

1.5 Střed, singulární body kvadriky

```
[ > restart;
[ > with(LinearAlgebra):
[ > X:=Vector[row]([x,y,z,1]);
                                     X := [x, y, z, 1]
```

Matice kvadriky:

```
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                     K :=  $\begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$ 
```

Rovnice kvadriky:

```
[ > Kv:=expand(evalm(X*K*Transpose(X)))=0;
Kv :=  $x^2 a(1,1) + 2xy a(1,2) + 2xz a(1,3) + 2x a(1,4) + y^2 a(2,2) + 2yz a(2,3) + 2y a(2,4) + z^2 a(3,3) + 2z a(3,4) + a(4,4) = 0$ 
```

Bod přímky:

```
[ > M:=[m,n,p];
                                     M := [m, n, p]
```

Parametrické rovnice přímky:

```
[ > Primka:=[x=m+t*u,y=n+t*v,z=p+t*w];
                                     Primka := [x = m + t u, y = n + t v, z = p + t w]
```

Parametrické rovnice přímky dosazené do rovnice kvadriky:

```
[ > Kv1:=simplify(eval(Kv,Primka));
Kv1 :=  $2 a(1,1) m t u + 2 a(1,2) m t v + 2 a(1,2) t u n + 2 a(1,2) t^2 u v + 2 a(1,3) m t w + 2 a(1,3) t u p + 2 a(1,3) t^2 u w + 2 a(2,2) n t v + 2 a(2,3) n t w + 2 a(2,3) t v p + 2 a(2,3) t^2 v w + 2 a(3,3) p t w + a(1,1) m^2 + 2 a(1,4) m + a(2,2) n^2 + 2 a(2,4) n + a(3,3) p^2 + 2 a(3,4) p + a(1,1) t^2 u^2 + 2 a(1,2) m n + 2 a(1,3) m p + 2 a(1,4) t u + a(2,2) t^2 v^2 + 2 a(2,3) n p + 2 a(2,4) t v + a(3,3) t^2 w^2 + 2 a(3,4) t w + a(4,4) = 0$ 
```

Koeficient B rovnice $At^2 + Bt + C = 0$ společných bodů kvadriky a přímky:

```
[ > B:=coeff(lhs(Kv1),t)/2;
B :=  $a(1,1) m u + a(1,2) m v + a(1,2) u n + a(1,3) m w + a(1,3) u p + a(2,2) n v + a(2,3) n w + a(2,3) v p + a(3,3) p w + a(1,4) u + a(2,4) v + a(3,4) w$ 
[ > B:=collect(B,[u,v,w]);
B :=  $(a(1,1) m + a(1,3) p + a(1,4) + a(1,2) n) u + (a(1,2) m + a(2,2) n + a(2,4) + a(2,3) p) v$ 
```

$$+ (a(3, 3)p + a(2, 3)n + a(1, 3)m + a(3, 4))w$$

Soustava rovnic - podmíněk pro určení bodu $M = [m, n, p]$ jako středu kvadriky :

```
> rS1:=sort(coeff(B,u),[m,n,p])=0;
rS2:=sort(coeff(B,v),[m,n,p])=0;
rS3:=sort(coeff(B,w),[m,n,p])=0;

rS1 := a(1, 1) m + a(1, 2) n + a(1, 3) p + a(1, 4) = 0
rS2 := a(1, 2) m + a(2, 2) n + a(2, 3) p + a(2, 4) = 0
rS3 := a(1, 3) m + a(2, 3) n + a(3, 3) p + a(3, 4) = 0
```

Determinant této soustavy - tzv. **hlavní minor** kvadriky:

```
> A44:=SubMatrix(K,[1..3],[1..3]);

A44 := 
$$\begin{bmatrix} a(1, 1) & a(1, 2) & a(1, 3) \\ a(1, 2) & a(2, 2) & a(2, 3) \\ a(1, 3) & a(2, 3) & a(3, 3) \end{bmatrix}$$

```

Příklad: Napište rovnici kvadriky, která prochází bodem A, má střed S a protíná rovinu $z = 0$ v dané kuželosečce.

```
> A:=[2,0,-1]; S:=[0,0,-1]; kz:=x^2-4*x*y-1=0;

A := [2, 0, -1]
S := [0, 0, -1]

kz := x2 - 4 x y - 1 = 0

> rS1:=eval(rS1,{m=S[1],n=S[2],p=S[3]});
rS2:=eval(rS2,{m=S[1],n=S[2],p=S[3]});
rS3:=eval(rS3,{m=S[1],n=S[2],p=S[3]});

rS1 := -a(1, 3) + a(1, 4) = 0
rS2 := -a(2, 3) + a(2, 4) = 0
rS3 := -a(3, 3) + a(3, 4) = 0

> res1:=solve({rS1,rS2,rS3},{a(1,3),a(2,3),a(3,3)});

res1 := { a(1, 3) = a(1, 4), a(2, 3) = a(2, 4), a(3, 3) = a(3, 4) }

> assign(res1);

> K1:=K;

K1 := 
$$\begin{bmatrix} a(1, 1) & a(1, 2) & a(1, 4) & a(1, 4) \\ a(1, 2) & a(2, 2) & a(2, 4) & a(2, 4) \\ a(1, 4) & a(2, 4) & a(3, 4) & a(3, 4) \\ a(1, 4) & a(2, 4) & a(3, 4) & a(4, 4) \end{bmatrix}$$


> z:=0;

z := 0

> Kv; kz;

x2 a(1, 1) + 2 x y a(1, 2) + 2 x a(1, 4) + y2 a(2, 2) + 2 y a(2, 4) + a(4, 4) = 0
x2 - 4 x y - 1 = 0

> res2:=solve({coeffs(lhs(Kv)-lhs(kz))},{a(1,1),a(1,2),a(1,4),a(2,2),a(2,4),a(4,4)});
```

```

[      res2 := { a(1, 1) = 1, a(1, 2) = -2, a(1, 4) = 0, a(2, 2) = 0, a(2, 4) = 0, a(4, 4) = -1 }
[ > assign(res2);
[ > K2:=map(x->eval(x),K1);
      K2 := 
$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ -2 & 0 & 0 & 0 \\ 0 & 0 & a(3,4) & a(3,4) \\ 0 & 0 & a(3,4) & -1 \end{bmatrix}$$

[ > z:='z';
      z := z
[ > Kv:=eval(Kv);
      Kv :=  $x^2 - 4xy - 1 + z^2 a(3,4) + 2z a(3,4) = 0$ 
[ > res3:=solve(eval(Kv,[x=2,y=0,z=-1]),{a(3,4)});
      res3 := { a(3,4) = 3 }
[ > assign(res3);
[ > Kv:=eval(Kv);
      Kv :=  $x^2 - 4xy - 1 + 3z^2 + 6z = 0$ 
[ > A44:=map(x->eval(x),A44);
      A44 := 
$$\begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

[ > Determinant(A44);
      -12

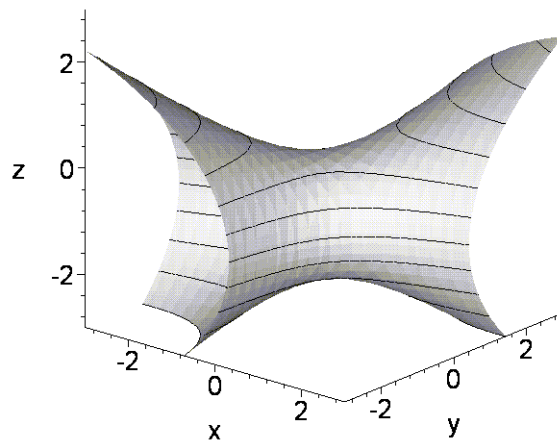
```

Hlavní minor je různý od nuly. Kvadrika je středová.

```

[ > plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);
[ > plots[implicitplot3d](Kv,x=-4..4,y=-4..4,z=-4..4,axes=frame,orientation=[-50,70],color=COLOR(RGB,205/255,205/255,205/255),style=patchcontour,grid=[30,30,40],light=[90,-5,1,1,1],tickmarks=[3,3,3],scaling=constrained,view=[-3..3,-3..3,-3..3]);

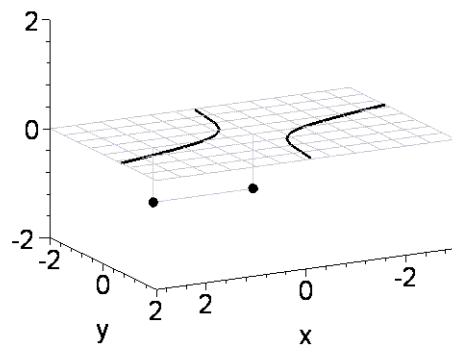
```



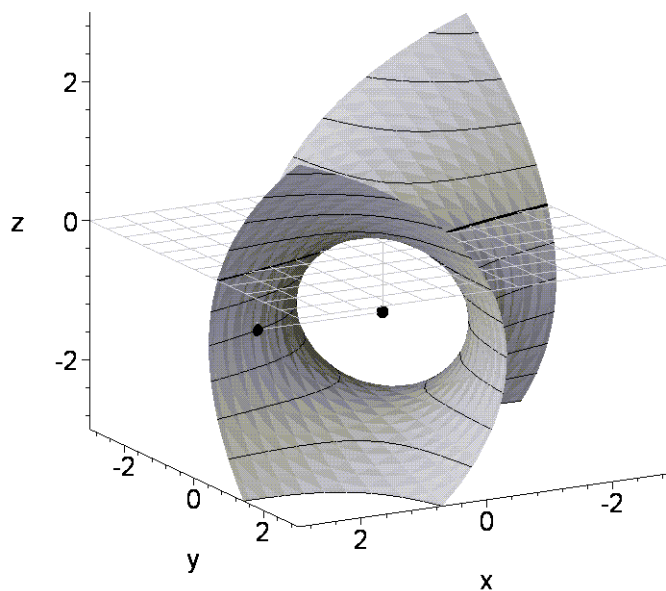
```

> Plocha_patch:=plots[implicitplot3d](Kv,x=-4..4,y=-4..4,z=-4..4,color=COLOR(
RGB,205/255,205/255,205/255),style=patchcontour,grid=[30,30,40],light=[90,-5,1,1,1],scaling=constrained):
> Plocha:=plots[implicitplot3d](Kv,x=-5..5,y=-5..5,z=-5..5,color=COLOR(
RGB,200/255,200/255,200/255),style=wireframe,grid=[30,30,30],scaling=constrained):
> PlochaColor:=plots[implicitplot3d](Kv,x=-8..8,y=-8..8,z=-5..5,style=wireframe,grid=[30,30,30],scaling=constrained):
> Krivka1:=plot3d([t,solve(eval(kz,x=t),y),0],t=-3..0,s=-3..3,thickness=3,color=red):
> Krivka2:=plot3d([t,solve(eval(kz,x=t),y),0],t=0..3,s=-3..3,thickness=3,color=red):
> Bod:=plottools[sphere]([2,0,-1],0.08,color=red):
> Stred:=plottools[sphere]([0,0,-1],0.08,color=red):
> Kvadr:=plottools[cuboid]([0,0,0],[2,0,-1],style=wireframe,color=grey):
> Rov_z:=plot3d(0,x=-6..6,y=-6..6,style=wireframe,color=grey):
> plots[display](Krivka1,Krivka2,Bod,Kvadr,Stred,Rov_z,axes=frame,orientation=[62,75],tickmarks=[3,2,2],scaling=constrained,view=[-3..3,-2..2,-2..2],light=[90,-20,1,1,1]);

```



```
> plots[display](Krivka1,Krivka2,Bod,
Kvadr,Plocha_patch,Stred,Rov_z,axes=frame,orientation=[62,75],ti
ckmarks=[3,3,3],scaling=constrained,view=[-3..3,-3..3,-3..3],lig
ht=[90,-20,1,1,1]);
```



```
[ >
```