On using automated deduction techniques in dynamic geometry environments

Francisco Botana
Vigo University, Spain
http://webs.uvigo.es/fbotana

What is Dynamic Geometry (DG)?

Models built by computer software that can be changed dynamically

King and Schattschneider (1997)

Properties of dynamical models

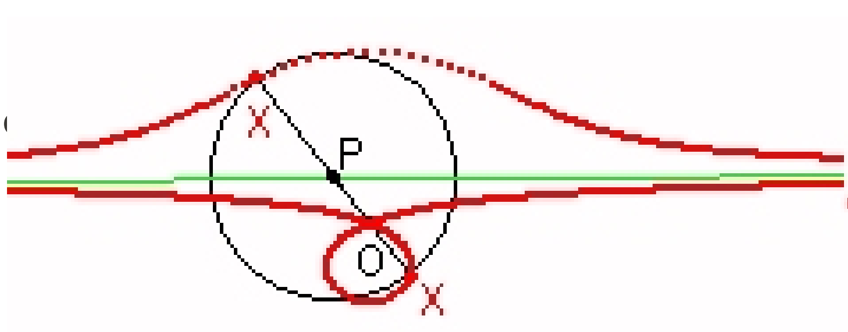
- Dynamic transformations
- Dynamic measurement
- Free dragging
- Animation
- Locus generation

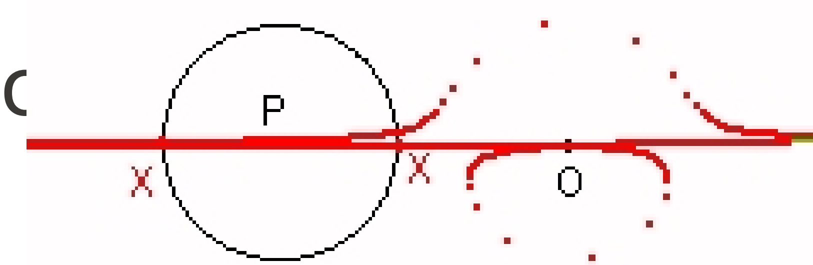
Common strategy for tracing loci

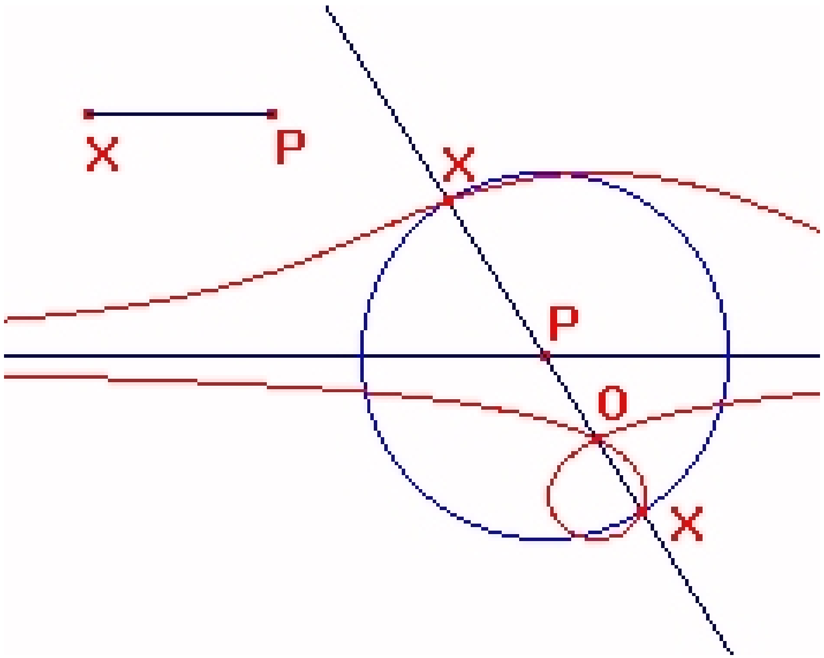
In order to trace the locus of an object X, depending somehow on another object P:

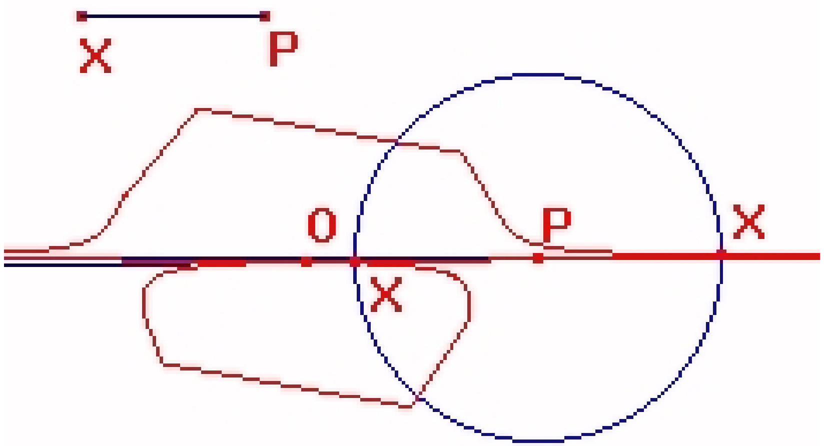
- Select the object P, the *driver object*, which must lie on a predefined path

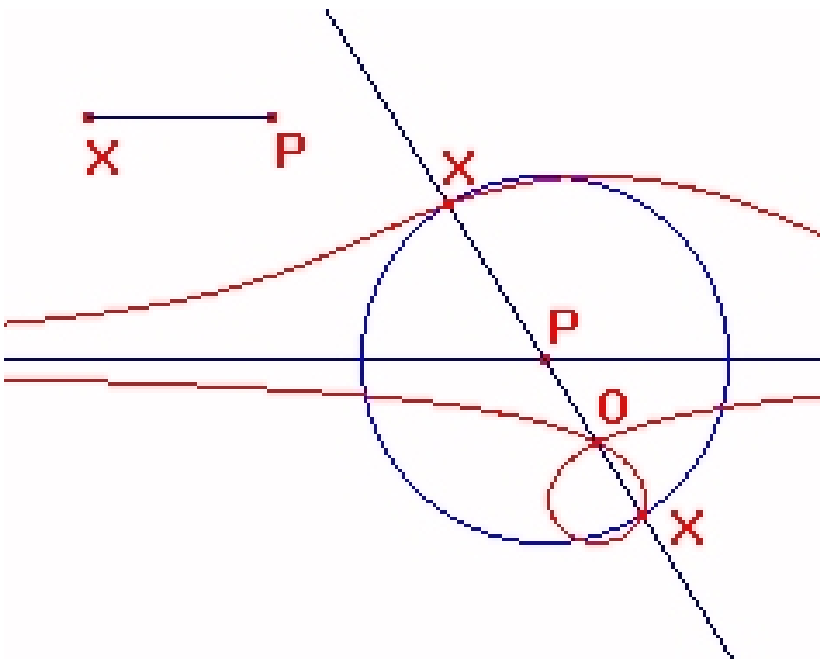
- Sample the path and plot X for each sample






X
P
X
P
O
X


X
P
O
P
X
X



P(x_1,x_2) lies on the x axis:

$$x_2=0$$

O(0,-1) is a fixed point

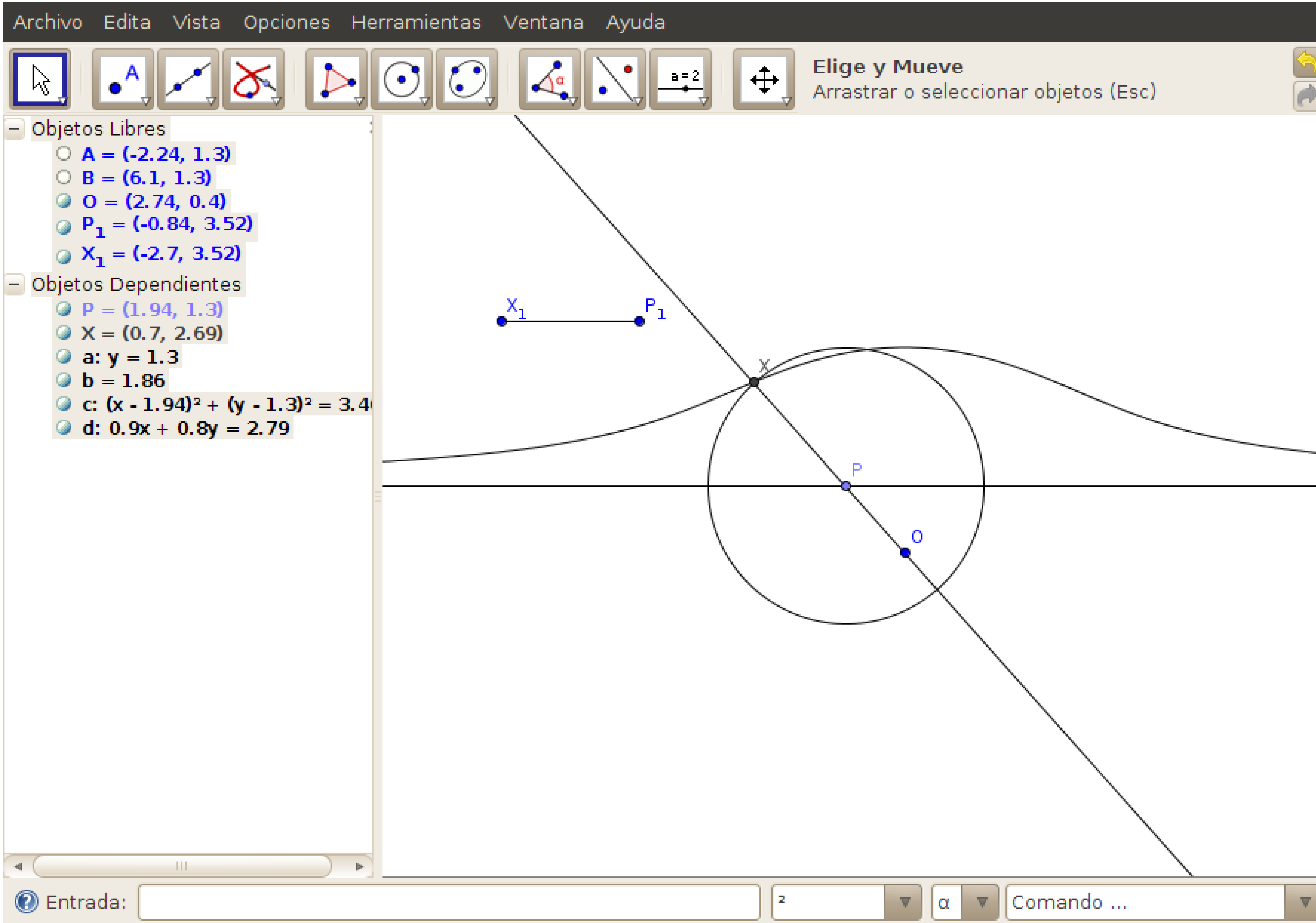
X(u,v) lies on the line OP:

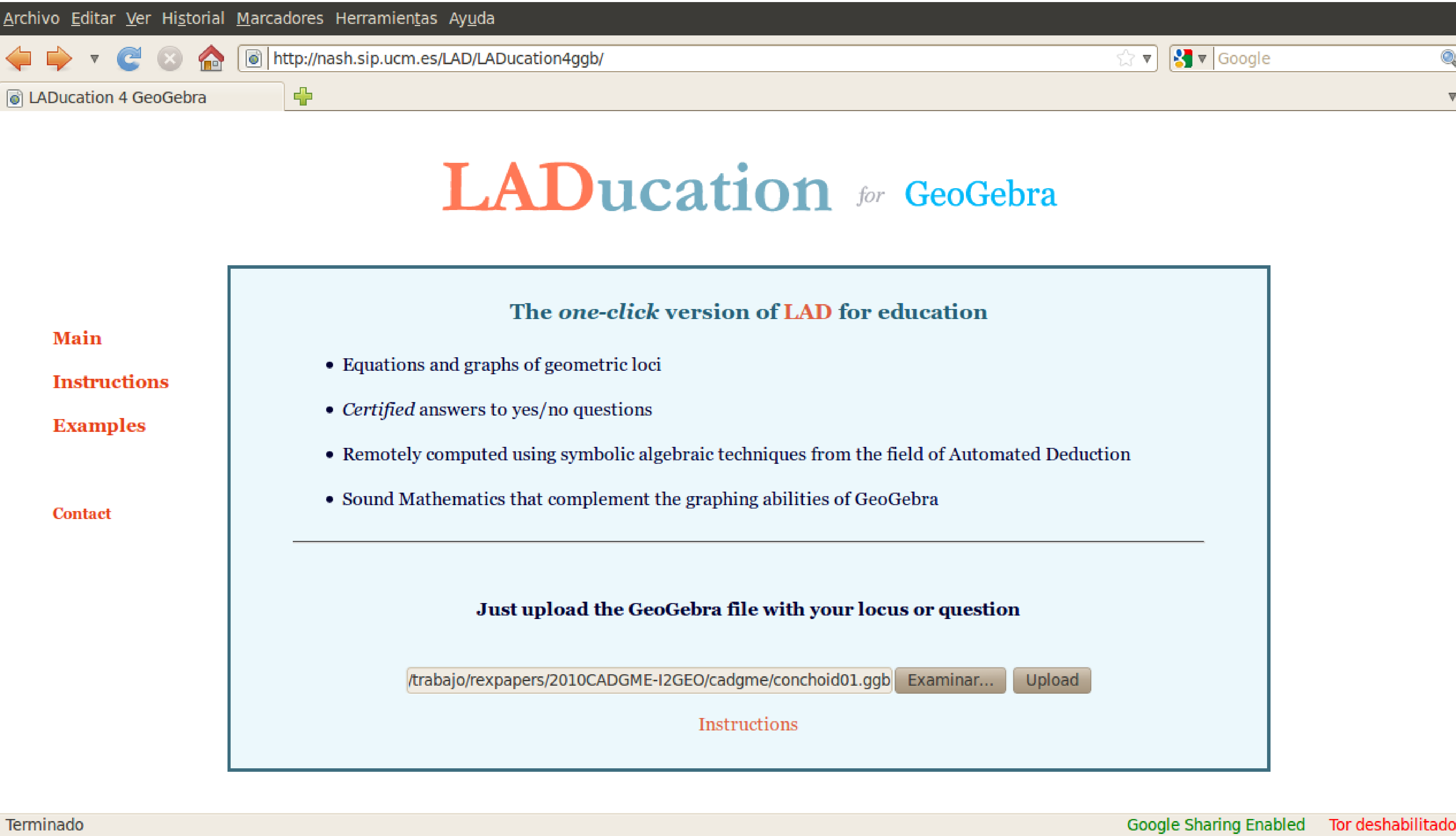
$$(v+1)/u=1/x_1$$

and on the circle (P,XP):

$$(u-x_1)^2+v^2=d^2$$

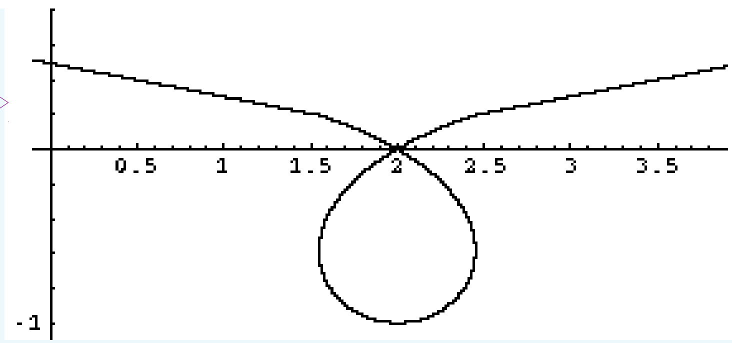
Eliminating x_1 in the above equations we get the conchoid's equation!


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve
Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (-2.24, 1.3)
B = (6.1, 1.3)
O = (2.74, 0.4)
P1 = (-0.84, 3.52)
X1 = (-2.7, 3.52)
Objetos Dependientes
P = (1.94, 1.3)
X = (0.7, 2.69)
a: y = 1.3
b = 1.86
c: (x - 1.94)² + (y - 1.3)² = 3.4
d: 0.9x + 0.8y = 2.79
X1
P1
X
P
O
Entrada:
α
Comando ...


Archivo Editar Ver Historial Marcadores Herramientas Ayuda
http://nash.sip.ucm.es/LAD/LADucation4ggb/
Google
LADucation 4 GeoGebra
LADucation for GeoGebra
Main
Instructions
Examples
Contact
The one-click version of LAD for education
Equations and graphs of geometric loci
Certified answers to yes/no questions
Remotely computed using symbolic algebraic techniques from the field of Automated Deduction
Sound Mathematics that complement the graphing abilities of GeoGebra
Just upload the GeoGebra file with your locus or question
/trabajo/rexpapers/2010CADGME-I2GEO/cadgme/conchoid01.ggb
Examinar...
Upload
Instructions
Terminado
Google Sharing Enabled
Tor deshabilitado

Continuity

GeoGebra allows you to turn the continuity heuristic *On* or *Off* in the *Options* menu. The software uses a near-to-heuristic to keep moving intersection points (e. g., line-conic, conic-conic) close to their old positions and avoid jumping intersection points.



The locus equation is

```
the quartic 4 - 4*x + x^2 - 8*y + 8*x*y - 2*x^2*y + y^2 - 4*x*y^2 + x^2*y^2 - 2*y^3 +
  y^4 == 0
```

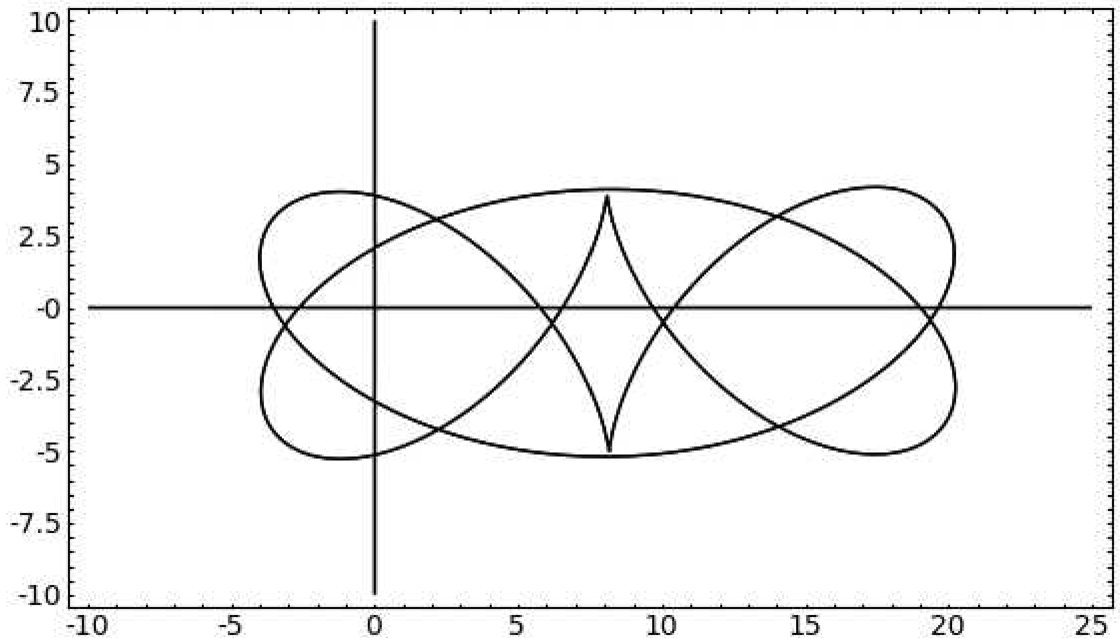
GeoGebra & Google Summer of Code 2010

Locus line equation

Find the equation for a locus line constructed in GeoGebra. The goal is an integrated solution that doesn't require upload of a ggb file to a server.

Contour and Implicit Plotting

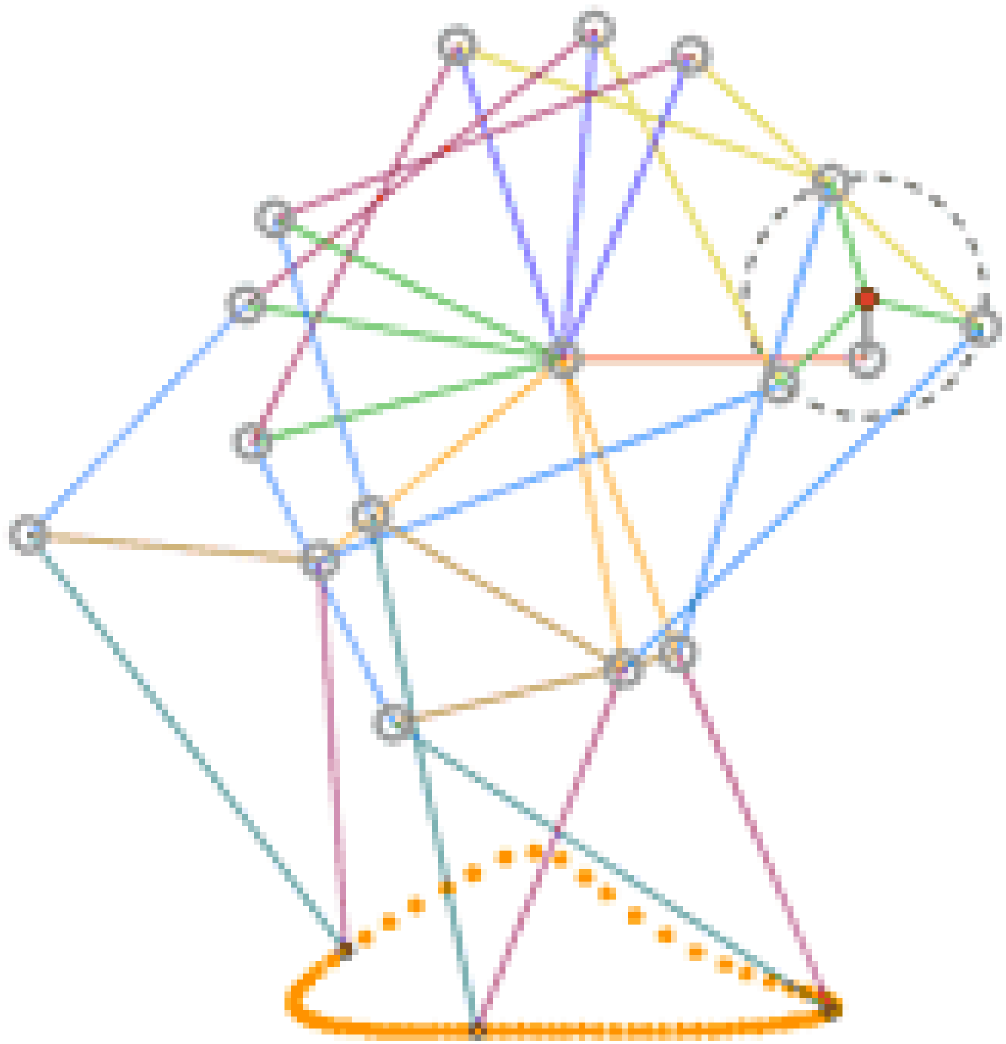
Implement contour and implicit plotting routines in GeoGebra.


10
7.5
5
2.5
-0
-2.5
-5
-7.5
-10
-10
-5
0
5
10
15
20
25

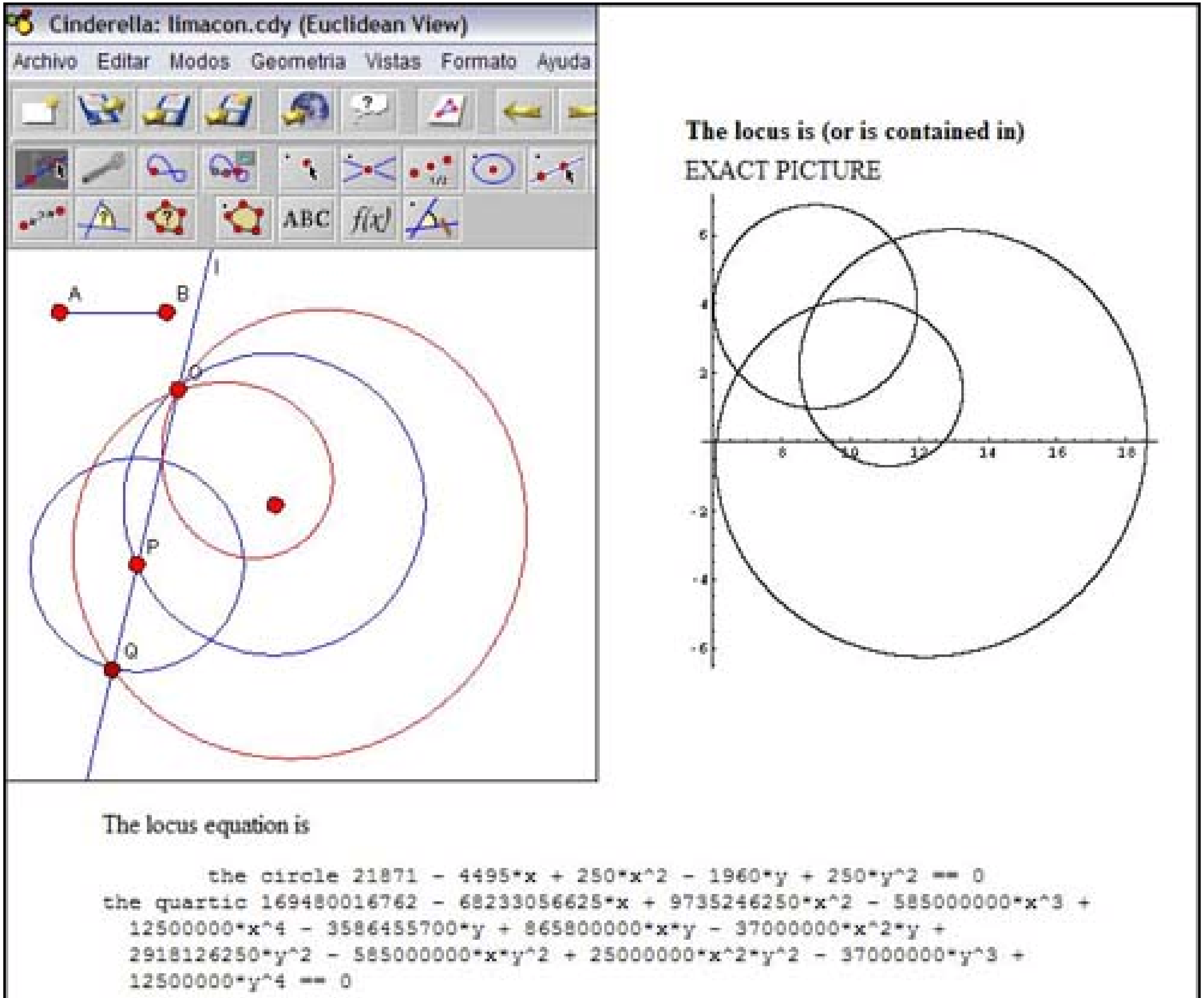
80232964351695482090052668589141175946115160321 +
2707034858032029512277843872719719601534605600*x -
12094748935307324983015398495218909628691240000*x^2 +
1361919898426493638759337505309527108856000000*x^3 +
394154177587603756005838424500972080600000000*x^4 -
104626220355981452458120738564213200000000000*x^5 +
9581959544774075721953221354940000000000000*x^6 -
392537657354404332373896300000000000000000*x^7 +
6045758945856809497506250000000000000000*x^8 -
(**30 terms**)
25919487370774725584999400000000000000000*y^7 +
6045758945856809497506250000000000000000*y^8=0

Should we hide this (kind of) result to students?

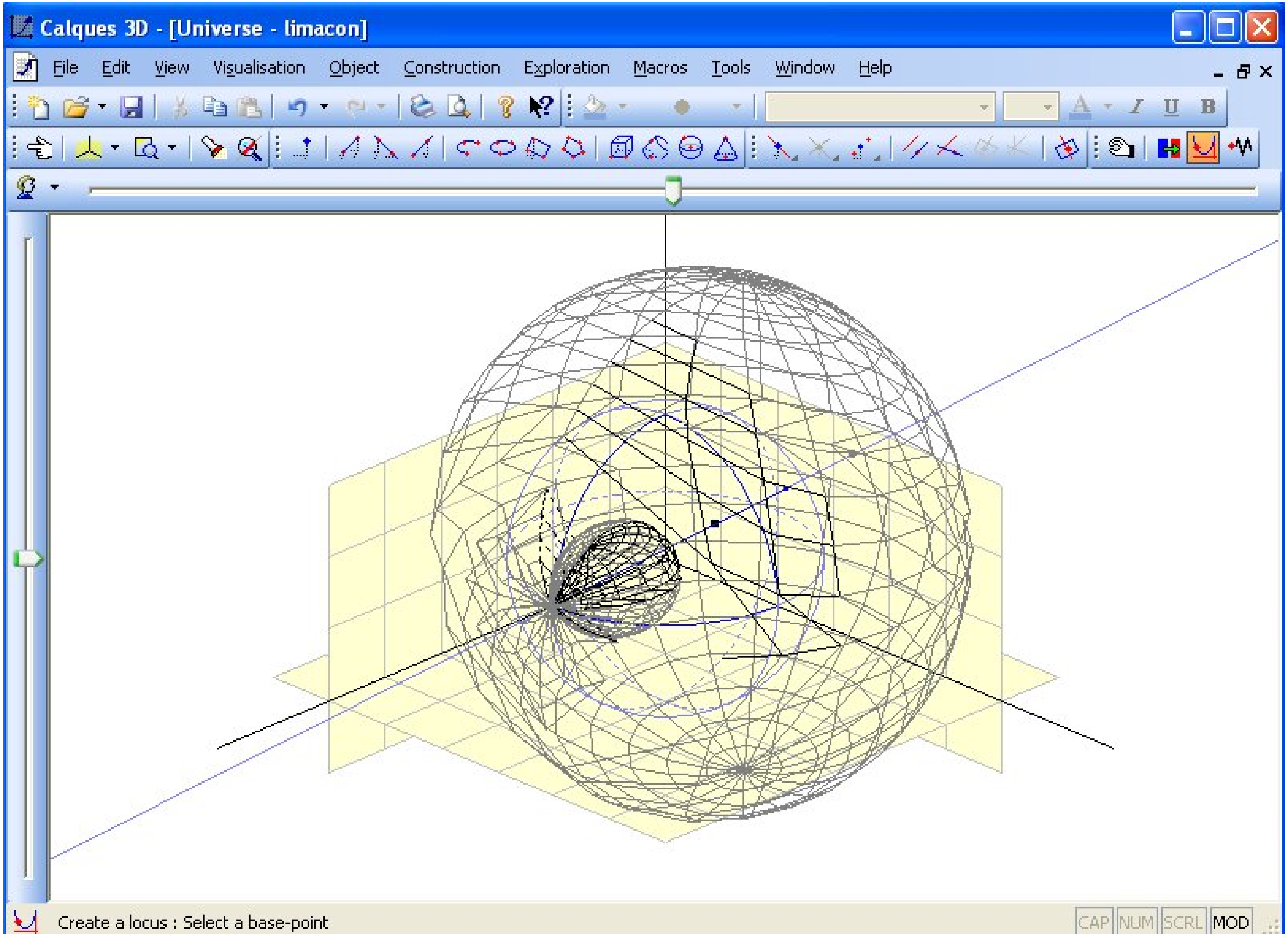
A case of failure



Loci and degenerated conditions



Extension to 3D



```
Range[-2.7,2.8,-4.4,2.8,-2.7,2.7];
FreePointD[Pt1,1,1,0];
FreePointD[Pt2,-31/49,16/9,0];
SphereD[Sp3,Pt1,Pt2];
PointOnSphere[Pt4,Sp3];
LineD[Ln5,Pt2,Pt4];
SphereD[Sp6,Pt4,Pt1];
Intersection1LineSphere[Pt8,Sp6,Ln5];
Intersection2LineSphere[Pt9,Sp6,Ln5];
Locus[O10,Pt8,Pt4];
```

OpenMath

- Extending the 3dgeo experimental CDs
- Requiring 3d-DGS developers OM-export

3D Discovery with Co...

http://193.146.36.49:8080/webMathematica/MSP/Examples/3Ddiscovery.msp

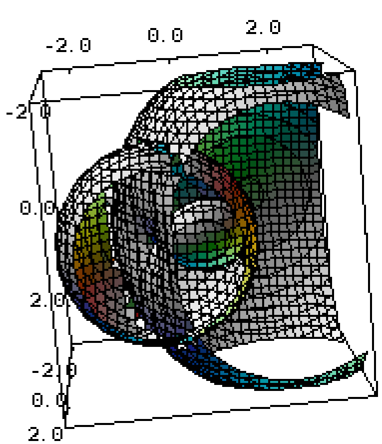
3D Automatic Discovery using CoCoA

POWERED BY webMATHEMATICA

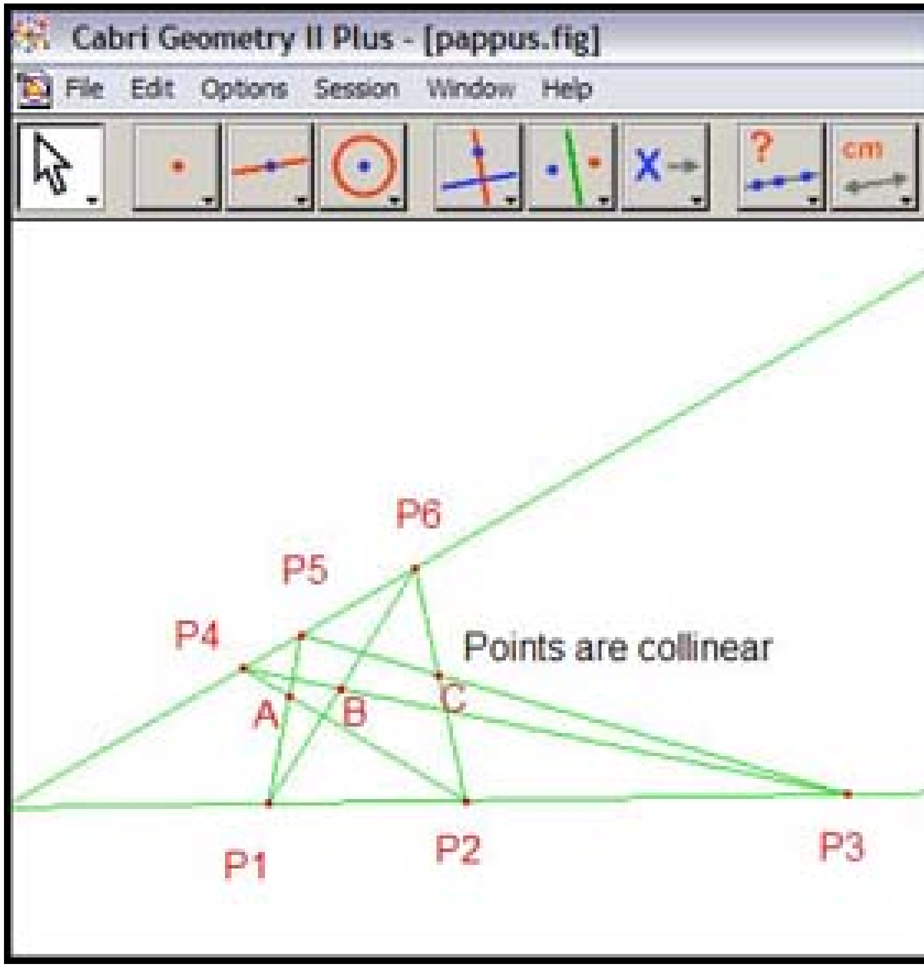
The locus of Pt8 in the given construction is

the sphere $441\,x^2 + 558\,x + 441\,y^2 + 441\,z^2 - 1568\,y + 128 = 0$

and $85766121\,x^4 - 343064484\,x^3 + 171532242\,y^2\,x^2 + 171532242\,z^2\,x^2 - 343064484\,y\,x^2 - 155363859\,x^2 - 343064484\,y^2\,x - 343064484\,z^2\,x + 686128968\,y\,x + 80946126\,x + 85766121\,y^4 + 85766121\,z^4 - 343064484\,y^3 - 155363859\,y^2 + 171532242\,y^2\,z^2 - 343064484\,y\,z^2 - 498428343\,z^2 + 1433186300\,y - 860340224 = 0$

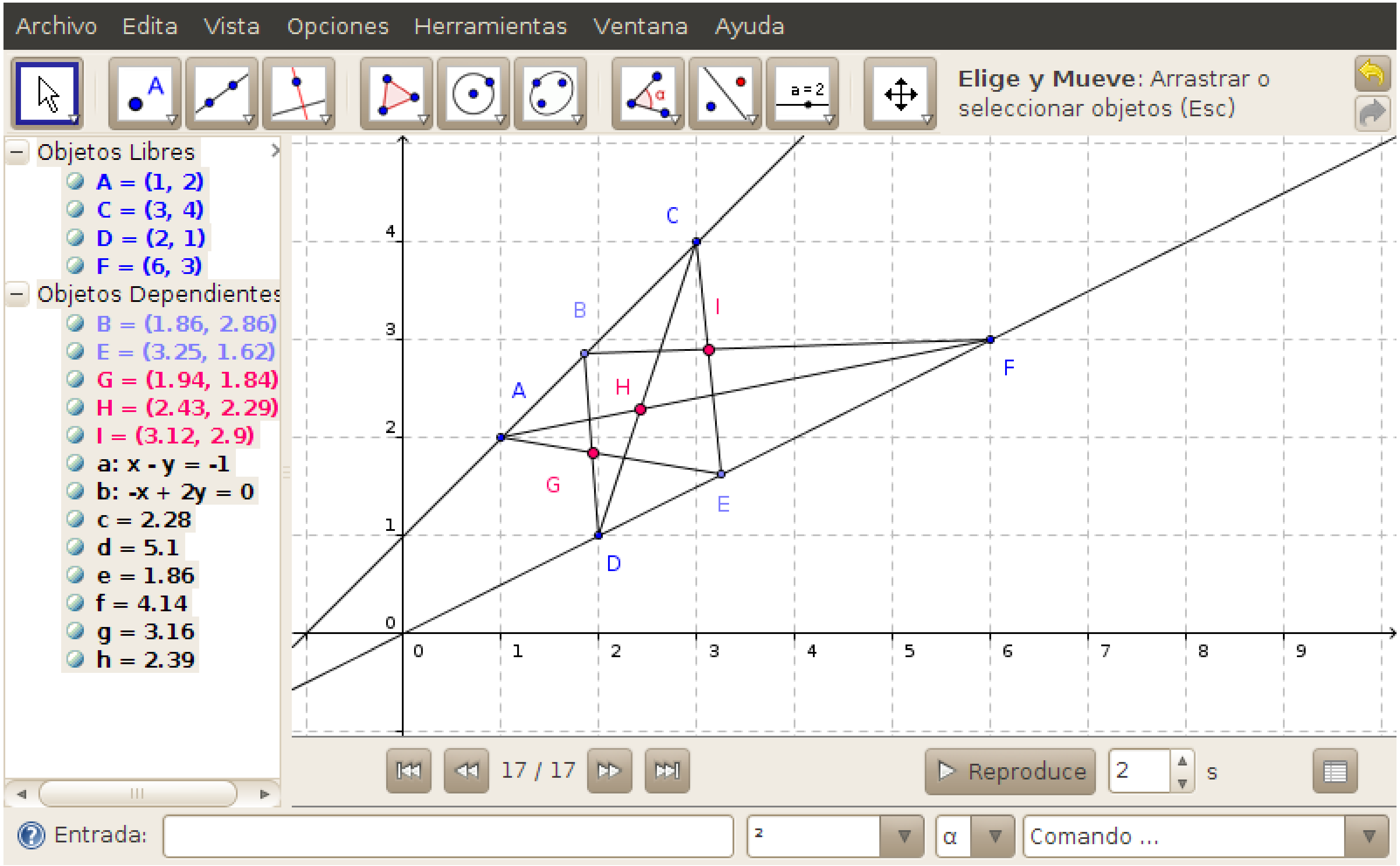


Proof



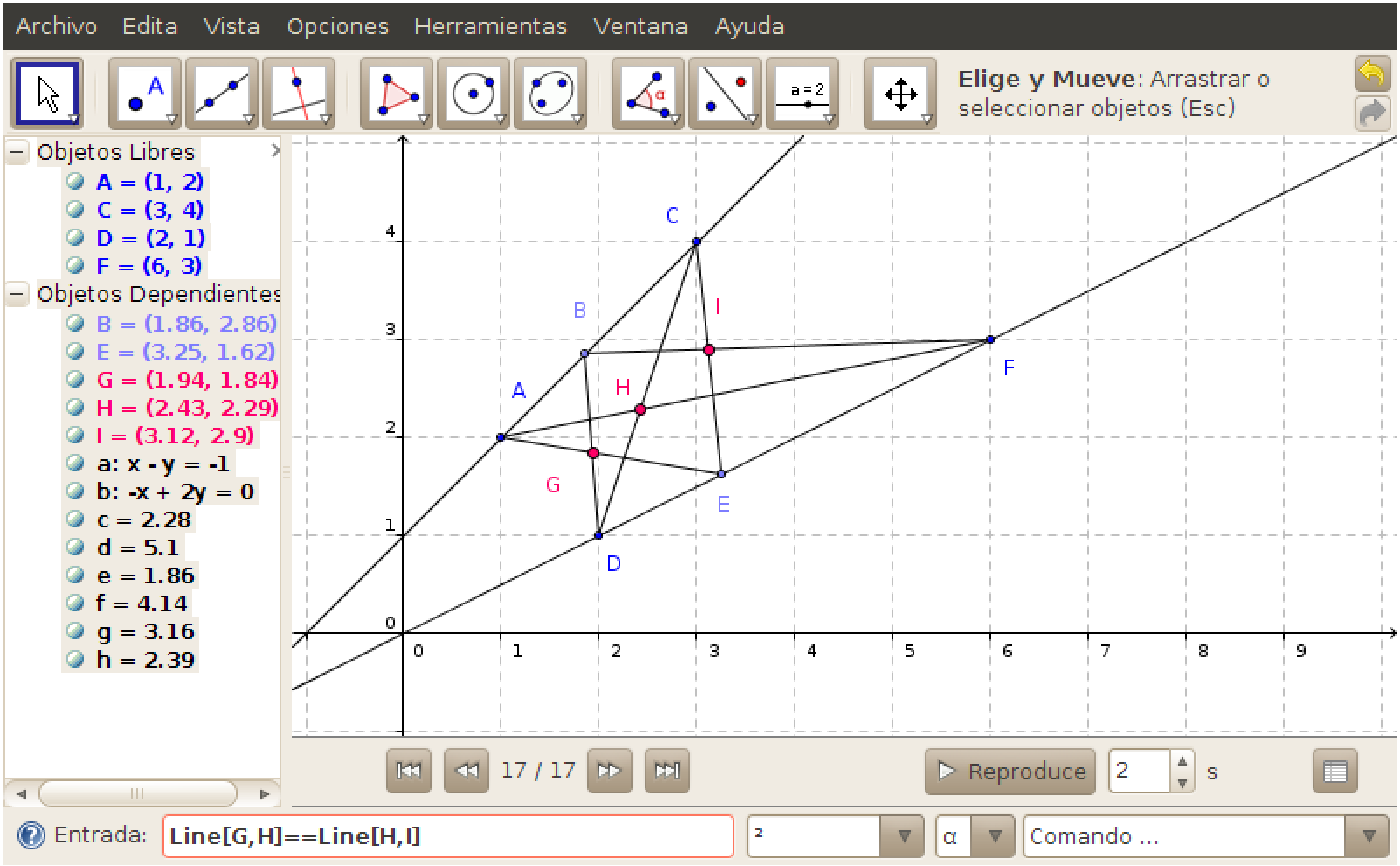

```
Given a construction with points
P4(-469/100,21/100)
P6(-274/100,134/100)
P1(-445/100,-111/100)
P3(210/100,-1)
P5(x[1],x[2])
P2(x[3],x[4])
A(x[5],x[6])
B(x[7],x[8])
C(x[9],x[10])

with constraints
Aligned(P5,P4,P6)
Aligned(P2,P1,P3)
Aligned(A,P5,P1)
Aligned(A,P2,P4)
Aligned(B,P4,P3)
Aligned(B,P6,P1)
Aligned(C,P5,P3)
Aligned(C,P6,P2)

the statement
 Aligned(A,B,C)
is true
```

Proof

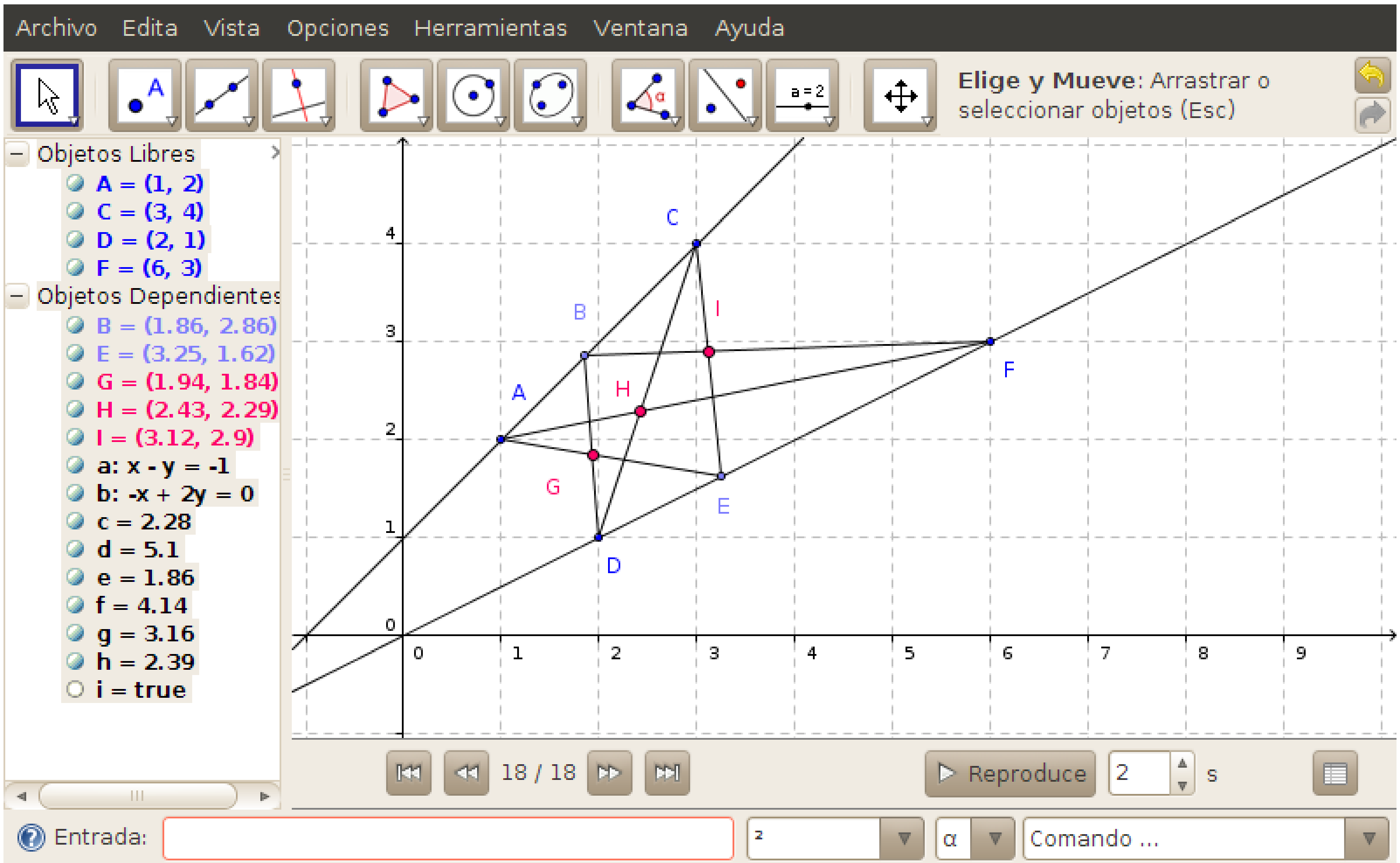


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (1, 2)
C = (3, 4)
D = (2, 1)
F = (6, 3)
Objetos Dependientes
B = (1.86, 2.86)
E = (3.25, 1.62)
G = (1.94, 1.84)
H = (2.43, 2.29)
I = (3.12, 2.9)
a: x - y = -1
b: -x + 2y = 0
c = 2.28
d = 5.1
e = 1.86
f = 4.14
g = 3.16
h = 2.39
A
B
C
D
E
F
G
H
I
17 / 17
Reproduce
2
s
Entrada:
Comando ...

Proof


Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (1, 2)
C = (3, 4)
D = (2, 1)
F = (6, 3)
Objetos Dependientes
B = (1.86, 2.86)
E = (3.25, 1.62)
G = (1.94, 1.84)
H = (2.43, 2.29)
I = (3.12, 2.9)
a: x - y = -1
b: -x + 2y = 0
c = 2.28
d = 5.1
e = 1.86
f = 4.14
g = 3.16
h = 2.39
17 / 17
Reproduce
Entrada: Line[G,H]==Line[H,I]
Comando ...

Proof



Archivo Edita Vista Opciones Herramientas Ventana Ayuda
Elige y Mueve: Arrastrar o seleccionar objetos (Esc)
Objetos Libres
A = (1, 2)
C = (3, 4)
D = (2, 1)
F = (6, 3)
Objetos Dependientes
B = (1.86, 2.86)
E = (3.25, 1.62)
G = (1.94, 1.84)
H = (2.43, 2.29)
I = (3.12, 2.9)
a: x - y = -1
b: -x + 2y = 0
c = 2.28
d = 5.1
e = 1.86
f = 4.14
g = 3.16
h = 2.39
i = true
18 / 18
Reproduce
Entrada:
Comando ...

LADucation 4 GeoGe...

http://nash.sip.ucm.es/LAD/LAducation4ggb/AuxFiles/Pappus-answer.htm

LADucation *for* GeoGebra

Your Geogebra file Pappus.ggb has been successfully uploaded.
It will be processed in 8 seconds.

```
Given a construction with points
A(1.,2.)
C(3.,4.)
D(2.,1.)
F(6.,3.)
B(x[1],x[2])
E(x[3],x[4])
H(x[5],x[6])
G(x[7],x[8])
I(x[9],x[10])

and with constraints
Aligned(B,A,C)
Aligned(E,D,F)
Aligned(H,A,F)
Aligned(H,C,D)
Aligned(G,A,E)
Aligned(G,B,D)
Aligned(I,B,F)
Aligned(I,C,E)

the statement
Aligned(G,H,I)
is true (except in certain degenerate cases)
```

Proof

(roughly) A proposition is declared to be True if

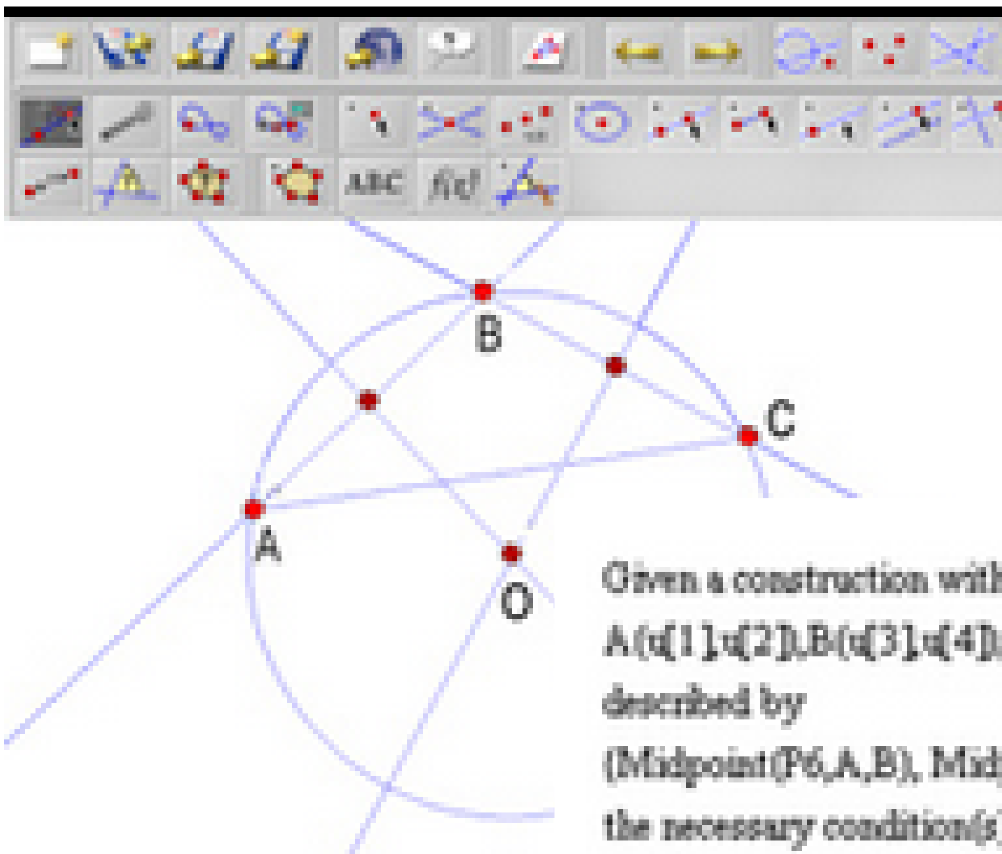
(any component of)

the hypotheses ideal is contained in the thesis ideal.

Be careful, this holds in the complex field, not **always** in the real one.

Sat(H,Sat(H,T)) <> (1)

Discovery



Given a construction with points

A(u[1],u[2]),B(u[3],u[4]),C(u[5],u[6]),P6(x[1],x[2]),P7(x[3],x[4]),O(x[5],x[6]),

described by

{Midpoint(P6,A,B), Midpoint(P7,B,C), Perpendicular(O,P6,A,B), Perpendicular(O,P7,B,C)},

the necessary condition(s) for

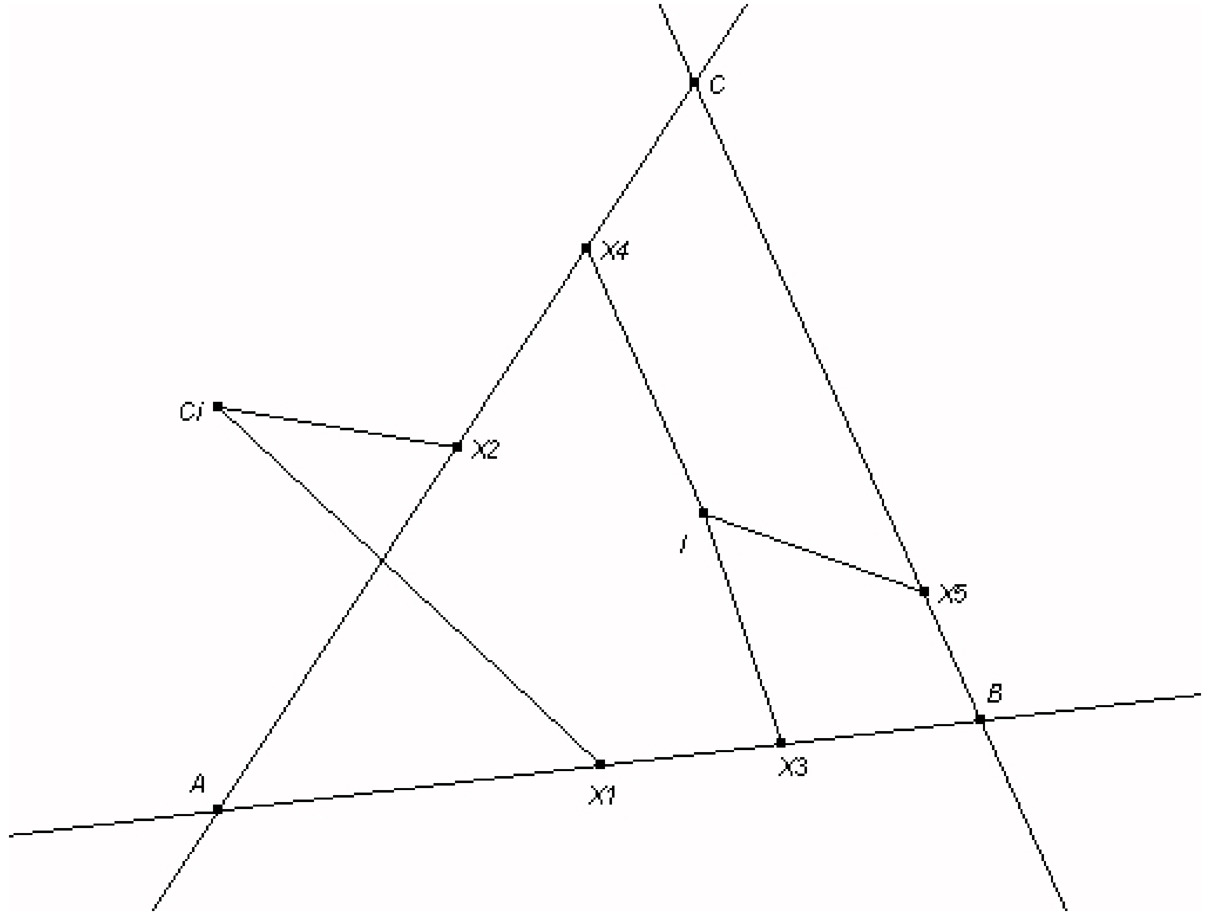
{Aligned(A,O,C)}

is(are)

{Perpendicular(A, B, B, C)} ∨ {Equal(A, C)}

A formula from Euler

concerning the relation between the radii of the incircle, r, and the circumcircle, c, of a triangle, and the distance d between their centers.



webDiscovery - Mozilla

File Edit View Go Bookmarks Tools Window Help

webDiscovery

POWERED BY webMATHEMATICA

Given a construction with points

A(0,0),B(1,0),C(u[5],u[6]),X1(x[1],x[2]),X2(x[3],x[4]),X3(x[5],x[6]),X4(x[7],x[8]) ,X5(x[9],x[10]),I(u[7],u[8]),Ci(u[9],u[10]) ,

described by

{Midpoint(X1,A,B) , Midpoint(X2,A,C), Aligned(X3,A,B), Aligned(X4,A,C), Aligned(X5,B,C) },

the necessary condition(s) for

{Perpendicular(A,B,X1,Ci), Perpendicular(A,C,X2,Ci) , Perpendicular(A,B,I,X3),
Perpendicular(A,C,I,X4), Perpendicular(B,C,I,X5), distance(I,X3)=distance(I,X4), distance(I,X3)=distance(I,X5) }

is(are)

$$2u(9)-1==0 \land u(5)^2-u(5)+u(6)^2-2u(6)u(10)==0 \land (\{\text{Aligned}(A,B,C)\} \lor -u(7)^2+2u(5)u(7)+u(8)^2-u(5)+2u(6)u(8)-4u(8)u(10)==0) \land$$
$$(\{\text{Aligned}(A,B,C)\} \lor 2u(7)u(6)-u(6)-2u(5)u(8)-2u(7)u(8)+2u(8)==0) \land$$
$$(\{\text{Aligned}(A,B,C)\} \lor 2u(8)^3-8u(10)u(8)^2+2u(7)^2u(8)-2u(7)u(8)-2u(8)+u(6)==0) \land$$
$$(\{\text{Aligned}(A,B,C)\} \lor 2u(7)^3-3u(7)^2+2u(8)^2u(7)-8u(8)u(10)u(7)-u(8)^2+u(5)+4u(8)u(10)==0) \land$$
$$(\{\text{Aligned}(A,B,C)\} \lor -4u(8)^4+16u(10)u(8)^3-8u(6)u(10)u(8)^2+4u(8)^2-4u(6)u(8)+u(6)^2==0) \land$$
$$(\{\text{Aligned}(A,B,C)\} \lor -8u(7)u(8)^3+4u(8)^3-16u(5)u(10)u(8)^2+16u(7)u(10)u(8)^2-6u(5)u(8)+2u(7)u(8)+2u(8)+2u(5)u(6)-u(6)==0)$$

Try another Discovery?

d^4 - 2*d^2*c^2 - 4*r^2*c^2 + c^4

Dík
Gracias