

1.8 Průměrová rovina

```
[ > restart;
  > with(LinearAlgebra):
  > with(plots):
  > X:=Vector[row]([x,y,z,1]);
                                     X := [x, y, z, 1]
```

Matice kvadriky:

```
[ > K:=Matrix(a,1..4,1..4,shape=symmetric);
                                     K :=
      [ a(1,1)  a(1,2)  a(1,3)  a(1,4)
      [ a(1,2)  a(2,2)  a(2,3)  a(2,4)
      [ a(1,3)  a(2,3)  a(3,3)  a(3,4)
      [ a(1,4)  a(2,4)  a(3,4)  a(4,4)]
```

Rovnice kvadriky:

```
[ > Kv:=sort(expand(X.K.Transpose(X)),[x,y,z])=0;
Kv := a(1,1)x2 + 2 a(1,2)xy + 2 a(1,3)xz + a(2,2)y2 + 2 a(2,3)yz + a(3,3)z2
      + 2 a(1,4)x + 2 a(2,4)y + 2 a(3,4)z + a(4,4) = 0
```

Parametrické rovnice tečny s neasymptotickým směrem $sm = [u, v, w]$ a s bodem dotyku

$M = [m, n, p]$

```
[ > sm:=[u,v,w]; M:=[m,n,p];
                                     sm := [u, v, w]
                                     M := [m, n, p]
  > Tecna:=[x=m+t*u,y=n+t*v,z=p+t*w];
                                     Tecna := [x = m + t u, y = n + t v, z = p + t w]
  > Kv1:=simplify(eval(Kv,Tecna));
Kv1 := 2 a(1,2)t2uv + 2 a(1,3)mtw + 2 a(1,3)tup + 2 a(1,3)t2uw + 2 a(2,2)ntv
      + 2 a(2,3)ntw + 2 a(2,3)tvp + 2 a(2,3)t2vw + 2 a(3,3)ptw + 2 a(1,1)mtu
      + 2 a(1,2)mtv + 2 a(1,2)tun + a(1,1)m2 + a(2,2)n2 + a(3,3)p2 + 2 a(1,4)m
      + 2 a(2,4)n + 2 a(3,4)p + a(1,1)t2u2 + 2 a(1,2)mn + 2 a(1,3)mp + a(2,2)t2v2
      + 2 a(2,3)np + a(3,3)t2w2 + 2 a(1,4)tu + 2 a(2,4)tv + 2 a(3,4)tw + a(4,4) = 0
  > A:=coeff(lhs(Kv1),t^2);
      A := 2 a(1,2)uv + 2 a(1,3)uw + 2 a(2,3)vw + a(1,1)u2 + a(2,2)v2 + a(3,3)w2
  > B:=1/2*coeff(lhs(Kv1),t);
      B := a(1,3)mw + a(1,3)up + a(2,2)nv + a(2,3)nw + a(2,3)vp + a(3,3)pw
      + a(1,1)mu + a(1,2)mv + a(1,2)un + a(1,4)u + a(2,4)v + a(3,4)w
  > C:=sort(coeff(lhs(Kv1),t,0),[r,s,u]);
      C := a(1,1)m2 + a(2,2)n2 + a(3,3)p2 + 2 a(1,4)m + 2 a(2,4)n + 2 a(3,4)p
      + 2 a(1,2)mn + 2 a(1,3)mp + 2 a(2,3)np + a(4,4)
```

```

> B:=collect(B,[u,v,w]);
B := (a(1,3)p + a(1,2)n + a(1,4) + a(1,1)m)u
      + (a(2,2)n + a(2,3)p + a(2,4) + a(1,2)m)v
      + (a(1,3)m + a(3,3)p + a(2,3)n + a(3,4))w
> n1:=sort(coeff(B,u),[m,n,p]); n2:=sort(coeff(B,v),[m,n,p]);
n3:=sort(coeff(B,w),[m,n,p]);
      n1 := a(1,1)m + a(1,2)n + a(1,3)p + a(1,4)
      n2 := a(1,2)m + a(2,2)n + a(2,3)p + a(2,4)
      n3 := a(1,3)m + a(2,3)n + a(3,3)p + a(3,4)
> BD:=linalg[dotprod](sm,[n1,n2,n3],orthogonal);
BD := (a(1,1)m + a(1,2)n + a(1,3)p + a(1,4))u
      + (a(1,2)m + a(2,2)n + a(2,3)p + a(2,4))v
      + (a(1,3)m + a(2,3)n + a(3,3)p + a(3,4))w
> BD:=collect(BD,[m,n,p]);
BD := (a(1,1)u + a(1,2)v + a(1,3)w)m + (a(1,2)u + a(2,2)v + a(2,3)w)n
      + (a(1,3)u + a(2,3)v + a(3,3)w)p + a(1,4)u + a(2,4)v + a(3,4)w
> ra:=sort(coeff(BD,m),[u,v,w]); rb:=sort(coeff(BD,n),[u,v,w]);
rc:=sort(coeff(BD,p),[u,v,w]);
rd:=sort(coeff(coeff(coeff(BD,m,0),n,0),p,0),[u,v,w]);
      ra := a(1,1)u + a(1,2)v + a(1,3)w
      rb := a(1,2)u + a(2,2)v + a(2,3)w
      rc := a(1,3)u + a(2,3)v + a(3,3)w
      rd := a(1,4)u + a(2,4)v + a(3,4)w
> PrumerR:=ra*x+rb*y+rc*z+rd=0;
PrumerR := (a(1,1)u + a(1,2)v + a(1,3)w)x + (a(1,2)u + a(2,2)v + a(2,3)w)y
          + (a(1,3)u + a(2,3)v + a(3,3)w)z + a(1,4)u + a(2,4)v + a(3,4)w = 0
> U:=Vector[row]([sm[1],sm[2],sm[3],0]);
      U := [u, v, w, 0]
> PrumerR:=collect(expand(U.K.Transpose(X)),[x,y,z])=0;
PrumerR := (a(1,1)u + a(1,2)v + a(1,3)w)x + (a(1,2)u + a(2,2)v + a(2,3)w)y
          + (a(1,3)u + a(2,3)v + a(3,3)w)z + a(1,4)u + a(2,4)v + a(3,4)w = 0

```

Příklad: Průměrová rovina kvadriky $x^2 + 4y^2 + 16z^2 - 144 = 0$ sružená se směrem $u = (-4, 2, 1)$.

```

> kv:=x^2+4*y^2+16*z^2-144=0;
      kv := x^2 + 4y^2 + 16z^2 - 144 = 0
> a(1,1):=coeff(lhs(kv),x^2); a(2,2):=coeff(lhs(kv),y^2);
a(3,3):=coeff(lhs(kv),z^2);
a(1,2):=1/2*coeff(coeff(lhs(kv),x),y);
a(1,3):=1/2*coeff(coeff(lhs(kv),x),z);
a(1,4):=1/2*coeff(coeff(coeff(lhs(kv),x,1),y,0),z,0);
a(2,3):=1/2*coeff(coeff(lhs(kv),y,1),z);

```

```

a(2,4):=1/2*coeff(coeff(coeff(lhs(kv),y,1),x,0),z,0);
a(3,4):=1/2*coeff(coeff(coeff(lhs(kv),z,1),y,0),x,0);
a(4,4):=coeff(coeff(coeff(lhs(kv),x,0),y,0),z,0);

```

```
a(1,1) := 1
```

```
a(2,2) := 4
```

```
a(3,3) := 16
```

```
a(1,2) := 0
```

```
a(1,3) := 0
```

```
a(1,4) := 0
```

```
a(2,3) := 0
```

```
a(2,4) := 0
```

```
a(3,4) := 0
```

```
a(4,4) := -144
```

```
> K;
```

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & -144 \end{bmatrix}$$

```
> PrumerR;
```

$$u x + 4 v y + 16 w z = 0$$

```
> Smer1:=[-4,2,1];
```

$$Smer1 := [-4, 2, 1]$$

```
> PrumerR1:=eval(PrumerR,[u=Smer1[1],v=Smer1[2],w=Smer1[3]]);
```

$$PrumerR1 := -4 x + 8 y + 16 z = 0$$

```
> PrumerR1_ex:=solve(PrumerR1,z);
```

$$PrumerR1_ex := \frac{x}{4} - \frac{y}{2}$$

```
> res1:=solve({kv,PrumerR1},{y,z});
```

$$res1 := \{y = \frac{x}{2} - \frac{1}{2} \text{RootOf}(x^2 - x_Z + _Z^2 - 72, \text{label} = _L2),$$

$$z = \frac{1}{4} \text{RootOf}(x^2 - x_Z + _Z^2 - 72, \text{label} = _L2)\}$$

```
> Kuz:=allvalues(res1);
```

$$Kuz := \{y = \frac{x}{4} - \frac{\sqrt{-3x^2 + 288}}{4}, z = \frac{x}{8} + \frac{\sqrt{-3x^2 + 288}}{8}\},$$

$$\{y = \frac{x}{4} + \frac{\sqrt{-3x^2 + 288}}{4}, z = \frac{x}{8} - \frac{\sqrt{-3x^2 + 288}}{8}\}$$

```
> Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);
```

$$Kuz1 := \left[x, \frac{x}{4} - \frac{\sqrt{-3x^2 + 288}}{4}, \frac{x}{8} + \frac{\sqrt{-3x^2 + 288}}{8} \right]$$

$$Kuz2 := \left[x, \frac{x}{4} + \frac{\sqrt{-3x^2 + 288}}{4}, \frac{x}{8} - \frac{\sqrt{-3x^2 + 288}}{8} \right]$$

```

> T1:=evalf(eval(Kuz1,x=-9)); T2:=evalf(eval(Kuz1,x=5));
T3:=evalf(eval(Kuz2,x=8)); T4:=evalf(eval(Kuz2,x=3));

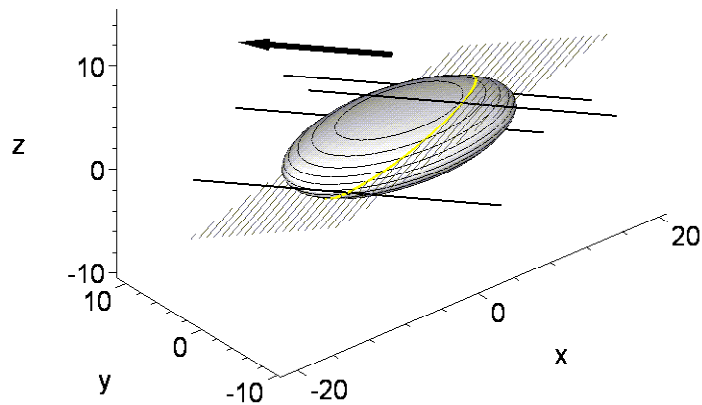
      T1 := [-9., -3.927050983, -0.2864745085]
      T2 := [5., -2.398629880, 2.449314940]
      T3 := [8., 4.449489743, -0.224744871]
      T4 := [3., 4.788873605, -1.644436802]

> tec1:=evalm(T1+t*Smer1); tec2:=evalm(T2+t*Smer1);
tec3:=evalm(T3+t*Smer1); tec4:=evalm(T4+t*Smer1);

      tec1 := [-9. - 4 t, -3.927050983 + 2 t, -0.2864745085 + t]
      tec2 := [5. - 4 t, -2.398629880 + 2 t, 2.449314940 + t]
      tec3 := [8. - 4 t, 4.449489743 + 2 t, -0.224744871 + t]
      tec4 := [3. - 4 t, 4.788873605 + 2 t, -1.644436802 + t]

> plotsetup(inline,plotoptions=`portrait,noborder,shrinkby=0`);
> tec1g:=plot3d(tec1,t=-3..3,s=-1..1,thickness=2):
tec2g:=plot3d(tec2,t=-3..3,s=0..1,thickness=2):
tec3g:=plot3d(tec3,t=-3..3,s=0..1,thickness=2):
tec4g:=plot3d(tec4,t=-3..3,s=0..1,thickness=2):
> kvg0:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,grid
=[30,30,30],style=patchcontour,color=red,lightmodel=light1,scaling=constrained):
> kvg:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,axes
=frame,color=COLOR(RGB,250/255,250/255,250/255),style=patchcontour,grid=[30,30,40],light=[90,-5,1,1,1],tickmarks=[3,3,3],orientation=[52,63],scaling=constrained):
> PrumerRov:=contourplot3d(PrumerR1_ex,x=-15..15,y=-10..10,grid=[2,2],contours=40,color=COLOR(RGB,50/255,50/255,50/255),filled=false):
> PrumerRov0:=implicitplot3d(PrumerR1,x=-15..15,y=-10..10,z=-10..10,style=patchnogrid,grid=[20,20,20],color=grey):
> Smerg:=plottools[arrow]([-5, -5, 12], vector(3*Smer1), 0.5, 1, 0.2, cylindrical_arrow,color=black):
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,color=yellow):
Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,color=yellow):
> display(kvg,PrumerRov,Smerg,Kuz1g,Kuz2g,tec1g,tec2g,tec3g,tec4g,axes=frame,orientation=[-130,60]);

```



```

> kvG:=plots[implicitplot3d](kv,x=-15..15,y=-10..10,z=-10..10,axes=frame,color=COLOR(
RGB,150/255,150/255,150/255),style=wireframe,grid=[20,20,40],light=[100,-20,1,1,1],tickmarks=[3,3,3],orientation=[52,63],scaling=constrained);
> St:=[1,2,eval(PrumerR1_ex,[x=1,y=2])]; Tet:=evalm(St+t*Smer1);
M1:=eval(Tet,solve(eval(kv,[x=Tet[1],y=Tet[2],z=Tet[3]]),{t}))[1]);
M2:=eval(Tet,solve(eval(kv,[x=Tet[1],y=Tet[2],z=Tet[3]]),{t}))[2]);

```

$$St := \begin{bmatrix} 1, 2, \frac{-3}{4} \end{bmatrix}$$

$$Tet := \begin{bmatrix} 1 - 4t, 2 + 2t, -\frac{3}{4} + t \end{bmatrix}$$

$$M1 := \begin{bmatrix} 1 - \frac{\sqrt{354}}{3}, 2 + \frac{\sqrt{354}}{6}, -\frac{3}{4} + \frac{\sqrt{354}}{12} \end{bmatrix}$$

$$M2 := \begin{bmatrix} 1 + \frac{\sqrt{354}}{3}, 2 - \frac{\sqrt{354}}{6}, -\frac{3}{4} - \frac{\sqrt{354}}{12} \end{bmatrix}$$

```

> Tetiva:=plot3d(evalm(M1+t*(M2-M1)),t=0..1,s=0..1,thickness=4):
> M11:=eval([x,y,z],[x=M1[1],y=M1[2],z=M1[3]]);

```

$$M11 := \left[1 - \frac{\sqrt{354}}{3}, 2 + \frac{\sqrt{354}}{6}, -\frac{3}{4} + \frac{\sqrt{354}}{12} \right]$$

> M22:=eval([x,y,z],[x=M2[1],y=M2[2],z=M2[3]]);

$$M22 := \left[1 + \frac{\sqrt{354}}{3}, 2 - \frac{\sqrt{354}}{6}, -\frac{3}{4} - \frac{\sqrt{354}}{12} \right]$$

> M1g:=plottools[sphere](M11,0.3):

M2g:=plottools[sphere](M22,0.3): Stg:=plottools[sphere](St,0.3):

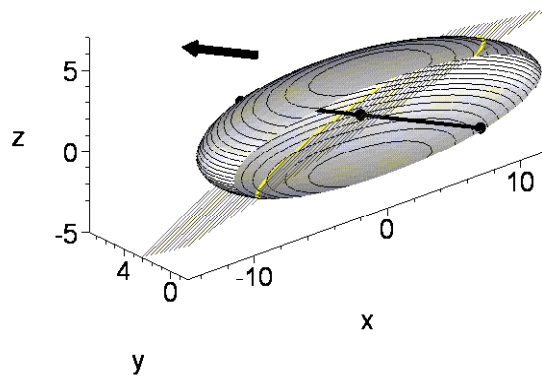
> Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,color=yellow):

Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,color=yellow):

> Smerg:=plottools[arrow]([-5, 4, 4], vector(Smer1), 0.5, 1, 0.2, cylindrical_arrow,color=black):

> PrumerRov:=contourplot3d(PrumerR1_ex,x=-15..15,y=-10..10,grid=[2,2],contours=40,color=COLOR(50/255,50/255,50/255),filled=false):

> display(Stg,M1g,M2g,Tetiva,kvg,PrumerRov,Kuz1g,Smerg,Kuz2g,axes=frame,orientation=[-131,66],view=[-15..12,-1.5..7,-5..7],scaling=constrained,light=[90,-5,1,1,1]);



Příklad: Průměrová rovina válcové plochy $x^2 + y^2 - 144 = 0$ sdružená se směrem $u = (1, 5, 2)$.

```
> kv2:=x^2+y^2-144=0;
```

```
kv2 := x^2 + y^2 - 144 = 0
```

```
> a(1,1):=coeff(lhs(kv2),x^2); a(2,2):=coeff(lhs(kv2),y^2);
```

```
a(3,3):=coeff(lhs(kv2),z^2);
```

```
a(1,2):=1/2*coeff(coeff(lhs(kv2),x),y);
```

```
a(1,3):=1/2*coeff(coeff(lhs(kv2),x),z);
```

```
a(1,4):=1/2*coeff(coeff(coeff(lhs(kv2),x,1),y,0),z,0);
```

```
a(2,3):=1/2*coeff(coeff(lhs(kv2),y,1),z);
```

```
a(2,4):=1/2*coeff(coeff(coeff(lhs(kv2),y,1),x,0),z,0);
```

```
a(3,4):=1/2*coeff(coeff(coeff(lhs(kv2),z,1),y,0),x,0);
```

```
a(4,4):=coeff(coeff(coeff(lhs(kv2),x,0),y,0),z,0);
```

```
a(1,1) := 1
```

```
a(2,2) := 1
```

```
a(3,3) := 0
```

$$a(1, 2) := 0$$

$$a(1, 3) := 0$$

$$a(1, 4) := 0$$

$$a(2, 3) := 0$$

$$a(2, 4) := 0$$

$$a(3, 4) := 0$$

$$a(4, 4) := -144$$

> **K;**

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -144 \end{bmatrix}$$

> **PrumerR;**

$$u x + v y = 0$$

> **Smer2:=[1,5,2];**

$$Smer2 := [1, 5, 2]$$

> **PrumerR2:=eval(PrumerR,[u=Smer2[1],v=Smer2[2],w=Smer2[3]]);**

$$PrumerR2 := x + 5 y = 0$$

> **res1:=solve({kv2,PrumerR2},{x,y});**

$$res1 := \{x = -30 \operatorname{RootOf}(13 _Z^2 - 2, \text{label} = _L5), y = 6 \operatorname{RootOf}(13 _Z^2 - 2, \text{label} = _L5)\}$$

> **Kuz:=allvalues(res1);**

$$Kuz := \left\{ x = -\frac{30\sqrt{26}}{13}, y = \frac{6\sqrt{26}}{13} \right\}, \left\{ x = \frac{30\sqrt{26}}{13}, y = -\frac{6\sqrt{26}}{13} \right\}$$

> **Kuz1:=eval([x,y,z],Kuz[1]); Kuz2:=eval([x,y,z],Kuz[2]);**

$$Kuz1 := \left[-\frac{30\sqrt{26}}{13}, \frac{6\sqrt{26}}{13}, z \right]$$

$$Kuz2 := \left[\frac{30\sqrt{26}}{13}, -\frac{6\sqrt{26}}{13}, z \right]$$

> **kvg:=plots[implicitplot3d](kv2,x=-15..15,y=-15..15,z=-10..10,grid=[30,30,30],style=patchcontour,color=red,lightmodel=light1,scaling=constrained);**

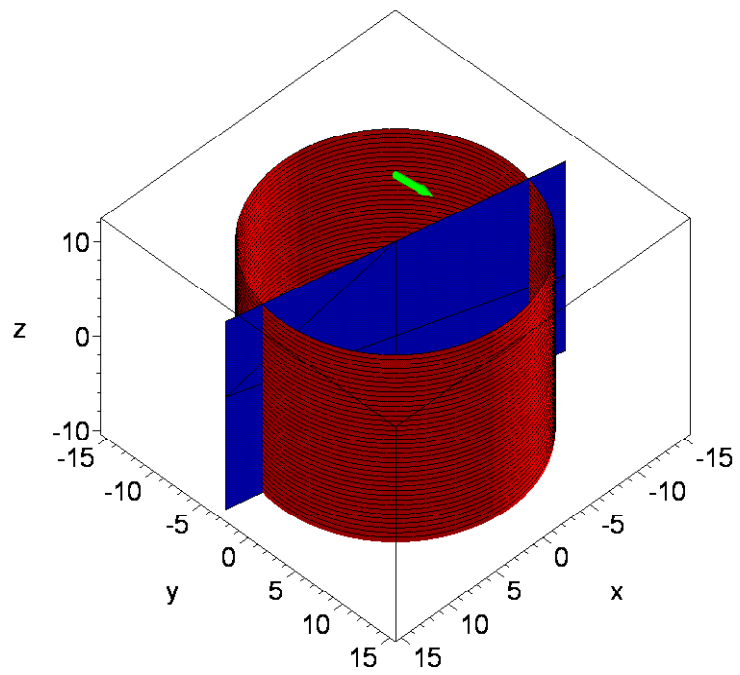
> **PrumerRg:=implicitplot3d(PrumerR2,x=-15..15,y=-15..15,z=-10..10,grid=[2,2,2],contours=40,color=blue);**

> **ug:=plottools[arrow]([-5, -5, 10], vector(Smer2), .6, .8, .3, cylindrical_arrow,color=green);**

> **Kuz1g:=spacecurve(Kuz1,x=-15..15,thickness=3,numpoints=1000,color=yellow);**

Kuz2g:=spacecurve(Kuz2,x=-15..15,thickness=3,numpoints=1000,color=yellow);

> **display(kvg,PrumerRg,ug,Kuz1g,Kuz2g,axes=boxed);**



[>
[>