Unexpected answers offered by computer algebra systems to school equations

Eno Tonisson, University of Tartu, Estonia

Equation solving constitutes an important part in school mathematics curricula, although there could be differences in emphases and range between different countries or school types. Equations also have a great importance in the case of using computer algebra systems (CASs). Most answers offered by CASs to equations are customary for school but there are still some answers that would be somewhat unexpected (or incompatible with the teaching practice) in school. The paper focuses on classification and mapping of such answers.

Types of unexpected answers vary in reason, weight and range. The issues of equivalence, domain, (hidden) assumptions, presentation of the branches could be emphasized. For example, the imaginary unit in the answer could be unexpected in the case of quadratic (or other) equations if only real numbers are in the school curriculum, or if CAS uses (hidden) assumptions and the solutions are provided within restricted interval in the case of trigonometric equations, or if the answer is equivalent to the expected one but not in the expected form.

The mapping includes linear, quadratic, fractional, equations that contain an absolute value of an expression, irrational, exponential, logarithmic, trigonometric and literal equations. The CASs Maxima, Wiris, WolframAlpha, Yacas, etc. are used in tests. In order to map the boundaries of the phenomena, the equations are subclassified in different ways.

As the CASs are in continuous improvement and the new versions and CASs appear, the situation in the case of a particular version of a particular CAS is not overly important. Rather, the idea about the possible range and essence of the phenomena could be useful, especially when a didactic value of an unexpected answer is capitalizable in the introduction or accentuation of several topics (e.g. equivalence, domain).