



ZÁPADOČESKÁ  
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# A Gentle Introduction to Geometric Modelling via Dynamic Geometry

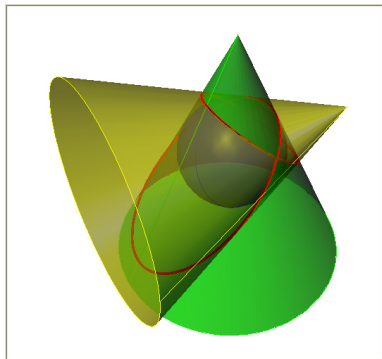
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KMA FAV ZČU Plzeň

2010

**CADGME 2010**

# Content



- ▶ Courses of Geometry, Tools of dynamic geometry
- ▶ Typical tasks for demonstration via dynamic geometry
  - ▶ Modelling of curves (de Casteljau, de Boor algorithms)
  - ▶ Modelling of surfaces and typical problems (surfaces of revolution, intersection of surfaces, helicoidal surfaces, envelope surfaces)

# Courses of Geometry

- ▶ Mathematics for six faculties or institutes
- ▶ 19 courses of geometry
- ▶ Modernisation and increasing of electronic support
- ▶ <http://geometrie.kma.zcu.cz/>

Oddělení Geometrie - Katedra matematiky ZČU

<http://geometrie.kma.zcu.cz/index.php?www/content/view/full/279/>

**ODDĚLENÍ GEOMETRIE**

**KATEDRA MATEMATIKY**

www.kma.zcu.cz

Geometrie / Materiály pro studenty / Materiály podle přednášky / APG1

**APG1**

APLIKACE GEOMETRIE I

**Harmonogram výuky**  
Program pro LS 2009/010, podmínky k udělení zápočtu a ke složení zkoušky - [Stáhnout](#) 69.01 kB [application/pdf]

**Minkowského suma**  
Promítané materiály - [Stáhnout](#) 1.01 MB [application/pdf]

**Voroného diagramy**  
Slajdy k přednášce - [Stáhnout](#) 440.23 kB [application/pdf]

**Kinematická geometrie**  
Promítané slajdy a soubory pro tvorbu obrázků - [Stáhnout](#) 274.96 kB [application/x-zip-compressed]

**Teorie offsetů**  
Slajdy k přednášce - [Stáhnout](#) 2.41 MB [application/pdf]

**Epipolární geometrie**  
Slajdy k přednášce - [Stáhnout](#) 4.15 MB [application/vnd.ms-powerpoint]

**Kvaterniony**  
Slajdy k přednášce - [Stáhnout](#) 778.52 kB [application/pdf]

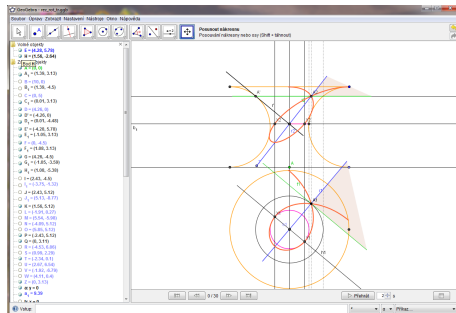
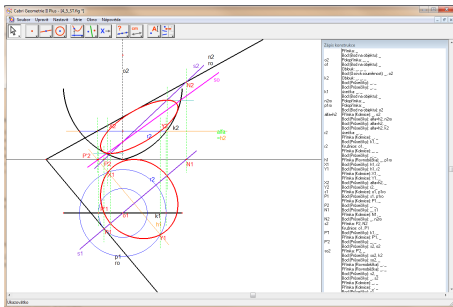
**Isogeometrická analýza**

**Pracovníci**  
**Kontakt**  
**Materiály pro studenty**  
**Materiály podle přednášky**

- APG1
- APG2
- AXG
- DEG1
- DEG2
- DEG3
- DG
- G1
- G2
- G3
- GE
- GPM4
- GS1
- GS2
- GVS
- IPO
- ITG
- NGM
- SG
- VKG
- ZKG

# Tools of Dynamic Geometry

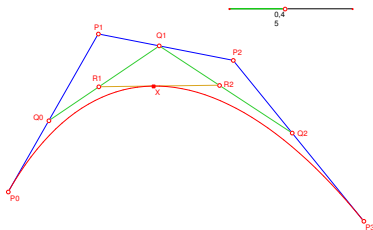
Cabri, Sketchpad, Cinderella, **GeoGebra**, Mathematica, Maple, RhinoCeros, ...



# Modelling of Bézier Curves

## The de Casteljau Algorithm

method for evaluating the point on a Bézier and rational Bézier curve corresponding to the parameter value  $t \in \langle 0, 1 \rangle$



Bézier curve given by four control points  $P_0, P_1, P_2, P_3$

- ▶ Linear interpolation to obtain three new points

$$Q_0(t) = (1 - t)P_0 + tP_1$$

$$Q_1(t) = (1 - t)P_1 + tP_2$$

$$Q_2(t) = (1 - t)P_2 + tP_3$$

- ▶ Linear interpolation to obtain two new points

$$R_0(t) = (1 - t)Q_0 + tQ_1$$

$$R_1(t) = (1 - t)Q_1 + tQ_2$$

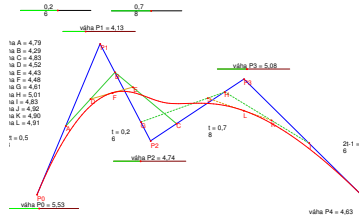
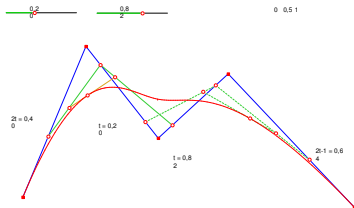
- ▶ Final calculation of the curve point

$$X(t) = (1 - t)R_0 + tR_1$$

# Modelling of Curves

## The De Boor Algorithm

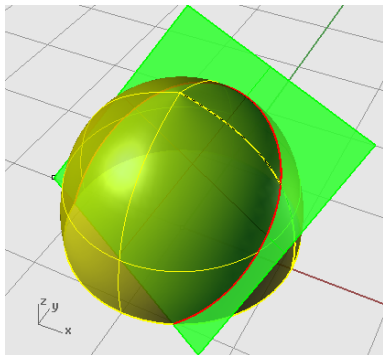
### Evaluation of points on a B-spline curve and NURBS curve



# Surfaces of Revolution

Intersection of surface of revolution and plane

Construct the intersection of surface of revolution and plane  $\sigma$

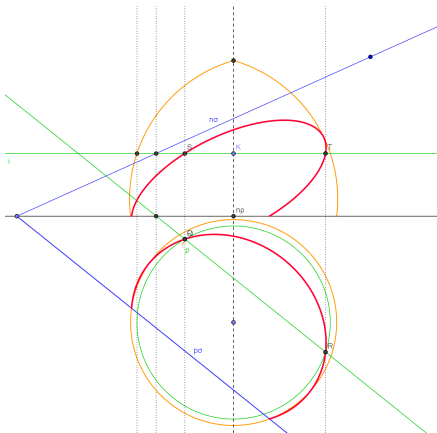


- ▶ choice of auxiliary plane  $\alpha : \alpha \parallel \pi$
- ▶ line  $h$ : the intersection of the plane  $\alpha$  and  $\sigma$
- ▶ circle  $k(S, r)$ : the intersection of the plane  $\alpha$  and surface of revolution
- ▶ points  $X, Y$ : intersection of  $h$  and  $k$
- ▶ choice of new plane  $\alpha'$

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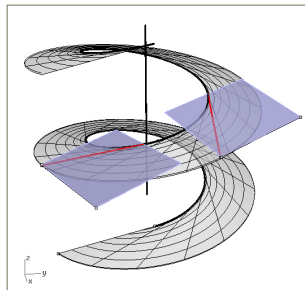
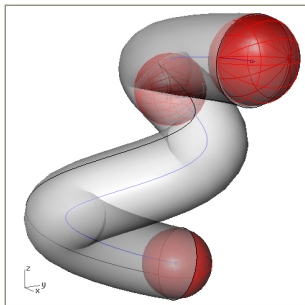
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# Envelope Surfaces

## The characteristic curve

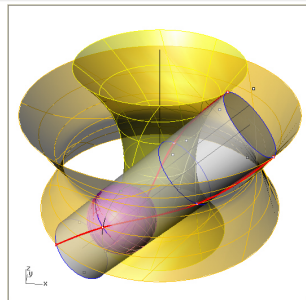
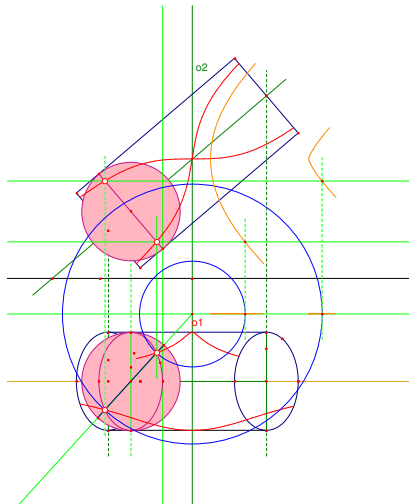
Let us a one-parametric system  $\kappa(t)$  of surfaces. **Envelope surface** of this system is the surface which contacts every surface of  $\kappa(t)$  along a curve  $k(t)$ . This contact curve is called **the characteristic curve** of the system.



# Envelope Surfaces generated by revolution of a surface of revolution

## The characteristic curve

- ▶ Construction of auxiliary sphere  $\kappa$  which contacts  $\kappa$  along  $k$ .
- ▶ Construction of characteristic curve  $c$  of the sphere  $\kappa$ .
- ▶ Intersection points  $X = k \cap c$ .
- ▶ Construction of new auxiliary sphere.





Thank you for your attention