An interactive learning activity for the formation of the concept of function based on representational transfer

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The concept of function is a central idea in mathematics. Functions have many facets, which often cause problems to students. On the one hand the nature of functional dependencies has various aspects: the mapping aspect, the aspect of change, and the object aspect. On the other hand there are various representations for functions. Each representation emphasizes different aspects and characteristics of the functional dependency. To establish subconcepts like injectivity within the function concept some representations are more suitable than others. Developing a certain subconcept within one representation does not mean that the student can transfer it to any other representation as well. Transfer means integration and building of a richer mental concept.

We present an interactive learning environment for the conceptualization of the notions of "function", "injectivity", "surjectivity", and "bijection", using a three-stage approach. In the first stage the notions mentioned above are initially developed by using finite arrow-diagrams. This focuses on the mapping aspect and the basic idea behind these notions. Within the second stage an extended dynagraph is used to integrate the aspects of change and continuity in the above subconcepts. Stage three links dynagraphs with common graphs of functions visualizing the transition between these representations.

For example strictly monotonic functions are injective. Monotony can be viewed best in the representation form graph, but cannot be displayed in arrow-diagrams. The learning environment "Squiggle-M" allows the integration of the connection injectivity-monotony in the concept by following these three stages.

"Squiggle-M" is a mathematical exploration tool which offers a bundle of experimentation laboratories regarding this purpose. Different representation forms of functions are implemented using the interactive geometry software "Cinderella". The software also presents a collection of open study questions that can be answered within the laboratories by making use of the different representation forms. The individual learning process of the student is reflected by the software's feedback module based on an intelligent (semi-)automated assessment system.