

## 1.5.1 Singulární body

```
[ > restart;
[ > with(LinearAlgebra):
[ > X:=Vector[row]([x,y,z,1]);
                                     X := [x, y, z, 1]
```

Matice kvadriky:

```
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                     K :=  $\begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$ 
```

Rovnice kvadriky:

```
[ > Kv:=expand(X.K.Transpose(X))=0;
Kv :=  $x^2 a(1,1) + 2xy a(1,2) + 2xz a(1,3) + 2x a(1,4) + y^2 a(2,2) + 2yz a(2,3) + 2y a(2,4) + z^2 a(3,3) + 2z a(3,4) + a(4,4) = 0$ 
```

Bod přímky:

```
[ > M:=[m,n,p];
                                     M := [m, n, p]
```

Parametrické rovnice přímky:

```
[ > Primka:=[x=m+t*u,y=n+t*v,z=p+t*w];
                                     Primka := [x = m + t u, y = n + t v, z = p + t w]
```

Parametrické rovnice přímky dosazené do rovnice kvadriky:

```
[ > Kv1:=simplify(eval(Kv,Primka));
Kv1 :=  $2 a(1,1) m t u + 2 a(1,2) m t v + 2 a(1,2) t u n + 2 a(1,2) t^2 u v + 2 a(1,3) m t w + 2 a(1,3) t u p + 2 a(1,3) t^2 u w + 2 a(2,2) n t v + 2 a(2,3) n t w + 2 a(2,3) t v p + 2 a(2,3) t^2 v w + 2 a(3,3) p t w + a(1,1) t^2 u^2 + 2 a(1,2) m n + 2 a(1,3) m p + 2 a(1,4) t u + a(2,2) t^2 v^2 + 2 a(2,3) n p + 2 a(2,4) t v + a(3,3) t^2 w^2 + 2 a(3,4) t w + a(4,4) + 2 a(3,4) p + a(1,1) m^2 + 2 a(1,4) m + a(2,2) n^2 + 2 a(2,4) n + a(3,3) p^2 = 0$ 
```

Koeficient B rovnice  $At^2 + Bt + C = 0$  společných bodů kvadriky a přímky:

```
[ > B:=coeff(lhs(Kv1),t)/2;
B :=  $a(1,1) m u + a(1,2) m v + a(1,2) u n + a(1,3) m w + a(1,3) u p + a(2,2) n v + a(2,3) n w + a(2,3) v p + a(3,3) p w + a(1,4) u + a(2,4) v + a(3,4) w$ 
[ > B:=collect(B,[u,v,w]);
B :=  $(a(1,1) m + a(1,3) p + a(1,4) + a(1,2) n) u$ 
```

$$\begin{aligned}
 &+ (a(1, 2) m + a(2, 2) n + a(2, 4) + a(2, 3) p) v \\
 &+ (a(3, 3) p + a(2, 3) n + a(1, 3) m + a(3, 4)) w
 \end{aligned}$$

Soustava rovnic - podmínek singularity bodu (bodů)  $M = [m, n, p]$ :

```

> r1:=sort(coeff(B,u),[m,n,p])=0; r2:=sort(coeff(B,v),[m,n,p])=0;
r3:=sort(coeff(B,w),[m,n,p])=0;
r4:=sort(simplify(coeff(lhs(Kv1),t,0)-m*lhs(r1)-n*lhs(r2)-p*lhs(
r3)),[m,n,p])=0;
      r1 := a(1, 1) m + a(1, 2) n + a(1, 3) p + a(1, 4) = 0
      r2 := a(1, 2) m + a(2, 2) n + a(2, 3) p + a(2, 4) = 0
      r3 := a(1, 3) m + a(2, 3) n + a(3, 3) p + a(3, 4) = 0
      r4 := a(1, 4) m + a(2, 4) n + a(3, 4) p + a(4, 4) = 0
> K;
      [a(1, 1)  a(1, 2)  a(1, 3)  a(1, 4)]
      [a(1, 2)  a(2, 2)  a(2, 3)  a(2, 4)]
      [a(1, 3)  a(2, 3)  a(3, 3)  a(3, 4)]
      [a(1, 4)  a(2, 4)  a(3, 4)  a(4, 4)]
> Delta:=Determinant(K):

```

**Příklad:** Volme  $M = [0, 0, 0]$ . Hledáme kvadriku s tímto singulárním bodem.

```

> M:=[0,0,0]:
> res1:=solve(eval({r1,r2,r3,r4},[m=0,n=0,p=0]),{a(1,4),a(2,4),a(3,4),a(4,4)});
      res1 := {a(1, 4) = 0, a(2, 4) = 0, a(3, 4) = 0, a(4, 4) = 0}
> assign(res1);
> Kv;
      x^2 a(1, 1) + 2 x y a(1, 2) + 2 x z a(1, 3) + y^2 a(2, 2) + 2 y z a(2, 3) + z^2 a(3, 3) = 0
> Kv1:=x^2+y^2-z^2=0;
      Kv1 := x^2 + y^2 - z^2 = 0
> Stred:=[0,0,0]:

```

Pro grafické znázornění kvadriky použijeme její parametrické vyjádření:

```

> Kvlp:=subs(x=t*cos(u),y=t*sin(u),Kv1);
      Kvlp := t^2 cos(u)^2 + t^2 sin(u)^2 - z^2 = 0
> Res_z:=solve(Kvlp,{z});
      Res_z := {z = t}, {z = -t}

```

Stačí uvažovat jenom jedno z řešení pro  $t$ :

```

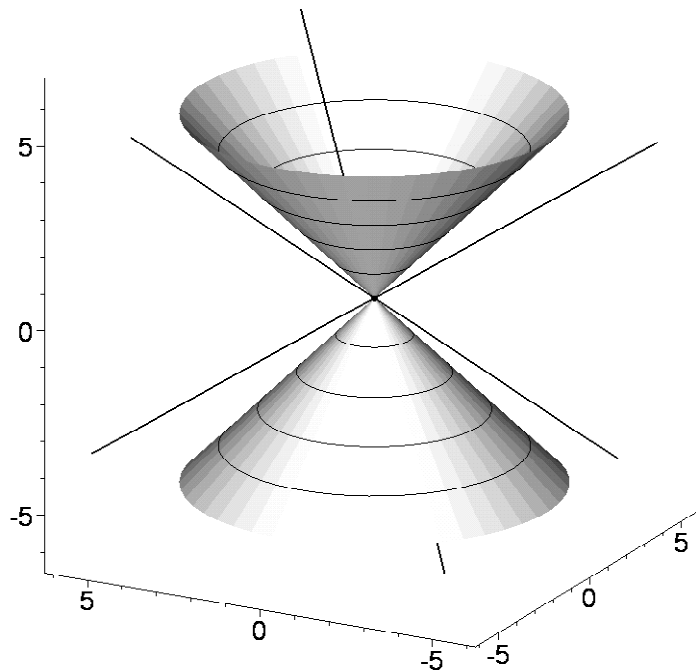
> Kvlp[1]:=eval([t*cos(u),t*sin(u),z],Res_z[1]);
Kvlp[2]:=eval([t*cos(u),t*sin(u),z],Res_z[2]);

```

$$Kvlp_1 := [t \cos(u), t \sin(u), t]$$

$$Kvlp_2 := [t \cos(u), t \sin(u), -t]$$

```
[ > plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);  
[ > Plocha:=plot3d({Kvlp[1],Kvlp[2]},t=0..5,u=0..2*Pi,orientation=[5  
2,63],grid=[50,50],style=patchcontour,axes=frame,color=COLOR(RGB  
,250/255,250/255,250/255),light=[90,-5,1,1,1],tickmarks=[3,3,3],  
scaling=constrained):  
[ > Bod:=plottools[sphere](Stred,0.08,color=black):  
[ > Primka1:=plot3d(evalm(Stred+t*([-1,3,2])),t=-2..2,s=-1..1,thickn  
ess=2):  
[ > Primka2:=plot3d(evalm(Stred+t*([1,1.5,3])),t=-2.1..2.2,s=-1..1,t  
hickness=2):  
[ > Primka3:=plot3d(evalm(Stred+t*([3,-2.5,1.6])),t=-2..2,s=-1..1,th  
ickness=2):  
[ > plots[display](Plocha,Bod,Primka1,Primka2,Primka3,orientation=[-  
152,71]);
```



```
[ >
```

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[ >
```