MORTGAGE CREDIT

Credit repayment with identical instalments – fixed annuity

TASK 2: A young married couple from example 1 considers another alternative for paying of the debt in the amount of CZK 2,500.000.00. They consider mortgage credit with the same interest rate, i.e. 4.9% but the date of maturity is extended from 20 years to 30 years. The married couple takes out life insurance when in reaching a specified age (after 20 years from signature of the policy) they will receive CZK 980,600.00. They assume that after 20 years with this amount they will repay a part of the debt. Determine

a) the amount of yearly overdue annuity instalment of mortgage credit,

b) which of the above alternatives is a better one, whether the alternative with the date of maturity 20 years or the alternative with the date of maturity 30 years and repayment of a part of the credit by means of life insurance.

Solution:

ad a) The amount of yearly overdue annuity instalment and curtail schedule

Equation $\boldsymbol{Dl = a\cdot v + a\cdot v^2 + a\cdot v^3 + ... + a\cdot v^n}$ holds true, where $\boldsymbol{v = \frac{1}{1+i}}$ is discount factor, ***Dl*** initial credit amount, ***a*** annuity, ***i*** annual interest rate in percentage expressed by decimal number.

On the right side of equation there is finite geometric progression with ratio *v* and with extreme *av*. After the application of formula for its total we get equation in form $Dl = \frac{a\cdot(1-v^n)}{i}$,

where $\frac{(1-v^n)}{i}$ is so-called overdue supplier.

Then relation $a = \frac{Dl\cdot i}{(1-v^n)}$ holds true for annuity a.

Equation adjustment in Maple

```
> restart;
> Dl := a*v*(Sum(v^i, i = 0 .. n-1));
```

$$Dl := a\, v \left(\sum_{i=0}^{n-1} v^i\right) \quad (1.1)$$

```
> a*v*(Sum(v^i, i = 0 .. n-1)) = a*v*(sum(v^i, i = 0 .. n-1));
```

$$a\, v \left(\sum_{i=0}^{n-1} v^i\right) = a\, v \left(\frac{v^n}{v-1} - \frac{1}{v-1}\right) \quad (1.2)$$

> Dl := factor(a*v*(sum(v^i, i = 0 .. n-1)));

$$Dl := \frac{a\,v\,(v^n - 1)}{v - 1} \tag{1.3}$$

Solution step by step:	
1. We define the functions complying with equation parameters	*restart*; **Discount factor:** $v := i \to \frac{1}{1+i}$: **Overdue supplier:** $ani := (n, i) \to \frac{(1 - v(i)^n)}{i}$: **Annuity:** $an := (Dl, i, n) \to \frac{Dl \cdot i}{(1 - v(i)^n)}$:
2. We set specific values of input variables of the task	**initial credit amount:** $Dl := 2500000$: **payment period (number of instalments):** $n := 30$: **annual interest rate:** $i := 0.049$:
3. We calculate annuity	$Annuity := an(Dl, i, n) =$ 160779,47 Kč

4. We form curtail schedule

The table of curtail schedule is represented as a matrix

For simplification we lay the value of annuity into variable a: $a := an(Dl, i, n)$;

160779,47 **(1)**

$UP := matrix([['period', 'annuity','interest','amortization', 'balance'], [0, ``, ``, ``, Dl], seq( [j, a, a*(1 - v(i)^(n-j+1)), a*v(i)^(n-j+1), a*ani(n-j, i)], j = 1..n-1), [n, a, a*(1 - v(i)), a*v(i), ``], [``, ``, ``, ``, ``], [``, n*a, n*a - Dl, Dl, ``]]);$

period	*annuity*	*interest*	*amortization*	*balance*
,00				2500000,00
1,00	160779,47	122500,00	38279,47	2461720,53
2,00	160779,47	120624,31	40155,17	2421565,36
3,00	160779,47	118656,70	42122,77	2379442,59
4,00	160779,47	116592,69	44186,79	2335255,80
5,00	160779,47	114427,53	46351,94	2288903,86
6,00	160779,47	112156,29	48623,18	2240280,68
7,00	160779,47	109773,75	51005,72	2189274,96
8,00	160779,47	107274,47	53505,00	2135769,96
9,00	160779,47	104652,73	56126,75	2079643,21
10,00	160779,47	101902,52	58876,96	2020766,25
11,00	160779,47	99017,55	61761,93	1959004,33
12,00	160779,47	95991,21	64788,26	1894216,07
13,00	160779,47	92816,59	67962,89	1826253,18
14,00	160779,47	89486,41	71293,07	1754960,11
15,00	160779,47	85993,04	74786,43	1680173,68
16,00	160779,47	82328,51	78450,96	1601722,72
17,00	160779,47	78484,41	82295,06	1519427,66
18,00	160779,47	74451,95	86327,52	1433100,14
19,00	160779,47	70221,91	90557,57	1342542,58
20,00	160779,47	65784,59	94994,89	1247547,69
21,00	160779,47	61129,84	99649,64	1147898,05
22,00	160779,47	56247,00	104532,47	1043365,58
23,00	160779,47	51124,91	109654,56	933711,02
24,00	160779,47	45751,84	115027,63	818683,39
25,00	160779,47	40115,49	120663,99	698019,40
26,00	160779,47	34202,95	126576,52	571442,88
27,00	160779,47	28000,70	132778,77	438664,10
28,00	160779,47	21494,54	139284,93	299379,17
29,00	160779,47	14669,58	146109,89	153269,28
30,00	160779,47	7510,19	153269,28	
	4823384,20	2323384,20	2500000,00	

(2)

Conclusion:
The credit amount is $u := UP[34, 3] = 2323384,20$ CZK. This means that for borrowing of CZK 2, 500,000.00 we will pay almost the same amount.

Notes

1. In the last line of the table of curtail schedule we can see how much we paid for respective credit on interest. It is interesting to watch how this amount depends on the term of credit reimbursement.

2. The payment period of mortgage credits is 5 – 30 years; we can pay back the credits in five-year cycles only.

Example: Dependence of amount of paid interest on term of reimbursement

Table:

matrix([['*term of reimbursement*' , *seq*(*j*, *j* = 5 ..30, 5)], ['*interest*', *seq*(*j* * *an* (*Dl*, *i*, *j*) − *Dl*, *j* = 5 ..30, 5)]]);

$$\begin{bmatrix} \textit{term of reimbursement} & 5{,}00 & 10{,}00 & 15{,}00 & 20{,}00 & 25{,}00 & 30{,}00 \\ \textit{interest} & 379208{,}54 & 721910{,}37 & 1088456{,}55 & 1478180{,}88 & 1890187{,}49 & 2323384{,}20 \end{bmatrix} \quad \textbf{(3)}$$

Solution:
ad b) Which of the above alternatives is a better one, whether the alternative with the date of maturity 20 years or the alternative with the date of maturity 30 years and repayment of a part of the credit by means of life insurance.

the date of maturity 20 years		**the date of maturity 30 years**	
Paid interest (see solution Task 1, curtail schedule)	1478180.88 $1.47818088\ 10^6$ **(4)** CZK.	**Paid interest** - the date of maturity 20 years	$\sum_{l=3}^{22} UP[l, 3] =$ 1963137,16 CZK
Balance	0 CZK	**Balance** after 20 years of repayment (see curtail schedule)	1247547.69 CZK
Difference of interest paid after 20 years	1963137.16 − 1478180.88 = 484956,28 CZK		

Conclusion:
From the given table we can see that the mortgage credit with the date of maturity 20 years is better than the one with the date of maturity 30 years.