

1.8 Polární rovina

```
[ > restart;
  > with(LinearAlgebra):
  > with(plots):
  > X:=Vector[row]([x,y,z,1]);
                                     X := [x, y, z, 1]
```

Matice kvadriky:

```
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                     K :=
      [ a(1,1)  a(1,2)  a(1,3)  a(1,4)
      [ a(1,2)  a(2,2)  a(2,3)  a(2,4)
      [ a(1,3)  a(2,3)  a(3,3)  a(3,4)
      [ a(1,4)  a(2,4)  a(3,4)  a(4,4)]
```

Rovnice kvadriky:

```
[ > Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
Kv := a(1,1)x2 + 2 a(1,2)xy + 2 a(1,3)xz + a(2,2)y2 + 2 a(2,3)yz + a(3,3)z2
      + 2 a(1,4)x + 2 a(2,4)y + 2 a(3,4)z + a(4,4) = 0
```

Parametrické rovnice tečny z bodu $R = [r, s, u]$ s bodem dotyku $T = [m, n, p]$

```
[ > R:=[r,s,u]; T:=[m,n,p];
                                     R := [r, s, u]
                                     T := [m, n, p]
  > Tecna:=[x=m+t*w1,y=n+t*w2,z=p+t*w3];
                                     Tecna := [x = m + t w1, y = n + t w2, z = p + t w3]
  > Kv1:=simplify(eval(Kv,Tecna));
Kv1 := 2 a(1,1) m t w1 + 2 a(1,2) m t w2 + 2 a(1,2) t w1 n + 2 a(1,2) t2 w1 w2
      + 2 a(1,3) m t w3 + 2 a(1,3) t w1 p + 2 a(1,3) t2 w1 w3 + 2 a(2,2) n t w2 + 2 a(2,3) n t w3
      + 2 a(2,3) t w2 p + 2 a(2,3) t2 w2 w3 + 2 a(3,3) p t w3 + a(4,4) + a(1,1) m2 + a(2,2) n2
      + a(3,3) p2 + 2 a(1,4) m + 2 a(2,4) n + 2 a(3,4) p + a(1,1) t2 w12 + 2 a(1,2) m n
      + 2 a(1,3) m p + a(2,2) t2 w22 + 2 a(2,3) n p + a(3,3) t2 w32 + 2 a(1,4) t w1
      + 2 a(2,4) t w2 + 2 a(3,4) t w3 = 0
  > Kv1 :=
a(2,2)*n2+a(4,4)+a(1,1)*m2+2*a(2,3)*n*p+2*a(1,3)*m*p+2*a(1,2)*
m*n+2*a(3,3)*p*t*w3+2*a(1,2)*m*t*w2+2*a(1,2)*t2*w1*w2+2*a(1,3)*
m*t*w3+2*a(1,3)*t*w1*p+a(1,1)*t2*w12+a(2,2)*t2*w22+2*a(1,4)*
t*w1+2*a(1,3)*t2*w1*w3+2*a(2,2)*n*t*w2+2*a(1,1)*m*t*w1+2*a(1,2)
)*t*w1*n+2*a(2,3)*n*t*w3+2*a(2,3)*t*w2*p+2*a(2,3)*t2*w2*w3+a(3,
3)*t2*w32+2*a(3,4)*t*w3+2*a(2,4)*t*w2+2*a(3,4)*p+2*a(2,4)*n+2*
a(1,4)*m+a(3,3)*p2 = 0;
Kv1 := 2 a(1,1) m t w1 + 2 a(1,2) m t w2 + 2 a(1,2) t w1 n + 2 a(1,2) t2 w1 w2
```

$$\begin{aligned}
& + 2 a(1, 3) m t w^3 + 2 a(1, 3) t w l p + 2 a(2, 2) n t w^2 + 2 a(2, 3) n t w^3 + 2 a(2, 3) t w^2 p \\
& + 2 a(2, 3) t^2 w^2 w^3 + 2 a(3, 3) p t w^3 + a(4, 4) + a(1, 1) m^2 + a(2, 2) n^2 + a(3, 3) p^2 \\
& + 2 a(1, 4) m + 2 a(2, 4) n + 2 a(3, 4) p + a(1, 1) t^2 w l^2 + 2 a(1, 2) m n + 2 a(1, 3) m p \\
& + a(2, 2) t^2 w^2 + 2 a(2, 3) n p + a(3, 3) t^2 w^3 + 2 a(1, 4) t w l + 2 a(2, 4) t w^2 \\
& + 2 a(3, 4) t w^3 + 2 a(1, 3) t^2 w l w^3 = 0
\end{aligned}$$

> **A:=coeff(lhs(Kv1),t^2);**

$$A := 2 a(1, 2) w l w^2 + 2 a(2, 3) w^2 w^3 + a(1, 1) w l^2 + a(2, 2) w^2 + a(3, 3) w^3$$

> **B:=1/2*coeff(lhs(Kv1),t);**

$$\begin{aligned}
B := & a(1, 1) m w l + a(1, 2) m w^2 + a(1, 2) w l n + a(1, 3) m w^3 + a(1, 3) w l p + a(2, 2) n w^2 \\
& + a(2, 3) n w^3 + a(2, 3) w^2 p + a(3, 3) p w^3 + a(1, 4) w l + a(2, 4) w^2 + a(3, 4) w^3
\end{aligned}$$

> **C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);**

$$\begin{aligned}
C := & a(4, 4) + a(1, 1) m^2 + a(2, 2) n^2 + a(3, 3) p^2 + 2 a(1, 4) m + 2 a(2, 4) n + 2 a(3, 4) p \\
& + 2 a(1, 2) m n + 2 a(1, 3) m p + 2 a(2, 3) n p + 2 a(1, 3) t^2 w l w^3
\end{aligned}$$

> **B:=collect(B,[w1,w2,w3]);**

$$\begin{aligned}
B := & (a(1, 1) m + a(1, 3) p + a(1, 4) + a(1, 2) n) w l \\
& + (a(1, 2) m + a(2, 2) n + a(2, 4) + a(2, 3) p) w^2 \\
& + (a(3, 3) p + a(2, 3) n + a(1, 3) m + a(3, 4)) w^3
\end{aligned}$$

> **n1:=sort(coeff(B,w1),[m,n,p]); n2:=sort(coeff(B,w2),[m,n,p]);**
n3:=sort(coeff(B,w3),[m,n,p]);

$$n1 := a(1, 1) m + a(1, 2) n + a(1, 3) p + a(1, 4)$$

$$n2 := a(1, 2) m + a(2, 2) n + a(2, 3) p + a(2, 4)$$

$$n3 := a(1, 3) m + a(2, 3) n + a(3, 3) p + a(3, 4)$$

> **BD:=linalg[dotprod](T-R,[n1,n2,n3],orthogonal);**

$$\begin{aligned}
BD := & (-r + m) (a(1, 1) m + a(1, 2) n + a(1, 3) p + a(1, 4)) \\
& + (-s + n) (a(1, 2) m + a(2, 2) n + a(2, 3) p + a(2, 4)) \\
& + (-u + p) (a(1, 3) m + a(2, 3) n + a(3, 3) p + a(3, 4))
\end{aligned}$$

> **BD:=collect(eval(lhs(Kv),[x=m,y=n,z=p])-expand(BD),[m,n,p]);**

$$\begin{aligned}
BD := & (r a(1, 1) + u a(1, 3) + s a(1, 2) + a(1, 4)) m \\
& + (a(2, 4) + s a(2, 2) + u a(2, 3) + r a(1, 2)) n \\
& + (u a(3, 3) + r a(1, 3) + s a(2, 3) + a(3, 4)) p + a(4, 4) + s a(2, 4) + u a(3, 4) + r a(1, 4)
\end{aligned}$$

> **ra:=sort(coeff(BD,m),[r,s,u]); rb:=sort(coeff(BD,n),[r,s,u]);**
rc:=sort(coeff(BD,p),[r,s,u]);
rd:=sort(coeff(coeff(coeff(BD,m,0),n,0),p,0),[r,s,u]);

$$ra := a(1, 1) r + a(1, 2) s + a(1, 3) u + a(1, 4)$$

$$rb := a(1, 2) r + a(2, 2) s + a(2, 3) u + a(2, 4)$$

$$rc := a(1, 3) r + a(2, 3) s + a(3, 3) u + a(3, 4)$$

$$rd := a(1, 4) r + a(2, 4) s + a(3, 4) u + a(4, 4)$$

> **pi:=ra*x+rb*y+rc*z+rd=0;**

$$\begin{aligned}
\pi := & (a(1, 1) r + a(1, 2) s + a(1, 3) u + a(1, 4)) x \\
& + (a(1, 2) r + a(2, 2) s + a(2, 3) u + a(2, 4)) y
\end{aligned}$$

```

+ (a(1, 3) r + a(2, 3) s + a(3, 3) u + a(3, 4) z + a(1, 4) r + a(2, 4) s + a(3, 4) u + a(4, 4) =
0
> R:=Vector[row]([r,s,u,1]);
R := [r, s, u, 1]
> PolR:=collect(expand(R.K.Transpose(X)),[x,y,z])=0;
PolR := (a(1, 1) r + a(1, 2) s + a(1, 3) u + a(1, 4) x
+ (a(1, 2) r + a(2, 2) s + a(2, 3) u + a(2, 4) y
+ (a(1, 3) r + a(2, 3) s + a(3, 3) u + a(3, 4) z + a(1, 4) r + a(2, 4) s + a(3, 4) u + a(4, 4) =
0

```

Příklad: Polární rovina bodu $R = [15, -4, 5]$ vzhledem ke kvadrice $x^2 + 4y^2 + 16z^2 - 144 = 0$

```

> kv:=x^2+4*y^2+16*z^2-144=0;
kv := x^2 + 4 y^2 + 16 z^2 - 144 = 0
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
a(3,3):=coeff(lhs(kv),z^2);
a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);
a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);
a(1, 1) := 1
a(2, 2) := 4
a(3, 3) := 16
a(1, 2) := 0
a(1, 3) := 0
a(1, 4) := 0
a(2, 3) := 0
a(2, 4) := 0
a(3, 4) := 0
a(4, 4) := -144
> K;
[ 1  0  0  0 ]
[ 0  4  0  0 ]
[ 0  0 16  0 ]
[ 0  0  0 -144 ]
> R:=Vector[row]([15,-4,5,1]);
R := [15, -4, 5, 1]
> PolR:=collect(expand(R.K.Transpose(X)),[x,y,z])=0;
PolR := -144 + 15 x - 16 y + 80 z = 0

```

```
> PolR_ex:=solve(PolR,z);
```

$$PolR_ex := \frac{9}{5} - \frac{3x}{16} + \frac{y}{5}$$

```
> {kv,PolR};
```

$$\{-144 + 15x - 16y + 80z = 0, x^2 + 4y^2 + 16z^2 - 144 = 0\}$$

```
> res1:=solve({kv,PolR},{y,z});
```

```
res1 := {
```

$$y = -9 + \frac{15x}{16} + 5 \operatorname{RootOf}(7424_Z^2 + (-23040 + 2400x)_Z + 289x^2 + 11520 - 4320x),$$

$$z = \operatorname{RootOf}(7424_Z^2 + (-23040 + 2400x)_Z + 289x^2 + 11520 - 4320x)}$$

```
> Kuz:=allvalues(res1);
```

```
Kuz := {
```

$$y = -\frac{36}{29} + \frac{15x}{116} + \frac{5\sqrt{46080 + 4320x - 689x^2}}{232}, z = \frac{45}{29} - \frac{75x}{464} + \frac{\sqrt{46080 + 4320x - 689x^2}}{232},$$

$$\{y = -\frac{36}{29} + \frac{15x}{116} - \frac{5\sqrt{46080 + 4320x - 689x^2}}{232}, z = \frac{45}{29} - \frac{75x}{464} - \frac{\sqrt{46080 + 4320x - 689x^2}}{232}$$

```
}
```

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

```
Kuz1 :=
```

$$\left[x, -\frac{36}{29} + \frac{15x}{116} + \frac{5\sqrt{46080 + 4320x - 689x^2}}{232}, \frac{45}{29} - \frac{75x}{464} + \frac{\sqrt{46080 + 4320x - 689x^2}}{232} \right]$$

```
Kuz2 :=
```

$$\left[x, -\frac{36}{29} + \frac{15x}{116} - \frac{5\sqrt{46080 + 4320x - 689x^2}}{232}, \frac{45}{29} - \frac{75x}{464} - \frac{\sqrt{46080 + 4320x - 689x^2}}{232} \right]$$

```
> T1:=evalf(eval(Kuz1,x=0)); T2:=evalf(eval(Kuz1,x=5));
```

```
T3:=evalf(eval(Kuz2,x=0)); T4:=evalf(eval(Kuz1,x=10));
```

$$T1 := [0., 3.384968229, 2.476993646]$$

$$T2 := [5., 4.246161736, 1.711732347]$$

$$T3 := [0., -5.867726849, 0.6264546302]$$

$$T4 := [10., 3.128416755, 0.5506833509]$$

```
> Rk:=[R[1],R[2],R[3]]; 
```

$$Rk := [15, -4, 5]$$

```
> t:='t':
```

```
> tec1:=evalm(Rk+t*(T1-Rk)); tec2:=evalm(Rk+t*(T2-Rk));
```

```
tec3:=evalm(Rk+t*(T3-Rk)); tec4:=evalm(Rk+t*(T4-Rk));
```

$$tec1 := [15 - 15. t, -4 + 7.384968229 t, 5 - 2.523006354 t]$$

$$tec2 := [15 - 10. t, -4 + 8.246161736 t, 5 - 3.288267653 t]$$

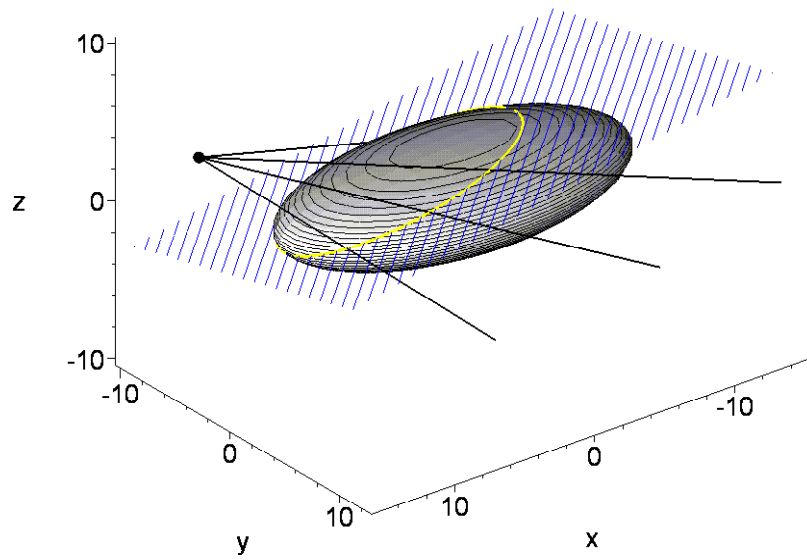
$$tec3 := [15 - 15. t, -4 - 1.867726849 t, 5 - 4.373545370 t]$$

$$tec4 := [15 - 5. t, -4 + 7.128416755 t, 5 - 4.449316649 t]$$

```

> tec1g:=plot3d(tec1,t=0..2,s=-1..1,thickness=2,color=red):
tec2g:=plot3d(tec2,t=0..2,s=0..1,thickness=2,color=red):
tec3g:=plot3d(tec3,t=0..2,s=0..1,thickness=2,color=red):
tec4g:=plot3d(tec4,t=0..2,s=0..1,thickness=2,color=red):
> Body:=plots[pointplot3d]({T1,T2,T3,T4}):
> kvg0:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,grid=[20,20,20],style=patchcontour,color=red,lightmodel=light1,scaling=constrained):
> kvg:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,axes=frame,color=COLOR(RGB,250/255,250/255,250/255),style=patchcontour,grid=[20,20,40],light=[100,-20,1,1,1],tickmarks=[3,3,3],orientation=[52,63],scaling=constrained):
> PolRg:=contourplot3d(PolR_ex,x=-15..15,y=-10..10,grid=[2,2],contours=60,color=blue,filled=false):
> PolRg2:=plot3d(PolR_ex,x=-15..15,y=-10..10,grid=[2,2],color=grey):
> Rg:=plottools[sphere]([R[1],R[2],R[3]],0.3):
> Rg2:=pointplot3d([R[1],R[2],R[3]],symbol=cross,symbolsize=20,color=red):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,color=yellow):
Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,color=yellow):
> display(kvg,PolRg,Rg,Kuz1g,Kuz2g,tec1g,tec2g,tec3g,tec4g,axes=frame,scaling=constrained);

```



```
> display(kvg,PolRg,Rg,Kuz1g,Kuz2g,tec1g,tec2g,tec3g,tec4g,axes=frame,scaling=constrained,orientation=[160,70]);
```

