

$\frac{am + ay}{\sqrt{(x^2 + m^2 - 2my) + y^2}}$ . Est vero:  $FQ^2 = \frac{y^2 + \frac{y^2 x^2}{(m-y)^2}}{\sqrt{(x^2 + m^2 - 2my) + y^2}}$ ; &  $FQ = \frac{y}{m-y}$ . Hinc:  $EQ = \frac{\sqrt{(x^2 + m^2 - 2my) + y^2}}{\sqrt{(x^2 + m^2 - 2my) + y^2} - am + ay}$ . Igitur ex conditione Problematis:  $(x^2 + m^2 - 2my) + y^2 - am + ay = am - ay^2$ ; Et:  $\frac{am^2 - mx^2 - m^3}{a + m} = y^2 - 2m^2y$ . Si utrique aequationis membro addatur  $\frac{m^2}{(a + m)^2}$ , &  $y - \frac{m^2}{a + m}$  fiat =  $z$ , obtinetur demum:  $z^2 = \frac{a^2 m^2}{(a + m)^2} - \frac{mx^2}{a + m}$ ; quæ est aequatio ad Ellipsum; cuius semiaxis minor est  $= \frac{am}{a + m}$  major  $= \sqrt{\frac{a^2 m}{a + m}}$ .

**45. SCHOLION:** Cum linea  $AF$ , secta in  $E$ , debeat ea lege secari in  $Q$ , ut sit  $A F : E Q = A E : Q F$ , patet, rectam  $AF$  secari harmonice.

#### PROBLEMA 34.

**46.** Data recta infinita  $AM$  (Fig. 25.), & puncto fixo  $Q$ , ductaque  $QN$  normali ad  $AM$ , & quavis  $AQ$  producta in  $O$ , ut sit  $QO$  aequalis quartæ proportionali ad  $\sqrt{4AQ^2 - AN^2}$ ,  $2BQ$ , &  $QN$ , invenire locum punctorum  $O$ .

*Solutio:* Sit  $NM = x$ ;  $MO = y$ ;  $QN = a$ ;  $AN = z$ ; Erit:  $z: a = z + x: y$ ; & ex conditione

ditione Problematis:  $QO = \frac{2a\sqrt{(z^2 + a^2)}}{\sqrt{(3z^2 + 4a^2)}}$ ;

Igitur  $z: x = \sqrt{(z^2 + a^2)}: \frac{2a\sqrt{(z^2 + a^2)}}{\sqrt{(3z^2 + 4a^2)}}$ .

Sed ex prima proportione eruitur  $z = \frac{ax}{y-a}$ ; Est ergo:

$\frac{ax}{y-a} : x = a\sqrt{(3x^2 + 4y^2 - 8ay + 4a^2)}: 2a$ .

Proinde:  $a: 1 = \sqrt{(3x^2 + 4y^2 - 8ay + 4a^2)}: 2a$ ;

Et:  $y^2 - 2ay + a^2 = a^2 - \frac{2}{3}x^2$ . Fiat  $y - a = u$ ;

erit:  $u^2 = a^2 - \frac{2}{3}x^2$ ; quæ est aequatio ad Ellipsum;

in qua semiaxis minor est  $a$ ; major  $= 2\sqrt{\frac{a^2}{3}}$ .

#### PROBLEMA 35.

**47.** Si in circulo fixa sit diameter  $MP$ , (Fig. 32.) & radii circuli  $AB$  producantur, ut ducta ad  $MP$  normali  $BQ$ ,  $BC$  semper sit quarta proportionalis ad  $MO$ ,  $AO$ , & radium, seu: ad finum verum, cosinum, & radium, invenire locum punctorum  $C$ .

*Solutio:* Ducta  $CD$  normali ad  $MP$ , sit  $AD = x$ ;  $DC = y$ ;  $AB = r$ ;  $OM = z$ ; erit:  $r = z$ ;  $OB = z$ ; hinc:  $OB = ry - zy = \sqrt{(2rz - z^2)}$ .

Inde eruitur  $z = \frac{rx}{\sqrt{(y^2 + x^2)}}$ . Est vero  $AO = \frac{rx}{\sqrt{(y^2 + x^2)}}$ .

Igitur ex conditione Problematis:

$r = \frac{rx}{\sqrt{(y^2 + x^2)}} : \frac{rx}{\sqrt{(y^2 + x^2)}} = r : BC$ .

C 5 Proinde:

*Proinde:*  $BC = \sqrt{(x^2 + y^2)} - r$ . Et:  $r: \frac{rx}{\sqrt{(y^2 + x^2)}}$   $= \sqrt{(x^2 + y^2)}: \sqrt{(x^2 + y^2)} - r$ ; seu:  $\sqrt{(y^2 + x^2)}: x = \sqrt{(x^2 + y^2)}: \sqrt{(x^2 + y^2)} - r$ . Itaque:  $x = \sqrt{(x^2 + y^2)} - r$ ; ac demum:  $y^2 = r^2 + 2rx$ ; quæ est aequatio ad Parabolam, cuius parameter aequalis est diametro circuli dati.

#### PROBLEMA 36.

**48.** Datis infinitis Parabolis (Fig. 33.), communem verticem  $A$ , & axem  $AC$  habentibus, datoque in communis omnium Paraboliarum tangente  $AG$  puncto  $G$ , invenire semianum minimarum linearum, quæ a puncto  $G$  ad Paraboliarum perimetros duci possunt.

*Solutio:* Sit  $GD$  omnium minima, quæ duci possunt ad perimetrum Parabolæ  $AD$ ; Hec coincidet cum recta  $DE$  ad tangentem  $DL$  normali. Per punctum  $D$  ducatur ad axem perpendicularis  $CO$ ; & ad hanc normalis  $GO$ . Sit  $GO$  ( $= AC$ )  $= x$ ;  $OD = y$ ;  $AG = a$ ; erit:  $CD = a - y$ ; &  $y: x = a - y: CE$ ;

*Proinde:*  $CE = \frac{y}{a - y}$ . Ob triangula autem  $ECD$ ,  $LCD$  similia, est:  $L C: CD = CD: CE$ ;

five:  $2x: a - y = a - y: \frac{y}{a - y}$ . Hinc:  $ay - y^2 = 2x^2$ ; quæ est aequatio ad Ellipsum exteram.

Et si (ducta  $DF$  normali ad  $AG$ )  $GF$  ducatur  $x$ ,  $DF$  vero  $y$ , erit  $y^2 = \frac{1}{4}ax - \frac{1}{2}x^2$ . Curva ergo quæsita est Ellipsis; cuius axis transversus est aequalis  $a$ , axis autem transversi parameter  $= \frac{1}{2}a$ .

Et certe: si Parabolæ latitudo augeatur in infinitum; punctum  $D$  infinite parum distabit a puncto  $G$ ; hinc

hinc curva pntum  $G$  transire debet; Et: si Parabolæ infinite contrahatur, coincidet hæc cum recta  $AE$ ; & perpendicularis ad illam ducta, erit ipsa recta  $GA$ ; ac propterea curva transit punctum  $A$ .

Ellipsis igitur dimidia super axe principali  $AG$ , altero axe  $= \sqrt{\frac{1}{2}a^2}$ , descripta est locus punctorum  $D$ .

#### PROBLEMA 37.

**49.** Si Parabolæ infinitæ communem habeant axem  $AC$ , & verticem  $A$ , ut ante; a dato autem in communis omnium Paraboliarum tangente  $AG$  puncto  $G$  ad quamvis Parabolam ducatur  $GD$  sub dato angulo, invenire locum punctorum  $D$ .

*Solutio:* Sit  $AC = x$ ;  $CD = y$ ;  $AG = CO = a$ ;

erit  $BM = \frac{1}{2}y$ ;  $LM = \frac{1}{2}\sqrt{(4x^2 + y^2)}$ ;  $DO = a - y$ ;

Hinc  $DG = \sqrt{(x^2 + a^2 + y^2 - 2ay)}$ ;  $MG = a - \frac{1}{2}y$ .

Duc  $Gr$  normalem ad tangentem  $DL$ ; dabiturque

specie triangulum  $GrD$ ; & ob triangula  $ALM$ ,  $Mrg$

similia, erit:  $L M: LA = MG: Gr$ ; five:

$\frac{1}{2}\sqrt{(4x^2 + y^2)}: x = a - \frac{1}{2}y: Gr$ . Igitur  $Gr = \frac{2ax - xy}{\sqrt{(4x^2 + y^2)}}$ .

Sit  $r G$ ;  $GD = m: n$ ; erit:

$\frac{2ax - xy}{\sqrt{(4x^2 + y^2)}} : \sqrt{(x^2 + a^2 - 2ay + y^2)}$

$= m: n$ . Proinde:  $m\sqrt{(x^2 + a^2 - 2ay + y^2)} =$

$2anx - ny$ ; &  $4a^2n^2x^2 + n^2x^2y^2 - 4an^2x^2y$

$= 4m^2x^4 + 4m^2a^2x^2 + 5m^2y^2x^2 - 8am^2y^2x^2$

$+ a^2m^2y^2 + m^2y^4 - 2am^2y^3$ .

Si in hac aequatione fiat  $m = n$ , seu, quod idem est: si angulus  $GD L$  sit rectus, qui est casus Problematis