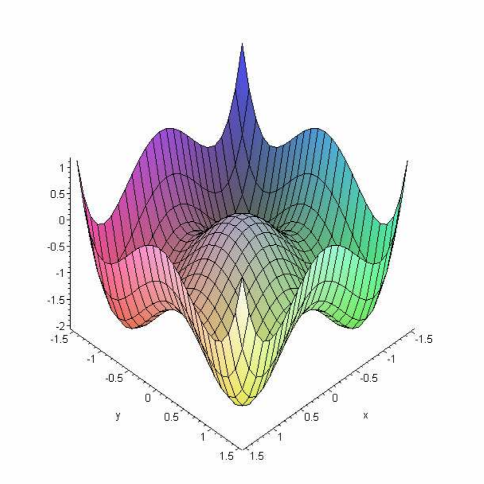
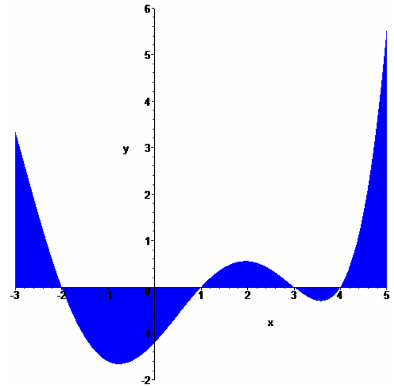
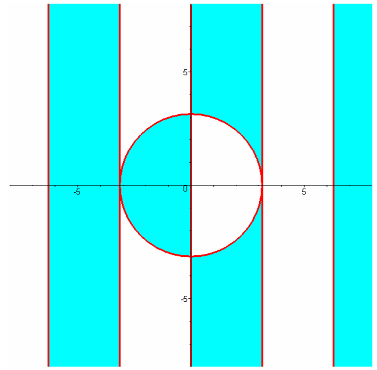



Calculus of one and more variables with Maple

Libuše Samková

Goal setting



- A guide of using Maple
  in teaching fundamental calculus
- Only several Maple commands →
  → suitable for Maple beginners
- Scans advantages and disadvantages of using Maple in relation to students /
  / to a teacher

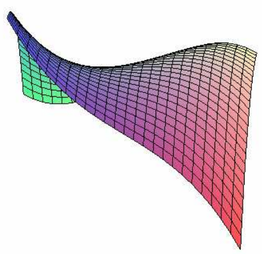
Calculus is a part of mathematics which still insists on student’s hard training --- continuous practice of logical, numerical and even memory skills.

The student has to test all the procedures by himself, to experience the process of calculation.

So, we must be careful in using Maple, we must not suppress the process of student’s training.

What is Maple efficient for ?

- Checking results
- Comparing different forms of results
- Creating a graphical preview of the situation



Checking and comparing numerical or functional results

Situation:

A student solves the problem himself, he lets Maple solve it, and then compare both results.

=

Procedures available:

- apply **simplify(%);** and/or **evalf(%);** command to Maple's result
- while using both commands, be sure to take **simplify(%);** first (to avoid rounding errors):

\> **ln(81/16)/ln(3/2);**

$$\frac{\ln\left(\frac{81}{16}\right)}{\ln\left(\frac{3}{2}\right)}$$

\> **simplify(%);**

$$4$$

\> **evalf(%);**

$$3.999999999$$

- subtract student's result from Maple's result (and expect zero):

> 1/cos(x)^2-1;

$$\frac{1}{\cos(x)^2} - 1$$

Student's result:

$$\tan(x)^2$$

> %-tan(x)^2;

$$\frac{1}{\cos(x)^2} - 1 - \tan(x)^2$$

> simplify(%);

$$0$$

- <u>Procedures for checking results of integration</u>

  (i) subtraction of student's and Maple's result may be any number:

$$\int 2\sin(x)\cos(x)\,dx$$

```
> int(2*sin(x)*cos(x),x);
```

$$-\cos(x)^2$$

Student's result:

$$\sin(x)^2$$

```
> simplify(%-sin(x)^2);
```

$$-1$$

(ii) the derivative of student's result should equal the integrand:

$$\int \frac{1}{x\sqrt{x+1}}\,dx$$

> int(1/(x*sqrt(1+x)),x);

$$-2\,\mathrm{arctanh}(\sqrt{1+x})$$

Student's result:

$$\ln(\sqrt{x+1}-1)-\ln(\sqrt{x+1}+1)$$

> diff(ln(sqrt(x+1)-1)-ln(sqrt(x+1)+1),x);

$$\frac{1}{2\sqrt{1+x}\,(\sqrt{1+x}-1)}-\frac{1}{2\sqrt{1+x}\,(\sqrt{1+x}+1)}$$

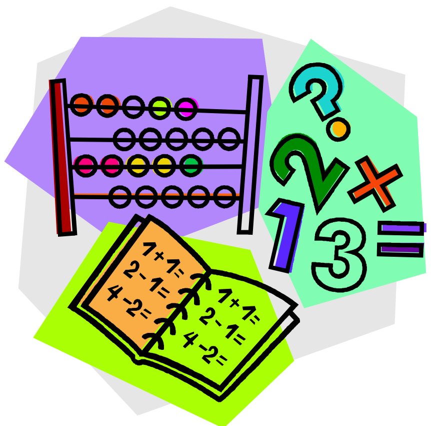
> simplify(%);

$$\frac{1}{x\sqrt{1+x}}$$

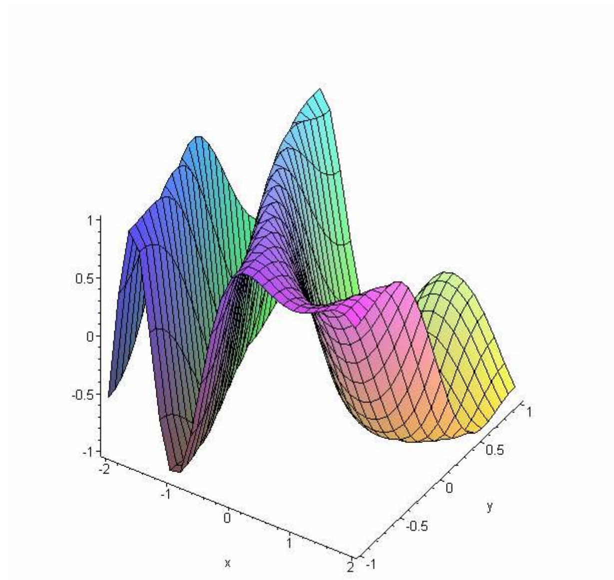
Graphical preview

Situation:

A student lets Maple draw
a picture of a given problem.



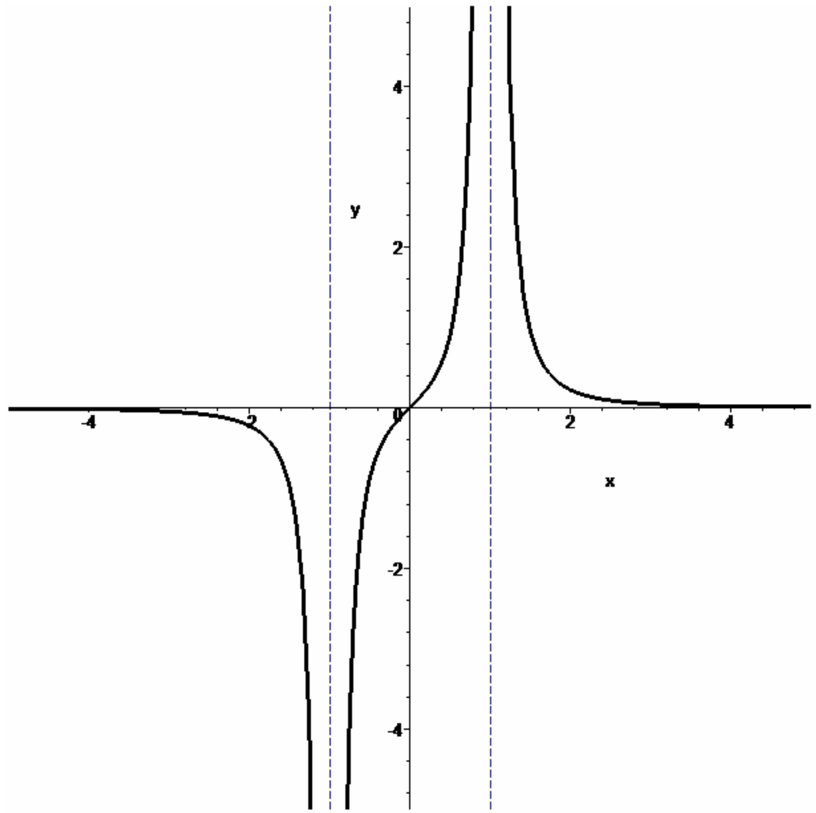
=>



- a graph of a function, with asymptotes:

$$f := \frac{x}{(1 - x^2)^2}$$

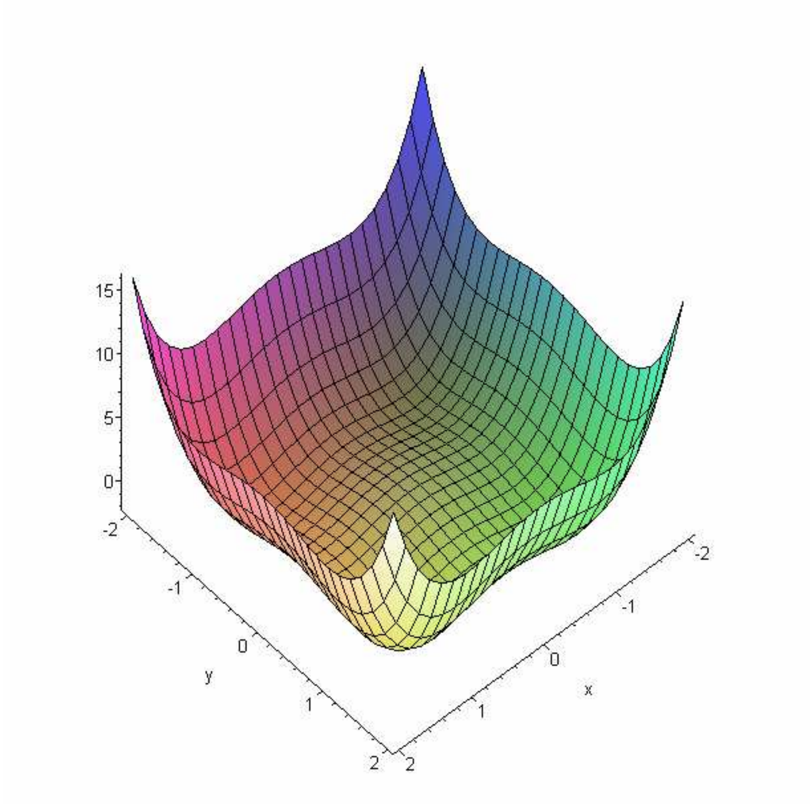
> **p1:=plot(f,x=-5..5,y=-5..5,color=black,**
> **thickness=3,discont=true,**
> **scaling=constrained):**
> **p2:=plot([-1,t,t=-5..5],color=navy,**
> **linestyle=3,numpoints=2):**
> **p3:=plot([1,t,t=-5..5],color=navy,**
> **linestyle=3,numpoints=2):**
> **display(p1,p2,p3);**

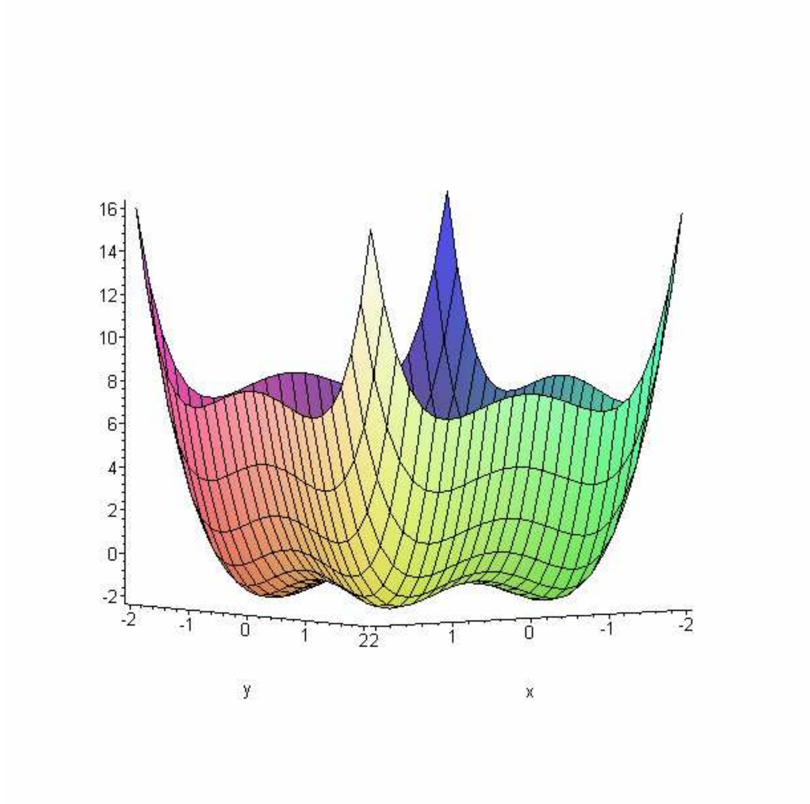


- a graph of a function of two variables:

$$F := x^4 - 2\,x^2 + y^4 - 2\,y^2$$

> **plot3d(F,x=-2..2,y=-2..2);**



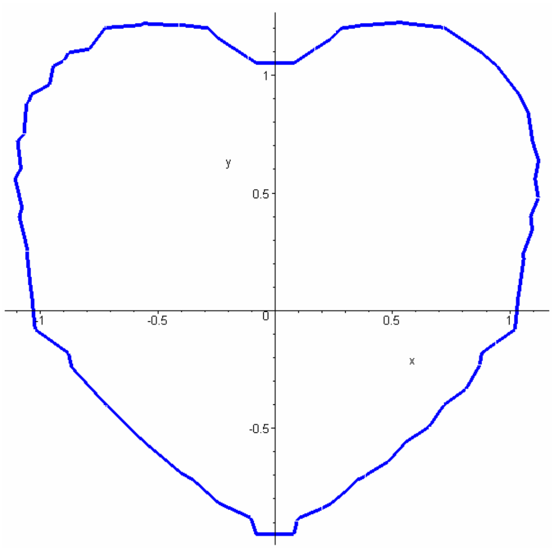


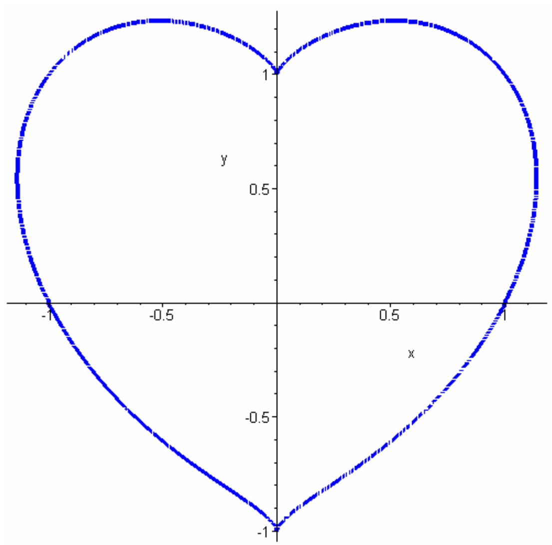
- a picture of a plane curve (an implicit function):

$$(x^2+y^2-1)^3 = x^2\,y^3$$

> **implicitplot((x^2+y^2-1)^3 =x^2*y^3,x=-2..2,y=-2..2, scaling=constrained,thickness=4);**

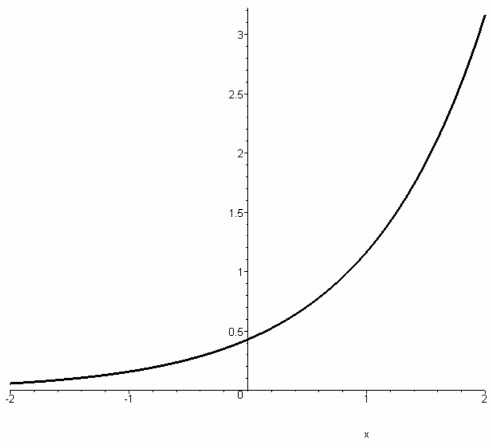
add **numpoints=100000**





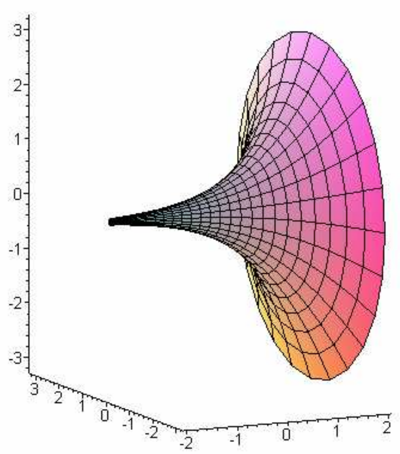
- a picture of a plane curve rotating about the *x*-axis:

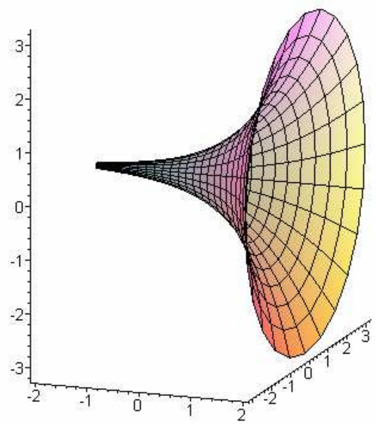
$$f := \frac{3}{7}\mathbf{e}^{x}$$



> plot3d([x,f*cos(t),f*sin(t)], x=-2..2,

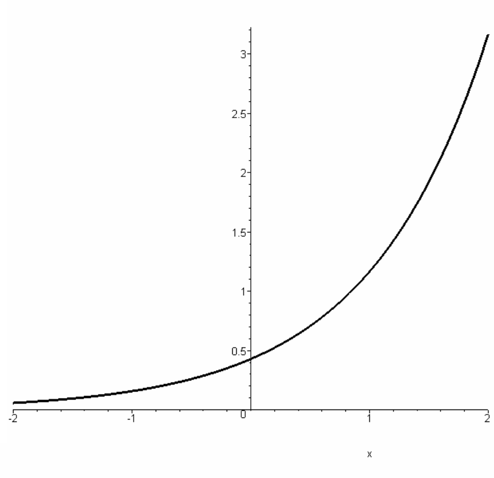
t=0..2*Pi, scaling=constrained);



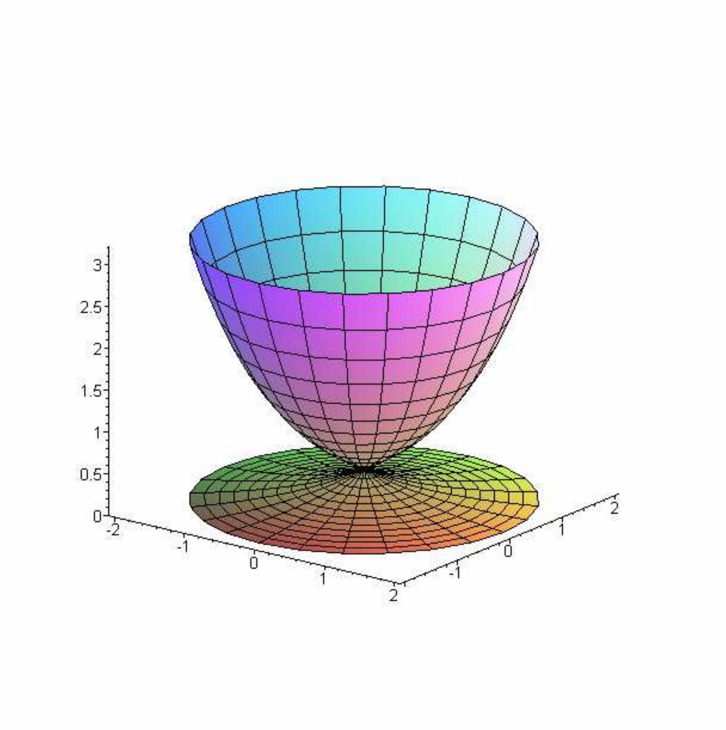


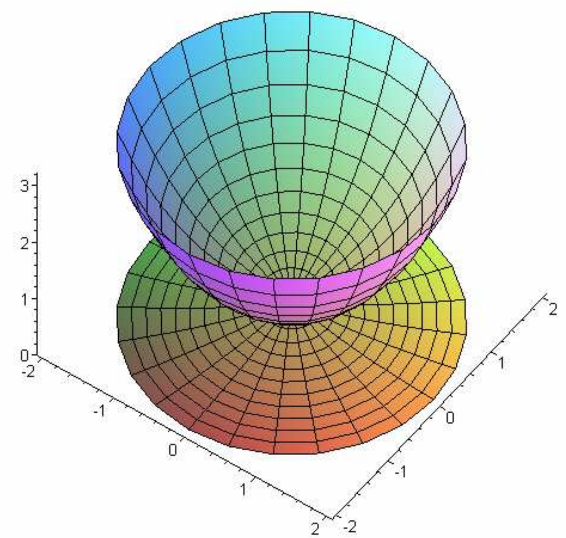
- a picture of a plane curve rotating about the *y*-axis:

$$f := \frac{3}{7}\,\mathbf{e}^{x}$$



```
> plot3d([x*cos(t),x*sin(t),f], x=-2..2,
  t=0..2*Pi, scaling=constrained);
```

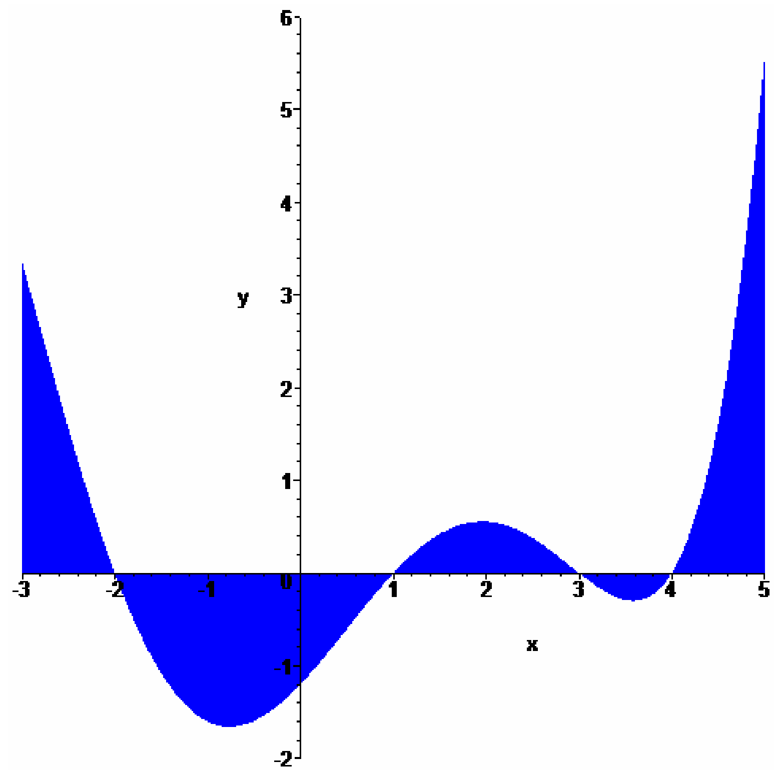




- an area between a graph of a function and the *x*-axis:

$$f := \frac{(x+2)(x-1)(x-3)(x-4)(x+5)}{100}$$

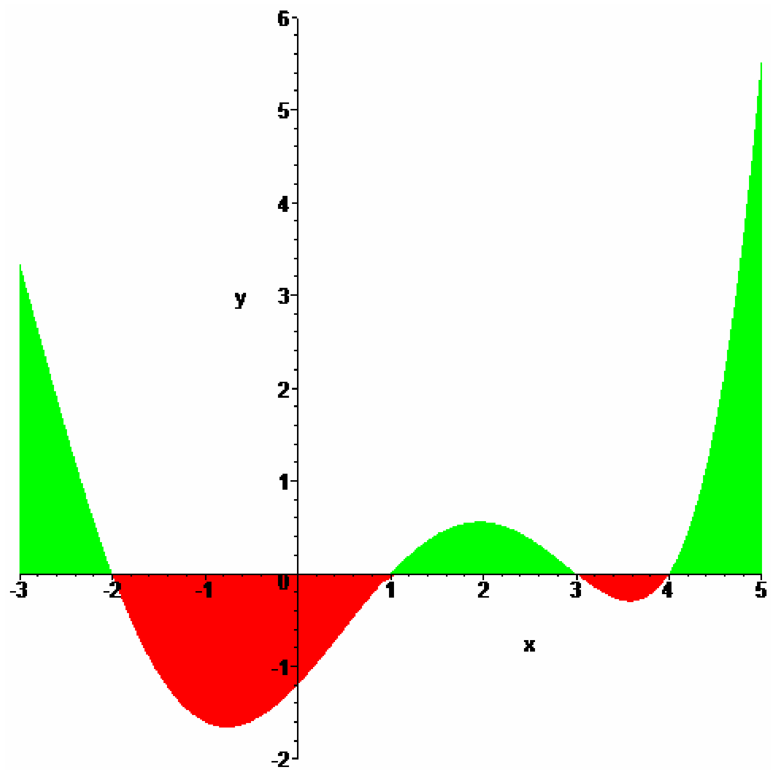
```
> plot(f,x=-3..5,y=-2..6,
  scaling=constrained,
  color=blue,thickness=3,
  filled=true);
```



- the value of a definite integral of a function:

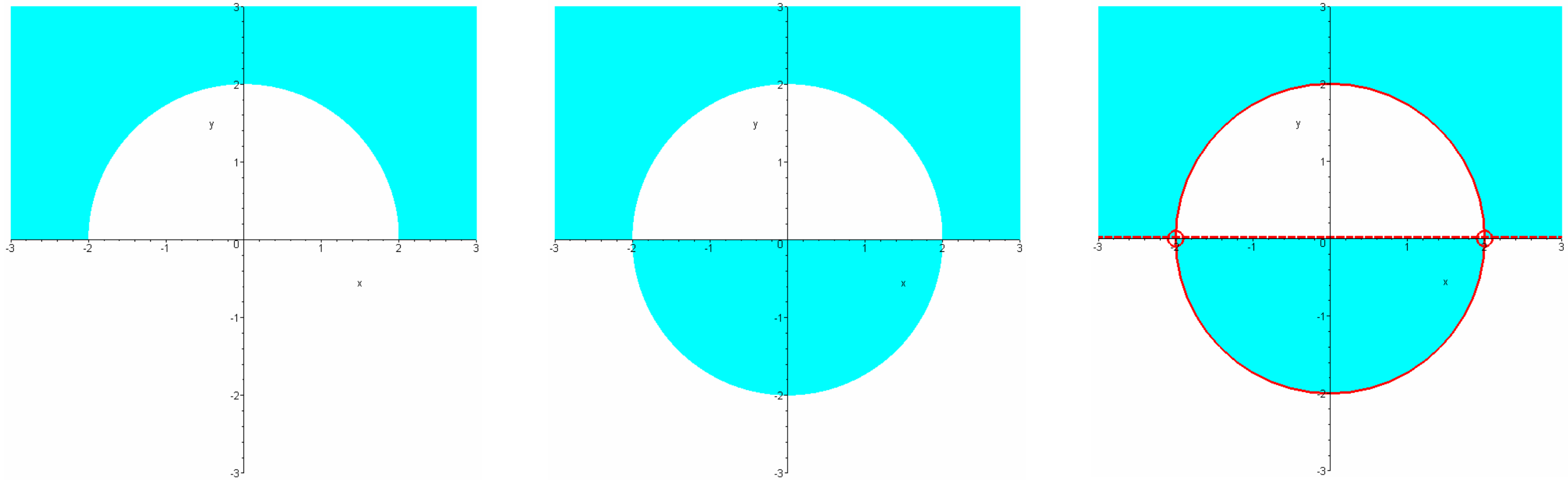
$$\int_{-3}^{5} f \, dx \qquad \text{for a function} \qquad f := \frac{(x+2)(x-1)(x-3)(x-4)(x+5)}{100}$$

```
> p1:=plot(max(f,0),x=-3..5, y=-2..6,
  scaling=constrained,
  thickness=3,filled=true,
  color=green):
> p2:=plot(min(f,0),x=-3..5, y=-2..6,
  scaling=constrained,
  thickness=3,filled=true,
  color=red):
> display(p1,p2);
```



- a domain of a function of two variables (this topic needs additional calculations; it is suitable for preparation of teaching materials):

$$F := \sqrt{\frac{x^2 + y^2 - 4}{y}}$$



The main principle (the **plot** command together with **filled=true** and **color**): once filled area is no longer possible to be re-filled.

- a tangent plane of a two-dimensional manifold
(another topic suitable for teachers and their materials)

$z = \sqrt{-x^2 - y^2 + 4}$ at the point $[1, 1, \sqrt{2}]$

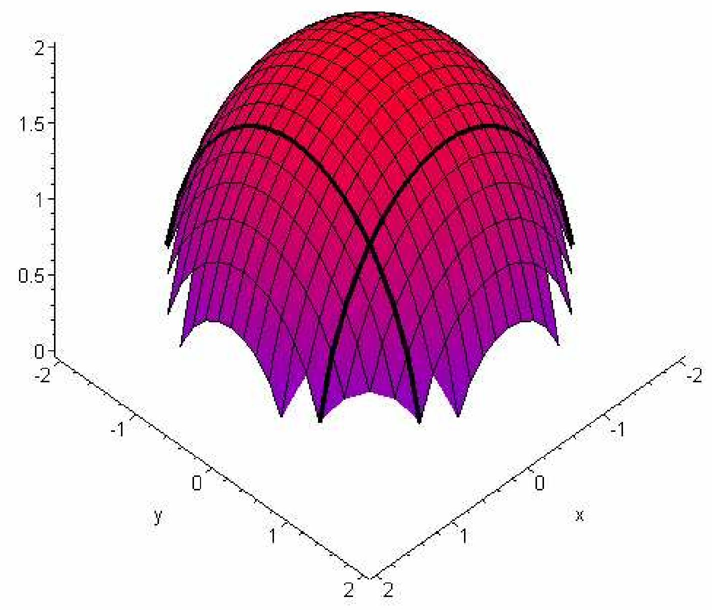
Intersections of the manifold
and planes *x*=1, *y*=1:

> **p1:=plot3d(sqrt(4-x^2-y^2),x=-2..2, y=-2..2):**

> **p2:=plot3d(sqrt(3-y^2),x=1..1.01, y=-2..2,thickness=3):**

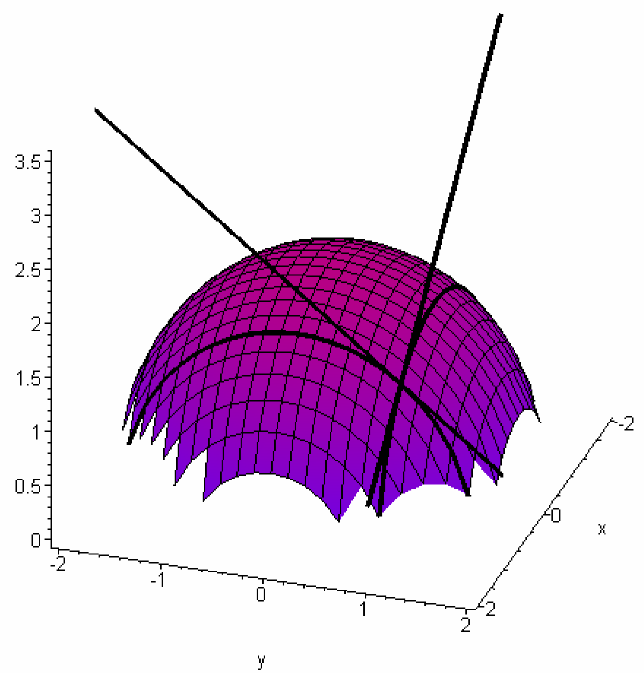
> **p3:=plot3d(sqrt(3-x^2),x=-2..2, y=1..1.01,thickness=3):**

> **display(p1,p2,p3);**



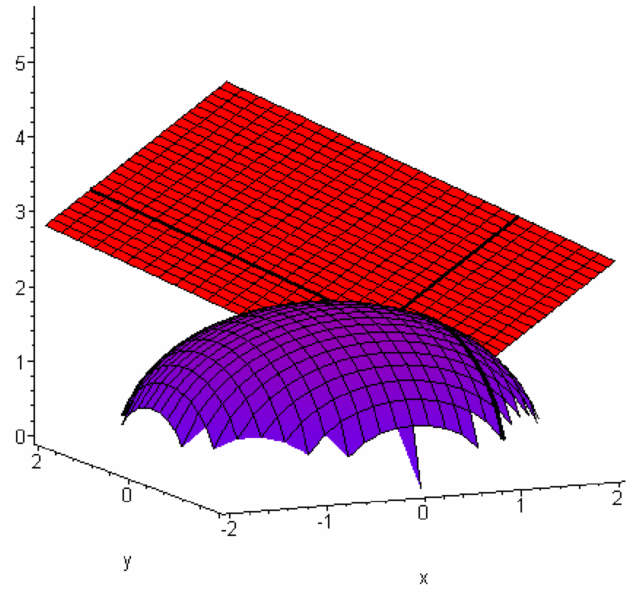
Tangent lines belonging
to these intersections:

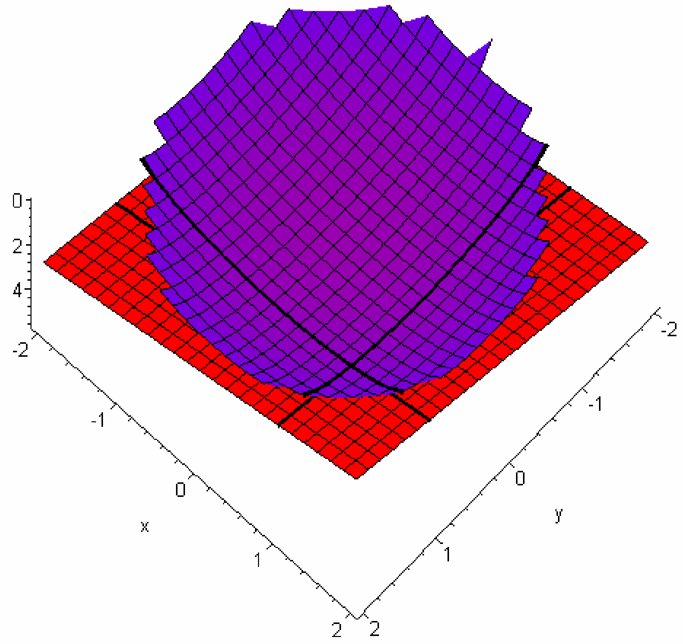
```
> p4:=plot3d((3-y)/sqrt(2),
    x=1..1.01,y=-2..2,thickness=3):
> p5:=plot3d((3-x)/sqrt(2),
    x=-2..2,y=1..1.01,thickness=3):
> display(p1,p2,p3,p4,p5);
```



The tangent plane:

```
> p6:=plot3d((4-x-y)/sqrt(2),x=-2..2,y=-2..2,color=red):
> display(p1,p2,p3,p4,p5,p6);
```



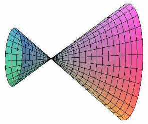


$\tan(x)^2$

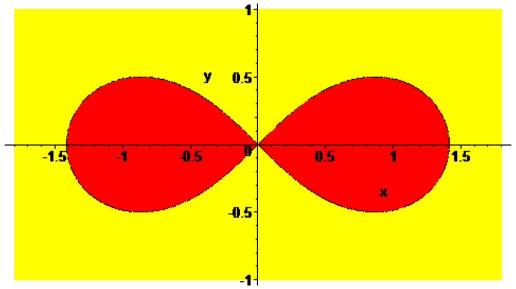
Conclusion

$\frac{1}{\cos(x)^2} - 1$

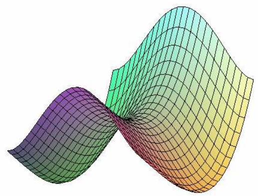
- Checking and comparing numerical and functional results
  - we must not replace the process of training calculations by the process of waiting for Maple’s result
  - very important information not available from Maple: indication of difficulty → → the teacher has to calculate all the examples manually to sort them properly

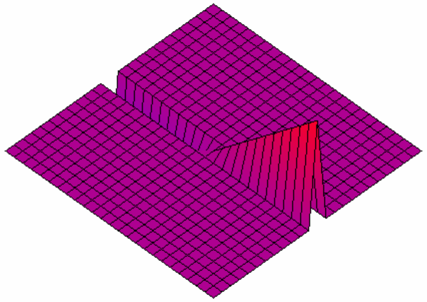


- Graphical previews

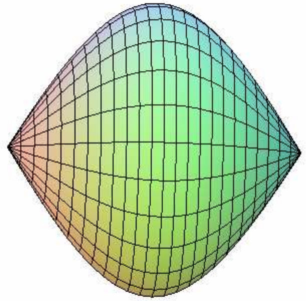


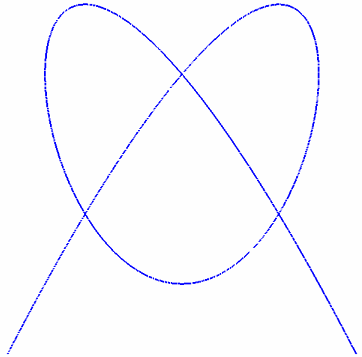
  - for students: a good opportunity to get a general idea of the situation studied, especially in 3D
  - for teachers: valuable tool for preparation of teaching materials





$$\ln(\sqrt{x+1}-1)-\ln(\sqrt{x+1}+1)$$





$$\int_{-3}^{5} f\,dx$$