

```

[ > restart;
[ > with(LinearAlgebra):
[ > X:=Vector[row]([x,y,z,1]);
                                X := [x, y, z, 1]
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                K :=  $\begin{bmatrix} a(1,1) & a(1,2) & a(1,3) & a(1,4) \\ a(1,2) & a(2,2) & a(2,3) & a(2,4) \\ a(1,3) & a(2,3) & a(3,3) & a(3,4) \\ a(1,4) & a(2,4) & a(3,4) & a(4,4) \end{bmatrix}$ 
[ > Kv:=expand(X.K.Transpose(X))=0;
Kv :=  $x^2 a(1,1) + 2xy a(1,2) + 2xz a(1,3) + 2x a(1,4) + y^2 a(2,2) + 2yz a(2,3) + 2y a(2,4) + z^2 a(3,3) + 2z a(3,4) + a(4,4) = 0$ 

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2) $h(K) = 2$

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[ > alpha:=a1*x+b1*y+c1*z+d1=0; beta:=a2*x+b2*y+c2*z+d2=0;
                                 $\alpha := a1 x + b1 y + c1 z + d1 = 0$ 
                                 $\beta := a2 x + b2 y + c2 z + d2 = 0$ 
[ > Kv2:=lhs(alpha)*lhs(beta)=0;
                                Kv2 :=  $(a1 x + b1 y + c1 z + d1)(a2 x + b2 y + c2 z + d2) = 0$ 
[ > Kv2:=expand(Kv2);
Kv2 :=  $a1 x^2 a2 + a1 x b2 y + a1 x c2 z + a1 x d2 + b1 y a2 x + b1 y^2 b2 + b1 y c2 z + b1 y d2 + c1 z a2 x + c1 z b2 y + c1 z^2 c2 + c1 z d2 + d1 a2 x + d1 b2 y + d1 c2 z + d1 d2 = 0$ 
[ > r1:=coeff(lhs(Kv-Kv2),x,2); r2:=coeff(lhs(Kv-Kv2),y,2);
r3:=coeff(lhs(Kv-Kv2),z,2); r4:=coeff(coeff(lhs(Kv-Kv2),x),y);
r5:=coeff(coeff(lhs(Kv-Kv2),x),z);
r6:=coeff(coeff(lhs(Kv-Kv2),y),z);
r7:=coeff(coeff(coeff(lhs(Kv-Kv2),x),y,0),z,0);
r8:=coeff(coeff(coeff(lhs(Kv-Kv2),y),x,0),z,0);
r9:=coeff(coeff(coeff(lhs(Kv-Kv2),z),x,0),y,0);
r10:=coeff(coeff(coeff(lhs(Kv-Kv2),x,0),y,0),z,0);
                                 $r1 := -a1 a2 + a(1,1)$ 
                                 $r2 := -b1 b2 + a(2,2)$ 
                                 $r3 := -c1 c2 + a(3,3)$ 
                                 $r4 := -a1 b2 - b1 a2 + 2 a(1,2)$ 
                                 $r5 := -a1 c2 - c1 a2 + 2 a(1,3)$ 
                                 $r6 := 2 a(2,3) - b1 c2 - c1 b2$ 
                                 $r7 := -a1 d2 - d1 a2 + 2 a(1,4)$ 
                                 $r8 := -b1 d2 - d1 b2 + 2 a(2,4)$ 
                                 $r9 := -c1 d2 - d1 c2 + 2 a(3,4)$ 
                                 $r10 := a(4,4) - d1 d2$ 
[ > res:=solve({r1,r2,r3,r4,r5,r6,r7,r8,r9,r10},{a(1,1),a(1,2),a(1,3)

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),a(1,4),a(2,2),a(2,3),a(2,4),a(3,3),a(3,4),a(4,4)});
```

$$res := \{ a(1, 1) = a1 a2, a(1, 2) = \frac{a1 b2}{2} + \frac{b1 a2}{2}, a(1, 3) = \frac{a1 c2}{2} + \frac{c1 a2}{2},$$

$$a(1, 4) = \frac{a1 d2}{2} + \frac{d1 a2}{2}, a(2, 2) = b1 b2, a(2, 3) = \frac{b1 c2}{2} + \frac{c1 b2}{2}, a(2, 4) = \frac{b1 d2}{2} + \frac{d1 b2}{2},$$

$$a(3, 3) = c1 c2, a(3, 4) = \frac{c1 d2}{2} + \frac{d1 c2}{2}, a(4, 4) = d1 d2 \}$$

```
> K2:=2*map(x->eval(x,res),K);
```

$$K2 := \begin{bmatrix} 2 a1 a2 & a1 b2 + b1 a2 & a1 c2 + c1 a2 & a1 d2 + d1 a2 \\ a1 b2 + b1 a2 & 2 b1 b2 & b1 c2 + c1 b2 & b1 d2 + d1 b2 \\ a1 c2 + c1 a2 & b1 c2 + c1 b2 & 2 c1 c2 & c1 d2 + d1 c2 \\ a1 d2 + d1 a2 & b1 d2 + d1 b2 & c1 d2 + d1 c2 & 2 d1 d2 \end{bmatrix}$$

Příklad: Dvě různoběžné roviny

```
> alpha:=x+2*y-z+1=0; beta:=2*x+y+3*z-2=0;
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$$\alpha := x + 2y - z + 1 = 0$$

$$\beta := 2x + y + 3z - 2 = 0$$

```
> plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);
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```
> r1:=solve(alpha,z); r2:=solve(beta,z);
```

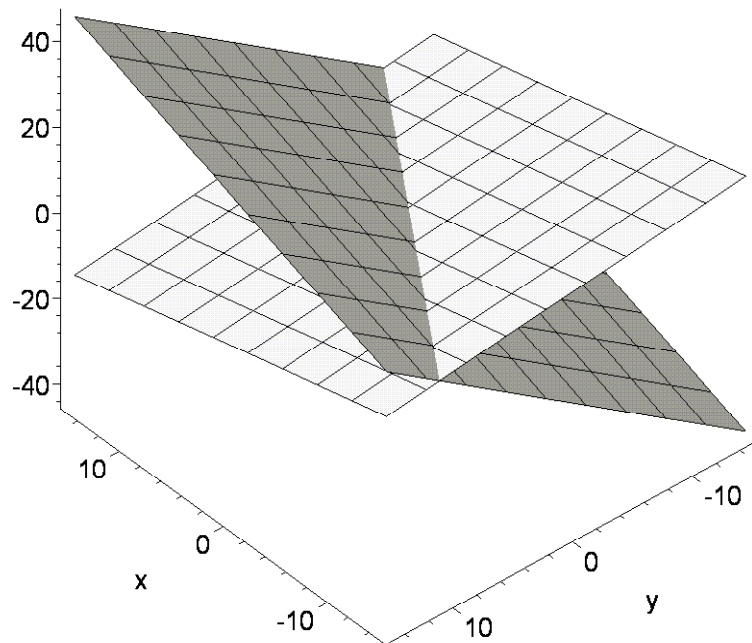
$$r1 := x + 2y + 1$$

$$r2 := -\frac{2x}{3} - \frac{y}{3} + \frac{2}{3}$$

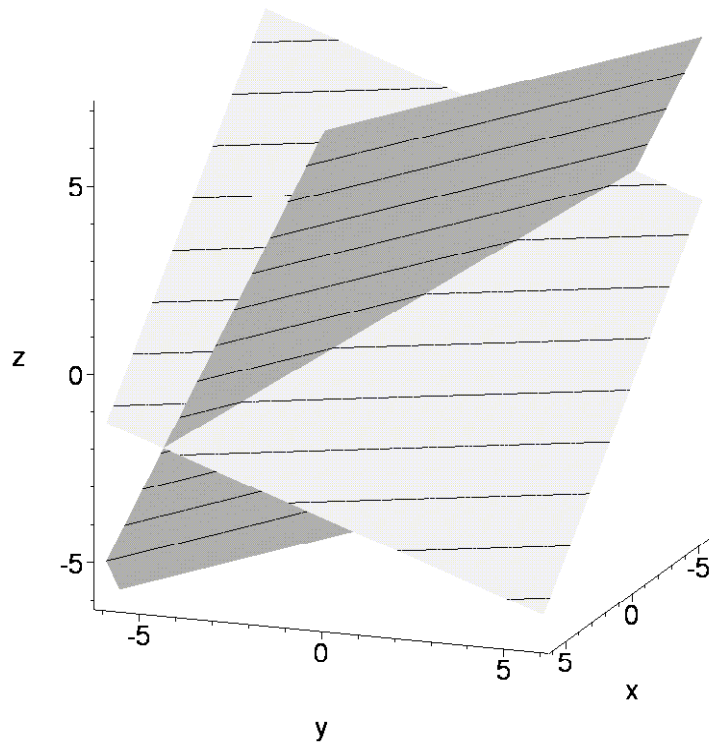
```
> r1:=plot3d(r1,x=-15..15,y=-15..15,axes=frame,orientation=[139,52],color=grey,grid=[10,10],color=COLOR(RGB,140/255,140/255,140/255),tickmarks=[3,3,3],numpoints=100000):
```

```
> r2:=plot3d(r2,x=-15..15,y=-15..15,axes=frame,orientation=[139,52],color=grey,grid=[10,10],color=COLOR(RGB,243/255,243/255,243/255),tickmarks=[3,3,3],numpoints=100000):
```

```
> plots[display](r1,r2);
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```
> plots[implicitplot3d]([alpha,beta],x=-6..6,y=-6..6,z=-6..7,orientation=[20,75],gridstyle=rectangular,axes=frame,light=[90,-5,1,1,1],grid=[30,30,30],style=patchcontour,color=[COLOR(RGB,240/255,240/255,240/255),COLOR(RGB,243/255,243/255,243/255)],scaling=constrained,tickmarks=[3,3,3]);
```



3) $h(K) = 1$

```

> Kv3:=(a*x+b*y+c*z+d)^2=0;
                                Kv3 := (ax+by+cz+d)^2=0
> Kv3:=expand(Kv3);
Kv3 :=
    a^2 x^2 + 2 a x b y + 2 a x c z + 2 a x d + b^2 y^2 + 2 b y c z + 2 b y d + c^2 z^2 + 2 c z d + d^2 = 0
> r1:=coeff(lhs(Kv-Kv3),x,2); r2:=coeff(lhs(Kv-Kv3),y,2);
r3:=coeff(lhs(Kv-Kv3),z,2); r4:=coeff(coeff(lhs(Kv-Kv3),x),y);
r5:=coeff(coeff(lhs(Kv-Kv3),x),z);
r6:=coeff(coeff(lhs(Kv-Kv3),y),z);
r7:=coeff(coeff(coeff(lhs(Kv-Kv3),x),y,0),z,0);
r8:=coeff(coeff(coeff(lhs(Kv-Kv3),y),x,0),z,0);
r9:=coeff(coeff(coeff(lhs(Kv-Kv3),z),x,0),y,0);
r10:=coeff(coeff(coeff(lhs(Kv-Kv3),x,0),y,0),z,0);
                                r1 := a(1,1) - a^2
                                r2 := a(2,2) - b^2
                                r3 := a(3,3) - c^2

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$$r4 := 2 a(1, 2) - 2 a b$$

$$r5 := 2 a(1, 3) - 2 a c$$

$$r6 := 2 a(2, 3) - 2 b c$$

$$r7 := 2 a(1, 4) - 2 a d$$

$$r8 := 2 a(2, 4) - 2 b d$$

$$r9 := 2 a(3, 4) - 2 c d$$

$$r10 := a(4, 4) - d^2$$

```
> res:=solve({r1,r2,r3,r4,r5,r6,r7,r8,r9,r10},{a(1,1),a(1,2),a(1,3),a(1,4),a(2,2),a(2,3),a(2,4),a(3,3),a(3,4),a(4,4)});
```

```
res := { a(1, 1) = a2, a(1, 2) = a b, a(1, 3) = a c, a(1, 4) = a d, a(2, 2) = b2, a(2, 3) = b c, a(2, 4) = b d, a(3, 3) = c2, a(3, 4) = c d, a(4, 4) = d2 }
```

```
> K3:=map(x->eval(x,res),K);
```

$$K3 := \begin{bmatrix} a^2 & a b & a c & a d \\ a b & b^2 & b c & b d \\ a c & b c & c^2 & c d \\ a d & b d & c d & d^2 \end{bmatrix}$$

Příklad: Dvojnásobná rovina

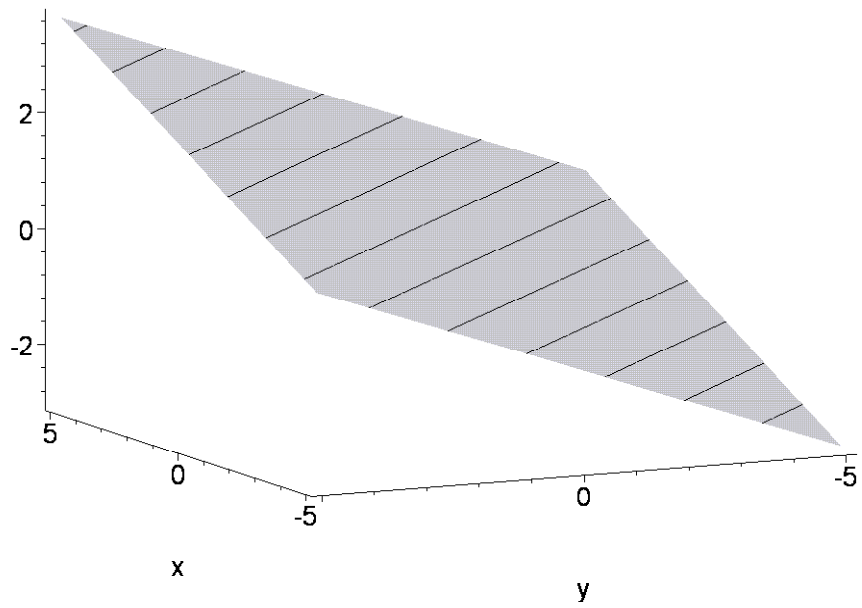
```
> Kv3:=(x+y-3*z+1)^2=0;
```

$$Kv3 := (x + y - 3 z + 1)^2 = 0$$

```
> op(lhs(Kv3))[1];
```

$$x + y - 3 z + 1$$

```
> plot3d(solve(op(lhs(Kv3))[1],z),x=-5..5,y=-5..5,axes=frame,orientation=[154,81],style=surfacecontour,color=grey,grid=[10,10],scaling=constrained,tickmarks=[3,3,3]);
```



```
> plots[implicitplot3d](op(lhs(Kv3))[1],x=-16..16,y=-16..16,z=-15.  
.15,orientation=[170,60],gridstyle=rectangular,axes=frame,grid=[  
10,10,10],style=patchcontour,color=grey,scaling=constrained);
```

