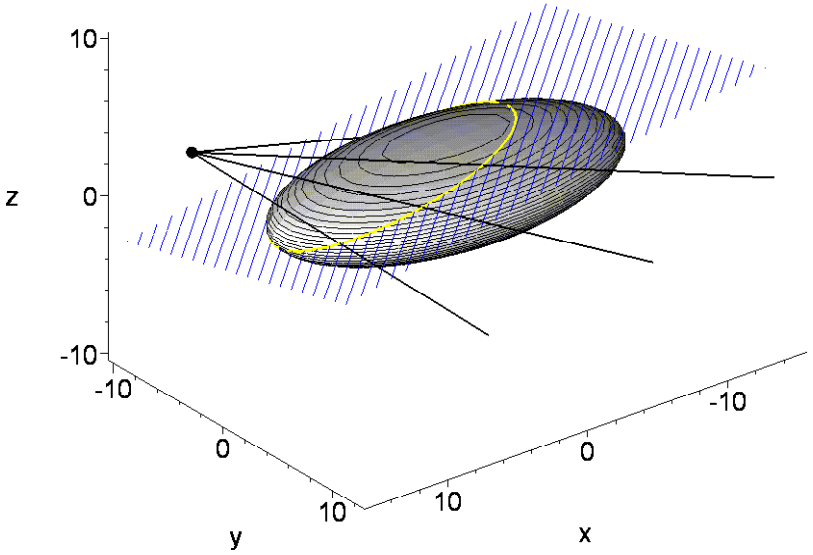
$$tec1 := [15 - 15.\,t,\ -4 + 7.384968229\,t,\ 5 - 2.523006354\,t]$$

$$tec2 := [15 - 10.\,t,\ -4 + 8.246161736\,t,\ 5 - 3.288267653\,t]$$

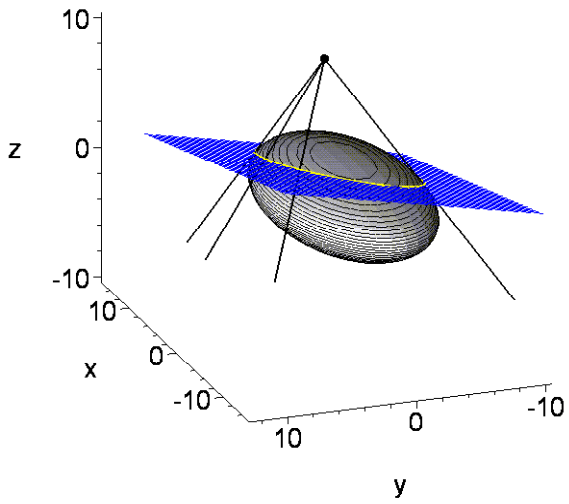
$$tec3 := [15 - 15.\,t,\ -4 - 1.867726849\,t,\ 5 - 4.373545370\,t]$$

$$tec4 := [15 - 5.\,t,\ -4 + 7.128416755\,t,\ 5 - 4.449316649\,t]$$

```
> tec1g:=plot3d(tec1,t=0..2,s=-1..1,thickness=2,color=red):
  tec2g:=plot3d(tec2,t=0..2,s=0..1,thickness=2,color=red):
  tec3g:=plot3d(tec3,t=0..2,s=0..1,thickness=2,color=red):
  tec4g:=plot3d(tec4,t=0..2,s=0..1,thickness=2,color=red):
> Body:=plots[pointplot3d]({T1,T2,T3,T4}):
> kvg0:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,gri
  d=[20,20,20],style=patchcontour,color=red,lightmodel=light1,scal
  ing=constrained):
> kvg:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,axes
  =frame,color=COLOR(RGB,250/255,250/255,250/255),style=patchconto
  ur,grid=[20,20,40],light=[100,-20,1,1,1],tickmarks=[3,3,3],orien
  tation=[52,63],scaling=constrained):
> PolRg:=contourplot3d(PolR_ex,x=-15..15,y=-10..10,grid=[2,2],cont
  ours=60,color=blue,filled=false):
> PolRg2:=plot3d(PolR_ex,x=-15..15,y=-10..10,grid=[2,2],color=grey
  ):
> Rg:=plottools[sphere]([R[1],R[2],R[3]],0.3):
> Rg2:=pointplot3d([R[1],R[2],R[3]],symbol=cross,symbolsize=20,col
  or=red):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
  Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,colo
  r=yellow):
> display(kvg,PolRg,Rg,Kuz1g,Kuz2g,tec1g,tec2g,tec3g,tec4g,axes=fr
  ame,scaling=constrained);
```




```
> display(kvg,PolRg,Rg,Kuz1g,Kuz2g,tec1g,tec2g,tec3g,tec4g,axes=fr
  ame,scaling=constrained,orientation=[160,70]);
```


10
z
0
-10
10
x
0
-10
10
0
-10
y