

Výsledky cvičení z kapitoly 1

Cvičení 1:

$$(x+3)^2 + (y-2)^2 + (z+5)^2 = 38.$$

Cvičení 2:

$$(x-5)^2 + (y+4)^2 + (z-7)^2 = 100, S = [5, -4, 7], r = 10.$$

Cvičení 3:

a) středová regulární, dvojdílný hyperboloid, $\Delta = -172872$, $A_{44} = -2058$,
 $\lambda_1 = 7$, $\lambda_2 = 14$, $\lambda_3 = -21$, $u_1 = [1, \frac{3}{2}, 3]$, $u_2 = [3, 1, \frac{-3}{2}]$, $u_3 = [1, -2, \frac{2}{3}]$,

kanonická rovnice: $\frac{x^2}{12} + \frac{y^2}{6} - \frac{z^2}{4} = -1$, $a = 2\sqrt{3}$, $b = \sqrt{6}$, $c = 2$,

$$\alpha: 7x + \frac{21}{2}y + 21z - \frac{21}{2} = 0, \beta: 42x + 14y - 21z - 63 = 0,$$

$$\gamma: -21x + 42y - 14z - 140 = 0, o_1: x = \frac{3}{2} + \frac{3}{2}r, y = -3r, z = r,$$

$$o_2: x = -9 + 3q, y = q, z = -\frac{3}{2}q + \frac{7}{2}, o_3: x = p, y = \frac{3}{2}p + 3,$$

$$z = 3p - 1, S = [0, 3, -1]$$

b) nestředová regulární, eliptický paraboloid, $\Delta = -27$, $A_{44} = 0$,

$$\lambda_1 = 10 - 2\sqrt{7}, \lambda_2 = 10 + 2\sqrt{7}, \lambda_3 = 0, u_1 = [1, 3 - \sqrt{7}, -5 + 2\sqrt{7}],$$

$$u_2 = [\frac{5}{3} - \frac{2}{3}\sqrt{7}, \frac{1}{3} - \frac{1}{3}\sqrt{7}, 1], u_3 = [1, -2, -1],$$

kanonická rovnice: $\frac{x^2}{\frac{1}{4} \frac{\sqrt{6}}{10-2\sqrt{7}}} + \frac{y^2}{\frac{1}{4} \frac{\sqrt{6}}{10+2\sqrt{7}}} = 2z$, $a = \frac{1}{2} \sqrt{\left(\frac{\sqrt{6}}{10-2\sqrt{7}}\right)}$,

$$b = \frac{1}{2} \sqrt{\left(\frac{\sqrt{6}}{10+2\sqrt{7}}\right)},$$

$$\alpha: (10 - 2\sqrt{7})x + (44 - 16\sqrt{7})y + (-78 + 30\sqrt{7})z - \frac{5}{2} + 2\sqrt{7} = 0,$$

$$\beta: \left(\frac{22}{3} - \frac{10}{3}\sqrt{7}\right)x + \left(\frac{-4}{3} - \frac{8}{3}\sqrt{7}\right)y + (10 + 2\sqrt{7})z + \frac{31}{6} - \frac{5}{3}\sqrt{7} = 0,$$

$$o_1: x = \frac{-2}{3} - r, y = \frac{5}{24} + 2r, z = r, V = \left[\frac{-137}{144}, \frac{7}{9}, \frac{41}{144}\right]$$

c) nestředová singulární, parabolická válcová plocha, $\Delta = 0$, $A_{44} = 0$,

$$\lambda_1 = 26, \lambda_2 = 0, \lambda_3 = 0, u_1 = \left[\frac{-1}{3}, 1, \frac{4}{3}\right], u_2 = [3, 1, 0], u_3 = [4, 0, 1],$$

kanonická rovnice: $\frac{x^2}{1} = \frac{4}{17}\sqrt{17}z$, $a = \frac{1}{26}\sqrt{26}$,

$$\alpha: -\frac{26}{3}x + 26y + \frac{104}{3}z - \frac{8}{3} = 0$$

d) nestředová singulární, přímka (imaginární různoběžné roviny), $\Delta = 0$,

$$A_{44} = 0, \lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 0, u_1 = [1, 0, 1], u_2 = [1, 2, -1],$$

$$u_3 = [1, -1, -1],$$

kanonická rovnice: $\frac{x^2}{1} + \frac{y^2}{3} = 0$, $a = 1$, $b = \frac{1}{3}\sqrt{3}$, $\alpha: x - 2 + z = 0$,

$$\beta: 3x + 6y - 3z - 12 = 0, o_1: x = p, y = -p + 3, z = -p + 2$$

e) středová regulární, vejčitý rotační elipsoid, $\Delta = -64$, $A_{44} = 4$,
 $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = 2$, $u_1 = [0,0,1]$, $u_2 = [1,0,0]$, $u_3 = [0,1,0]$,
kanonická rovnice: $\frac{x^2}{16} + \frac{y^2}{8} + \frac{z^2}{8} = 1$, $a = 4$, $b = 2\sqrt{2}$, $c = 2\sqrt{2}$,
 $\alpha: z = 0$, $\beta: 2x - 2 = 0$, $\gamma: 2 + 2y = 0$, $o_1: x = 1$, $y = q$,
 $z = 0$, $o_2: x = p$, $y = -1$, $z = 0$, $o_3: x = 1$, $y = -1$, $z = r$,
 $S = [1, -1, 0]$

f) středová singulární, kuželová plocha, $\Delta = 0$, $A_{44} = -2.25$,
 $\lambda_1 = 3.17$, $\lambda_2 = -0.942$, $\lambda_3 = 0.762$, $u_1 = [1, -2.17, -1.02]$, $u_2 = [1, 1.94, -3.11]$,
 $u_3 = [1, 0.242, 0.468]$,
kanonická rovnice: $\frac{x^2}{0.316} - \frac{y^2}{1.06} + \frac{z^2}{1.32} = 0$, $a = 0.562$, $b = 1.03$, $c = 1.15$,
 $\alpha: 3.17x - 6.89y - 3.26z - 13.7 = 0$, $\beta: -0.942x - 1.80y + 2.91z - 5.56 = 0$,
 $\gamma: 0.762x + 0.180y + 0.35z - 0.76 = 0$, $o_1: x = 4.19q + 9.36$,
 $y = q$, $z = 1.96q + 4.93$, $o_2: x = 0.526q + 2.05$, $y = q$, $z = -1.60q - 2.20$,
 $o_3: x = 0.08 - 0.466q$, $y = q$, $z = 0.471q + 1.94$, $S = [1, -2, 1]$

g) nestředová regulární, hyperbolický paraboloid, $\Delta = 64$, $A_{44} = 0$,
 $\lambda_1 = 1 - 3\sqrt{5}$, $\lambda_2 = 1 + 3\sqrt{5}$, $\lambda_3 = 0$, $u_1 = [-\sqrt{5}, -\sqrt{5} - 3, 1]$,
 $u_2 = [\sqrt{5}, \sqrt{5} - 3, 1]$, $u_3 = [1, -1, -3]$,

kanonická rovnice: $-\frac{x^2}{\frac{4}{11} \frac{\sqrt{11}}{-1+3\sqrt{5}}} + \frac{y^2}{\frac{4}{11} \frac{\sqrt{11}}{1+3\sqrt{5}}} = 2z$, $a = \frac{2}{11} \sqrt{11} \sqrt{\left(\frac{\sqrt{11}}{-1+3\sqrt{5}}\right)}$,

$b = \frac{2}{11} \sqrt{11} \sqrt{\left(\frac{\sqrt{11}}{1+3\sqrt{5}}\right)}$,

$\alpha: (-\sqrt{5} + 15)x + (8\sqrt{5} + 12)y + (1 - 3\sqrt{5})z + 3\sqrt{5} + 15 = 0$,

$\beta: (\sqrt{5} + 15)x + (-8\sqrt{5} + 12)y + (1 + 3\sqrt{5})z - 3\sqrt{5} + 15 = 0$,

$o_1: x = p$, $y = -p - \frac{12}{11}$, $z = -3p - \frac{21}{11}$, $V = \left[\frac{-1433}{968}, \frac{377}{968}, \frac{2451}{968}\right]$

h) středová regulární, trojosý elipsoid, $\Delta = -5648$, $A_{44} = 324$, $\lambda_1 = 3$,
 $\lambda_2 = 6$, $\lambda_3 = 18$, $u_1 = [\frac{1}{2}, 1, 1]$, $u_2 = [2, 1, -2]$, $u_3 = [2, -2, 1]$,

kanonická rovnice: $\frac{x^2}{\frac{1412}{243}} + \frac{y^2}{\frac{706}{243}} + \frac{z^2}{\frac{706}{729}} = 1$, $a = \frac{2}{27} \sqrt{1059}$, $b = \frac{1}{27} \sqrt{2118}$,

$c = \frac{1}{27} \sqrt{706}$, $\alpha: \frac{3}{2}x + 3y + 3z - \frac{11}{2} = 0$, $\beta: 12x + 6y - 12z - 22 = 0$,

$\gamma: 36x - 36y + 18z - 22 = 0$, $o_1: x = \frac{11}{9} + 2r$, $y = \frac{11}{9} - 2r$, $z = r$,

$o_2: x = \frac{-11}{27} + 2q$, $y = q$, $z = -2q + \frac{55}{27}$, $o_3: x = p$, $y = 2p - \frac{55}{27}$,

$z = 2p - \frac{77}{27}$, $S = \left[\frac{121}{81}, \frac{77}{81}, \frac{11}{81}\right]$

i) nestředová singulární, dvě rovnoběžné roviny, $\Delta = 0$, $A_{44} = 0$,

$\lambda_1 = 14$, $\lambda_2 = 0$, $\lambda_3 = 0$, $u_1 = \left[\frac{-3}{2}, 1, \frac{-1}{2}\right]$, $u_2 = [1, 0, -3]$, $u_3 = [0, 1, 2]$,

kanonická rovnice: $\frac{x^2}{8} = 1$, $a = \frac{1}{4}\sqrt{2}$, $\alpha: -21x + 14y - 7z - \frac{7}{2} = 0$

j) nestředová singulární, eliptická válcová plocha, $\Delta = 0$, $A_{44} = 0$,

$\lambda_1 = \frac{7}{2} - \frac{1}{2}\sqrt{13}$, $\lambda_2 = \frac{7}{2} + \frac{1}{2}\sqrt{13}$, $\lambda_3 = 0$, $u_1 = \left[1, \frac{-5}{2} + \frac{1}{2}\sqrt{13}, 3 - \sqrt{13}\right]$,

$u_2 = \left[1, \frac{-5}{2} - \frac{1}{2}\sqrt{13}, 3 + \sqrt{13}\right]$, $u_3 = [2, 2, 1]$,

kanonická rovnice: $\frac{x^2}{\frac{7}{2} - \frac{1}{2}\sqrt{13}} + \frac{y^2}{\frac{7}{2} + \frac{1}{2}\sqrt{13}} = 1$, $a = \frac{\sqrt{5}}{\sqrt{\left(\frac{7}{2} - \frac{1}{2}\sqrt{13}\right)}}$, $b = \frac{\sqrt{5}}{\sqrt{\left(\frac{7}{2} + \frac{1}{2}\sqrt{13}\right)}}$,

$\alpha: \left(\frac{7}{2} - \frac{1}{2}\sqrt{13}\right)x + (-12 + 3\sqrt{13})y + (17 - 5\sqrt{13})z + \frac{7}{2} - \frac{1}{2}\sqrt{13} = 0$,

$$\beta: \left(\frac{7}{2} + \frac{1}{2}\sqrt{13}\right)x + (-12 - 3\sqrt{13})y + (17 + 5\sqrt{13})z + \frac{7}{2} + \frac{1}{2}\sqrt{13} = 0,$$

$$o_1: x = -1 + 2r, y = 2r, z = r$$

k) nestředová singulární, hyperbolická válcová plocha, $\Delta = 0$, $A_{44} = 0$,
 $\lambda_1 = -\frac{1}{2}\sqrt{66}$, $\lambda_2 = \frac{1}{2}\sqrt{66}$, $\lambda_3 = 0$, $u_1 = \left[\frac{20}{19} - \frac{7}{38}\sqrt{66}, 1, \frac{-8}{19} - \frac{1}{38}\sqrt{66}\right]$,
 $u_2 = \left[1, \frac{-40}{43} + \frac{7}{43}\sqrt{66}, \frac{29}{43} - \frac{4}{43}\sqrt{66}\right]$, $u_3 = [1, -4, -7]$,

$$\text{kanonická rovnice: } -\frac{x^2}{\frac{2}{33}\sqrt{66}} + \frac{y^2}{\frac{2}{33}\sqrt{66}} = 1, \quad a = \frac{1}{33}66^{\frac{3}{4}}, \quad b = \frac{1}{33}66^{\frac{3}{4}},$$

$$\alpha: \left(\frac{231}{38} - \frac{10}{19}\sqrt{66}\right)x - \frac{1}{2}\sqrt{66}y + \left(\frac{33}{38} + \frac{4}{19}\sqrt{66}\right)z - \frac{33}{38} + \frac{15}{19}\sqrt{66} = 0,$$

$$\beta: \frac{1}{2}\sqrt{66}x + \left(\frac{231}{43} - \frac{20}{43}\sqrt{66}\right)y + \left(\frac{-132}{43} + \frac{29}{86}\sqrt{66}\right)z - \frac{330}{43} + \frac{51}{86}\sqrt{66} = 0,$$

$$o_1: x = p, y = -4p + 2, z = -7p + 1$$

l) středová regulární, imaginární elipsoid, $\Delta = 750$, $A_{44} = \frac{375}{4}$, $\lambda_1 = \frac{5}{2}$,
 $\lambda_2 = 5$, $\lambda_3 = \frac{15}{2}$, $u_1 = \left[\frac{4}{3}, 1, \frac{-5}{3}\right]$, $u_2 = \left[1, \frac{-4}{3}, 0\right]$, $u_3 = \left[\frac{4}{3}, 1, \frac{5}{3}\right]$,

$$\text{kanonická rovnice: } \frac{x^2}{\frac{16}{5}} + \frac{y^2}{\frac{8}{5}} + \frac{z^2}{\frac{16}{15}} = -1, \quad a = \frac{4}{5}\sqrt{5}, \quad b = \frac{2}{5}\sqrt{10}, \quad c = \frac{4}{15}\sqrt{15},$$

$$\alpha: \frac{10}{3}x + \frac{5}{2}y - \frac{25}{6}z - \frac{5}{3} = 0, \quad \beta: 5x - \frac{20}{3}y + \frac{5}{3} = 0,$$

$$\gamma: 10x + \frac{15}{2}y + \frac{25}{2}z - 30 = 0, \quad o_1: x = \frac{-1}{3} + \frac{4}{3}q, y = q, z = \frac{5}{3}q - \frac{2}{3},$$

$$o_2: x = p, y = -\frac{4}{3}p + \frac{7}{3}, z = 1, \quad o_3: x = \frac{-1}{3} + \frac{4}{3}q, y = q, z = -\frac{5}{3}q + \frac{8}{3},$$

$$S = [1, 1, 1]$$

m) nestředová singulární, rotační válcová plocha, $\Delta = 0$, $A_{44} = 0$,

$$\lambda_1 = 3, \lambda_2 = 3, \lambda_3 = 0, \quad u_1 = [1, 0, -1], \quad u_2 = [0, 1, 1], \quad u_3 = [1, -1, 1],$$

$$\text{kanonická rovnice: } \frac{x^2}{26} + \frac{y^2}{26} = 1, \quad a = \sqrt{26}, \quad b = \sqrt{26}, \quad \alpha: 3x - 15 - 3z = 0,$$

$$\beta: 21 + 3y + 3z = 0, \quad o_1: x = p, y = -p - 2, z = -5 + p$$

n) nestředová singulární, dvojnásobná rovina, $\Delta = 0$, $A_{44} = 0$, $\lambda_1 = 11$,

$$\lambda_2 = 0, \lambda_3 = 0, \quad u_1 = [1, -3, 1], \quad u_2 = [1, 0, -1], \quad u_3 = [0, 1, 3],$$

$$\text{kanonická rovnice: } \frac{x^2}{11} = 0, \quad a = \frac{1}{11}\sqrt{11}, \quad \alpha: 11x - 33y + 11z + 44 = 0$$

o) nestředová singulární, hyperbolická válcová plocha, $\Delta = 0$, $A_{44} = 0$,

$$\lambda_1 = -4, \lambda_2 = 9, \lambda_3 = 0, \quad u_1 = [1, -1, 0], \quad u_2 = [1, 1, 1], \quad u_3 = [1, 1, -2],$$

$$\text{kanonická rovnice: } \frac{x^2}{9} - \frac{y^2}{4} = 1, \quad a = 3, \quad b = 2, \quad \alpha: -4x + 4y - 4 = 0,$$

$$\beta: 9x + 9y + 9z - 9 = 0, \quad o_1: x = p, y = p + 1, z = -2p$$

p) středová regulární, rotační dvojdílný hyperboloid, $\Delta = \frac{-1}{4}$, $A_{44} = \frac{1}{4}$,

$$\lambda_1 = 1, \lambda_2 = \frac{-1}{2}, \lambda_3 = \frac{-1}{2}, \quad u_1 = [1, 1, 1], \quad u_2 = [-1, 1, 0], \quad u_3 = [-1, 0, 1],$$

$$\text{kanonická rovnice: } -\frac{x^2}{1} + \frac{y^2}{2} + \frac{z^2}{2} = -1, \quad a = 1, \quad b = \sqrt{2}, \quad c = \sqrt{2},$$

$$\alpha: x + y + z = 0, \quad \beta: \frac{1}{2}x - \frac{1}{2}y = 0, \quad \gamma: \frac{1}{2}x - \frac{1}{2}z = 0, \quad o_1: x = p,$$

$$y = p, z = -2p, \quad o_2: x = p, y = -2p, z = p, \quad o_3: x = p, y = p,$$

$$z = p, \quad S = [0, 0, 0]$$

q) nestředová regulární, hyperbolický paraboloid, $\Delta = 16$, $A_{44} = 0$,

$$\lambda_1 = -5, \lambda_2 = 6, \lambda_3 = 0, \quad u_1 = [1, -2, 0], \quad u_2 = [2, 1, -1], \quad u_3 = [2, 1, 5],$$

$$\text{kanonická rovnice: } -\frac{x^2}{\frac{2}{75}\sqrt{30}} + \frac{y^2}{\frac{1}{45}\sqrt{30}} = -2z, \quad a = \frac{1}{15}\sqrt{6}\sqrt[4]{30}, \quad b = \frac{1}{15}\sqrt{5}\sqrt[4]{30},$$

$$\alpha: -5x + 10y - 8 = 0, \quad \beta: 12x + 6y - 6z - 2 = 0, \quad o_1: x = \frac{-8}{5} + 2q,$$

$$y = q, z = 5q - \frac{53}{15}, \quad V = \left[\frac{-739}{900}, \frac{701}{1800}, \frac{-571}{360}\right]$$

r) středová singulární, rotační kuželová plocha, $\Delta = 0$, $A_{44} = 128$,
 $\lambda_1 = 32$, $\lambda_2 = -2$, $\lambda_3 = -2$, $u_1 = [-1, 1, 0]$, $u_2 = [1, 1, 0]$, $u_3 = [0, 0, 1]$,
kanonická rovnice: $-\frac{x^2}{\frac{1}{32}} + \frac{y^2}{\frac{1}{2}} + \frac{z^2}{\frac{1}{2}} = 0$, $a = \frac{1}{8}\sqrt{2}$, $b = \frac{1}{2}\sqrt{2}$, $c = \frac{1}{2}\sqrt{2}$,
 $\alpha: -32x + 32y + 32 = 0$, $\beta: -2x - 2y + 2 = 0$, $\gamma: -2z = 0$,
 $o_1: x = 1, y = 0, z = r$, $o_2: x = q + 1, y = q, z = 0$,
 $o_3: x = -q + 1, y = q, z = 0$, $S = [1, 0, 0]$

s) nestředová regulární, rotační paraboloid, $\Delta = -13$, $A_{44} = 0$, $\lambda_1 = 13$,
 $\lambda_2 = 13$, $\lambda_3 = 0$, $u_1 = [1, 0, 0]$, $u_2 = [0, 1, \frac{3}{2}]$, $u_3 = [0, \frac{-3}{2}, 1]$,
kanonická rovnice: $\frac{x^2}{\frac{1}{169}\sqrt{13}} + \frac{y^2}{\frac{1}{169}\sqrt{13}} = 2z$, $a = \frac{1}{13}\sqrt[4]{13}$, $b = \frac{1}{13}\sqrt[4]{13}$,
 $\alpha: 13x + \frac{5}{2} = 0$, $\beta: \frac{-3}{4} + 13y + \frac{39}{2}z = 0$, $o_1: x = \frac{-5}{26}, y = \frac{3}{52} - \frac{3}{2}r$,
 $z = r$, $V = [\frac{-5}{26}, \frac{-501}{676}, \frac{90}{169}]$