11. cvičení

Exponenciální a logaritmické rovnice

Řešte rovnice:

$\boxed{1}$ $e^x = \dfrac{1}{e}$ $\qquad$ $\boxed{2}$ $e^x = e$ $\qquad$ $\boxed{3}$ $e^x = 1$

$\boxed{4}$ $e^x = -e^2$ $\qquad$ $\boxed{5}$ $e^x = 0$ $\qquad$ $\boxed{6}$ $\sqrt[3]{e} = e^x$

$\boxed{7}$ $(e^{2x})^x = e^8$ $\qquad$ $\boxed{8}$ $e^{3x} \cdot e^{4x} = e^{12}$ $\qquad$ $\boxed{9}$ $e^{x^2} = (e^x)^3$

$\boxed{10}$ $e^x \cdot \sqrt{e^3} = 1$ $\qquad$ $\boxed{11}$ $(e^{x+1})^2 = e^{2(x+1)}$ $\qquad$ $\boxed{12}$ $\dfrac{e^{3x}}{e^{x+1}} = e^2$

$\boxed{13}$ $\ln x = 1$ $\qquad$ $\boxed{14}$ $\ln x = 0$ $\qquad$ $\boxed{15}$ $\ln x = \dfrac{1}{2}$

$\boxed{16}$ $\ln x = -\dfrac{1}{3}$ $\qquad$ $\boxed{17}$ $\ln(e^3) = x$ $\qquad$ $\boxed{18}$ $\ln\left(\frac{1}{\sqrt{e}}\right) = x$

$\boxed{19}$ $\ln(-2) = x$ $\qquad$ $\boxed{20}$ $3\ln x + 5 = 0$ $\qquad$ $\boxed{21}$ $2 - 5\ln x = 0$

$\boxed{22}$ $\ln(x^2 - 2) = \ln x$ $\qquad$ $\boxed{23}$ $\ln^2 x - 3\ln x = 0$ $\qquad$ $\boxed{24}$ $\ln|x| = 3$

$\boxed{25}$ $\big|\ln x\big| = 3$ $\qquad$ $\boxed{26}$ $e^{2\ln x} = 4$ $\qquad$ $\boxed{27}$ $e^{\ln x^2} = 4$

Výsledky: $\boxed{1}$ -1; $\boxed{2}$ 1; $\boxed{3}$ 0; $\boxed{4}$ nemá řešení; $\boxed{5}$ nemá řešení; $\boxed{6}$ $\frac{1}{3}$; $\boxed{7}$ ± 2; $\boxed{8}$ $\frac{12}{7}$; $\boxed{9}$ $0,\ 3$; $\boxed{10}$ $-\frac{3}{2}$; $\boxed{11}$ $x \in \mathbb{R}$; $\boxed{12}$ $\frac{3}{2}$; $\boxed{13}$ e; $\boxed{14}$ 1; $\boxed{15}$ $\sqrt{e}$; $\boxed{16}$ $\frac{1}{\sqrt[3]{e}}$; $\boxed{17}$ 3; $\boxed{18}$ $-\frac{1}{2}$; $\boxed{19}$ nemá řešení; $\boxed{20}$ $\frac{1}{\sqrt[3]{e^5}}$; $\boxed{21}$ $\sqrt[5]{e^2}$; $\boxed{22}$ 2; $\boxed{23}$ $1,\ e^3$; $\boxed{24}$ $\pm e^3$; $\boxed{25}$ $e^3,\ \frac{1}{e^3}$; $\boxed{26}$ 2; $\boxed{27}$ ± 2.

Goniometrické rovnice

Řešte rovnice.

$\boxed{1}$ $\cos x = 0$ $\qquad$ $\boxed{2}$ $\sin x = -\dfrac{1}{2}$ $\qquad$ $\boxed{3}$ $\cos x = \dfrac{\sqrt{3}}{2}$

$\boxed{4}$ $\sin x = 0$ $\qquad$ $\boxed{5}$ $\cos x = -\dfrac{\sqrt{2}}{2}$ $\qquad$ $\boxed{6}$ $\sin x = -\dfrac{\sqrt{3}}{2}$

$\boxed{7}$ $\cos 0 = x$ $\qquad$ $\boxed{8}$ $\sin x = 1$ $\qquad$ $\boxed{9}$ $\operatorname{tg} x = \sqrt{3}$

$\boxed{10}$ $\operatorname{cotg} x = -1$ $\qquad$ $\boxed{11}$ $\operatorname{tg} x = -\dfrac{\sqrt{3}}{3}$ $\qquad$ $\boxed{12}$ $\operatorname{cotg} x = \sqrt{3}$

$\boxed{13}$ $2\cos^2 x = \cos x$ $\qquad$ $\boxed{14}$ $\sin x - \sqrt{3} \cdot \cos x = 0$ $\qquad$ $\boxed{15}$ $\sin x + \cos x = 0$

$\boxed{16}$ $\sqrt{2}\cos x + 2\sin x \cos x = 0$ $\qquad$ $\boxed{17}$ $\sin^2 x - \cos x - 1 = 0$

$\boxed{18}$ $\operatorname{cotg}^2 x = \sqrt{3} \cdot \operatorname{cotg} x$ $\qquad$ $\boxed{19}$ $2\sin^2 x + 3\sin x + 1 = 0$

■■ U příkladů z minulého odstavce určete navíc všechna řešení ležící v intervalu a) $\langle 0, 2\pi \rangle$, b) $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$.

Úpravy výrazů

Zjednodušte výrazy. Určete, pro jaká x mají dané výrazy smysl.

$\boxed{1}$ $\dfrac{\frac{3}{x}}{\frac{2}{x-1}} + 2$

$\boxed{2}$ $\dfrac{1}{1 + \left(\frac{x}{3}\right)^2}$

$\boxed{3}$ $\dfrac{1}{x+1} - \dfrac{x}{x-2}$

$\boxed{4}$ $\dfrac{1}{(x-1)^2} + \dfrac{1}{(x+1)^2}$

$\boxed{5}$ $\dfrac{1}{(x+1)^2} - \dfrac{1}{(x-1)^2}$

$\boxed{6}$ $\dfrac{1}{\frac{x+2}{x-2}} \cdot \dfrac{4}{(x-2)^2}$

$\boxed{7}$ $\dfrac{1}{3\frac{2x}{x^2+1} + \frac{x^2-1}{x^2+1} - 1} \cdot \dfrac{2}{1+x^2}$

$\boxed{8}$ $\dfrac{1 - \frac{x^2-1}{x^2+1}}{\frac{2x}{x^2+1} + 2} \cdot \dfrac{2}{x^2+1}$

$\boxed{9}$ $\dfrac{\sqrt{x}}{\sqrt{x}+3} + \dfrac{\sqrt{x}-3}{\sqrt{x}}$

$\boxed{10}$ $\dfrac{1}{\sqrt{x^2+1}-x}\left(1 - \dfrac{x}{\sqrt{x^2+1}}\right)$

$\boxed{11}\quad \dfrac{1}{\sqrt[3]{x}} + \sqrt[3]{x^2}$

$\boxed{12}\quad \sqrt[3]{x^4} + (x+7)\cdot\dfrac{4}{3}\cdot\sqrt[3]{x}$

$\boxed{13}\quad \sqrt[3]{x} + (x+4)\cdot\dfrac{1}{3}\cdot\dfrac{1}{\sqrt[3]{x^2}}$

$\boxed{14}\quad \dfrac{\sqrt{x^2+1} - (x-3)\cdot\frac{x}{\sqrt{x^2+1}}}{x^2+1}$

$\boxed{15}\quad \dfrac{\frac{2}{3\sqrt[3]{x}}\cdot(2x+1) - 2\sqrt[3]{x^2}}{(2x+1)^2}$

$\boxed{16}\quad \dfrac{\sin x\cdot(2+\sin x) + \cos^2 x}{(2+\sin x)^2}$

$\boxed{17}\quad \dfrac{\cos x\cdot(1-\cos x) - \sin^2 x}{(1-\cos x)^2}$

$\boxed{18}\quad \dfrac{(\operatorname{cotg}^2 x + 1)(1-\cos x)}{\sin^2 x}$

$\boxed{19}\quad \dfrac{x^3}{(1+x)^2} - x$

$\boxed{20}\quad \dfrac{x(x+3)^2 + (2-x^2)(x+3)}{(x+3)^4}$

$\boxed{21}\quad \dfrac{3x^2(2-x)^2 + 2x^3(2-x)}{(2-x)^4}$

$\boxed{22}\quad \dfrac{3(x+1)^2(1-x)^5 + 5(x+1)^3(1-x)^4}{(1-x)^{10}}$

$\boxed{23}\quad \dfrac{\sqrt[3]{x^6 - x^3}}{x^2}$

$\boxed{24}\quad \sqrt{4+x^2}\cdot\sqrt{1 + \dfrac{1}{4+x^2}}$

$\boxed{25}\quad \dfrac{x-7}{3 - \sqrt{x+2}}$

$\boxed{26}\quad \dfrac{x-1}{\sqrt{x^2+1} - \sqrt{x+1}}$

Výsledky: $\boxed{1}$ $\frac{7x-3}{2x}$, $x\in(-\infty,0)\cup(0,1)\cup(1,\infty)$; $\boxed{2}$ $\frac{9}{9+x^2}$, $x\in\mathbb{R}$; $\boxed{3}$ $-\frac{x^2+2}{(x+1)(x-2)}$, $x\in(-\infty,-1)\cup(-1,2)\cup(2,\infty)$; $\boxed{4}$ $\frac{2(x^2+1)}{(x-1)^2(x+1)^2}$, $x\in(-\infty,-1)\cup(-1,1)\cup(1,\infty)$; $\boxed{5}$ $-\frac{4x}{(x+1)^2(x-1)^2}$, $x\in(-\infty,-1)\cup(-1,1)\cup(1,\infty)$; $\boxed{6}$ $\frac{4}{(x+2)(x-2)}$, $x\in(-\infty,-2)\cup(-2,2)\cup(2,\infty)$; $\boxed{7}$ $\frac{1}{3x-1}$, $x\in(-\infty,\frac{1}{3})\cup(\frac{1}{3},\infty)$; $\boxed{8}$ $\frac{2}{(x^2+x+1)(x^2+1)}$, $x\in\mathbb{R}$; $\boxed{9}$ $\frac{2x-9}{\sqrt{x}(\sqrt{x}+3)}$, $x\in(0,\infty)$; $\boxed{10}$ $\frac{1}{\sqrt{x^2+1}}$, $x\in\mathbb{R}$; $\boxed{11}$ $\frac{1+x}{\sqrt[3]{x}}$, $x\in(-\infty,0)\cup(0,\infty)$; $\boxed{12}$ $\frac{7}{3}\sqrt[3]{x}(x+4)$, $x\in\mathbb{R}$; $\boxed{13}$ $\frac{4(x+1)}{3\sqrt[3]{x^2}}$, $x\in(-\infty,0)\cup(0,\infty)$; $\boxed{14}$ $\frac{3x+1}{\sqrt{(x^2+1)^3}}$, $x\in\mathbb{R}$; $\boxed{15}$ $\frac{2(1-x)}{3\sqrt[3]{x}(2x+1)^2}$, $x\in(-\infty,-\frac{1}{2})\cup(-\frac{1}{2},0)\cup(0,\infty)$; $\boxed{16}$ $\frac{2\sin x+1}{(2+\sin x)^2}$, $x\in\mathbb{R}$; $\boxed{17}$ $\frac{1}{\cos x-1}$, $x\in\bigcup_{k\in\mathbb{Z}}(2k\pi, 2\pi+2k\pi)$; $\boxed{18}$ $\frac{1}{\sin^2 x(1+\cos x)}$, $x\in\bigcup_{k\in\mathbb{Z}}(k\pi, \pi+k\pi)$; $\boxed{19}$ $-\frac{x(1+2x)}{(1+x)^2}$, $x\in(-\infty,-1)\cup(-1,\infty)$; $\boxed{20}$ $\frac{3x+2}{(x+3)^3}$, $x\in(-\infty,-3)\cup(-3,\infty)$; $\boxed{21}$ $\frac{x^2(6-x)}{(2-x)^3}$, $x\in(-\infty,2)\cup(2,\infty)$; $\boxed{22}$ $\frac{2(x+1)^2(x+4)}{(1-x)^6}$, $x\in(-\infty,1)\cup(1,\infty)$; $\boxed{23}$ $\frac{\sqrt[3]{x^3-1}}{x}$, $x\in(-\infty,0)\cup(0,\infty)$; $\boxed{24}$ $\sqrt{x^2+5}$, $x\in\mathbb{R}$; $\boxed{25}$ $-3-\sqrt{x+2}$, $x\in\langle-2,7)\cup(7,\infty)$; $\boxed{26}$ $\frac{\sqrt{x^2+1}+\sqrt{x+1}}{x}$, $x\in\langle-1,0)\cup(0,1)\cup(1,\infty)$.