16. Cvičení

Vyšetřete průběh funkce.

1) $f(x) = x^3 - 3x^2$

2) $f(x) = (2-x)\cdot(x+1)^2$

3) $f(x) = \operatorname{arctg}\left(\frac{1}{x+1}\right)$

4) $f(x) = \sin x + \cos x$

5) $f(x) = x^2 \cdot e^{-x}$

6) $f(x) = x \cdot \sqrt{1-x}$

7) $f(x) = \frac{x^3}{(x-1)^2}$

8) $f(x) = \ln(x + \sqrt{x^2+1})$

9) $f(x) = \frac{\sin x}{2+\cos x}$

10) $f(x) = (x+5)\cdot \sqrt[3]{x^2}$

11) $f(x) = \frac{\ln x}{\sqrt{x}}$

12) $f(x) = \ln(2\cos x - 1)$

13) $f(x) = \frac{x}{(x-2)^3}$

14) $f(x) = \frac{2-x^2}{(x+3)^2}$

Výsledky:
1) $D(f) = \mathbb{R}$; $\lim_{x\to\pm\infty} f(x) = \pm\infty$; $f'(x) = 3x^2 - 6x$, $f''(x) = 6x - 6$.
2) $D(f) = \mathbb{R}$; $\lim_{x\to\pm\infty} f(x) = \mp\infty$, $f'(x) = 3(x+1)(1-x)$; $f''(x) = -6x$.
3) $D(f) = (-\infty,-1)\cup(-1,\infty)$; $\lim_{x\to\pm\infty} f(x) = 0$, $\lim_{x\to-1-} f(x) = -\frac{\pi}{2}$, $\lim_{x\to-1+} f(x) = \frac{\pi}{2}$; $f'(x) = \frac{-1}{x^2+2x+2}$, $f''(x) = \frac{2(x+1)}{(x^2+2x+2)^2}$.
4) $D(f) = \mathbb{R}$; 2π-periodická funkce; $f'(x) = \cos x - \sin x$, $f''(x) = -\sin x - \cos x$.
5) $D(f) = \mathbb{R}$; $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 0$; $f'(x) = x\cdot e^{-x}\cdot(2-x)$, $f''(x) = e^{-x}\cdot(x^2-4x+2)$.
6) $D(f) = (-\infty,1\rangle$; $\lim_{x\to-\infty} f(x) = -\infty$; $f'(x) = \frac{2-3x}{2\sqrt{1-x}}$, $f''(x) = \frac{3x-4}{4(1-x)\sqrt{1-x}}$.
7) $D(f) = (-\infty,1)\cup(1,\infty)$; $\lim_{x\to\pm\infty} f(x) = \pm\infty$, $\lim_{x\to1} f(x) = \infty$; $f'(x) = \frac{x^2\cdot(x-3)}{(x-1)^3}$, $f''(x) = \frac{6x}{(x-1)^4}$.
8) $D(f) = \mathbb{R}$; lichá funkce; $\lim_{x\to\pm\infty} f(x) = \pm\infty$; $f'(x) = \frac{1}{\sqrt{x^2+1}}$, $f''(x) = -\frac{x}{(x^2+1)\sqrt{x^2+1}}$.
9) $D(f) = \mathbb{R}$; lichá funkce, 2π-periodická; $f'(x) = \frac{1+2\cos x}{(2+\cos x)^2}$, $f''(x) = \frac{\sin x\cdot(\cos x-1)}{(2+\cos x)^3}$.
10) $D(f) = \mathbb{R}$; $\lim_{x\to\pm\infty} f(x) = \pm\infty$; $f'(x) = \frac{5(x+2)}{3\sqrt[3]{x}}$, $f''(x) = \frac{10(x-1)}{9\sqrt[3]{x^4}}$.
11) $D(f) = (0,\infty)$; $\lim_{x\to0+} f(x) = -\infty$, $\lim_{x\to\infty} f(x) = 0$; $f'(x) = \frac{2-\ln x}{2x\sqrt{x}}$, $f''(x) = \frac{3\ln x-8}{4x^2\sqrt{x}}$.
12) $D(f) = \bigcup_{k\in\mathbb{Z}}(-\frac{\pi}{3}+2k\pi, \frac{\pi}{3}+2k\pi)$; sudá funkce, 2π-periodická; $\lim_{x\to\frac{\pi}{3}-} f(x) = -\infty$, $f'(x) = \frac{-2\sin x}{2\cos x-1}$, $f''(x) = \frac{2(\cos x-2)}{(2\cos x-1)^2}$.
13) $D(f) = (-\infty,2)\cup(2,\infty)$; $\lim_{x\to\pm\infty} f(x) = 0$, $\lim_{x\to2-} f(x) = -\infty$, $\lim_{x\to2+} f(x) = \infty$; $f'(x) = \frac{-2(x+1)}{(x-2)^4}$, $f''(x) = \frac{6(x+2)}{(x-2)^5}$.
14) $D(f) = (-\infty,-3)\cup(-3,\infty)$; $\lim_{x\to\pm\infty} f(x) = -1$, $\lim_{x\to-3} f(x) = -\infty$; $f'(x) = \frac{-2(3x+2)}{(x+3)^3}$, $f''(x) = \frac{6(2x-1)}{(x+3)^4}$.