9. cvičení

Určete derivaci funkce F ve směru s v bodě A :

1 $F(x,y) = x^3 + y^3 - xy,\ s = (1,2),\ A = [3,1]$

2 $F(x,y) = \cos(x + \pi y),\ s = (-1,1),\ A = [\frac{\pi}{2}, 1]$

3 $F(x,y) = \sin(x - y),\ s = (2,1),\ A = [\frac{\pi}{2}, \frac{\pi}{4}]$

4 $F(x,y) = \dfrac{x(x+y)}{x-y},\ s = (1,-2),\ A = [3,2]$

5 $F(x,y) = \ln(x^2 + y^4),\ s = (-3,2),\ A = [4,1]$

6 $F(x,y) = e^{x+3y},\ s = (1,-4),\ A = [3,2]$

7 $F(x,y) = \operatorname{arctg}\left(\dfrac{x}{y}\right),\ s = (2,1),\ A = [1,2]$

8 $F(x,y) = \sqrt{x-y},\ s = (3,-1),\ A = [2,2]$

9 $F(x,y) = \ln(y - x),\ s = (5,1),\ A = [2,1]$

10 $F(x,y) = \ln(x - y),\ s = (5,1),\ A = [2,1]$

11 $F(x,y) = \ln(x - y),\ s = (1,1),\ A = [3,1]$

12 $F(x,y) = \sqrt{xy},\ s = (1,3),\ A = [-2,-2]$

Mezivýsledky — parciální derivace: **1** $3x^2 - y,\ 3y^2 - x$; **2** $-\sin(x+\pi y),\ -\pi\cdot\sin(x+\pi y)$; **3** $\cos(x-y),\ -\cos(x-y)$; **4** $\frac{x^2-y^2-2xy}{(x-y)^2},\ \frac{2x^2}{(x-y)^2}$; **5** $\frac{2x}{x^2+y^4},\ \frac{4y^3}{x^2+y^4}$; **6** $e^{x+3y},\ 3e^{x+3y}$; **7** $\frac{y}{y^2+x^2},\ \frac{-x}{y^2+x^2}$; **8** $\frac{1}{2\sqrt{x-y}}$, více nepotřebujeme, tato derivace není definovaná v bodě $[2,2]$; **9** nepotřebujeme, bod $[2,1]$ neleží v definičním oboru fce F; **10** $\frac{1}{x-y},\ \frac{-1}{x-y}$; **11** $\frac{1}{x-y},\ \frac{-1}{x-y}$; **12** $\frac{y}{2\sqrt{xy}},\ \frac{x}{2\sqrt{xy}}$.

Výsledky: **1** 26; **2** $\pi - 1$; **3** $\frac{\sqrt{2}}{2}$; **4** -43; **5** $-\frac{16}{17}$; **6** $-11e^9$; **7** $\frac{3}{5}$; **8** neexistuje; **9** neexistuje; **10** 4; **11** 0; **12** -2.

Spočtěte limity:

1 $\displaystyle\lim_{[x,y]\to[0,0]} \ln(1 - xy)$

2 $\displaystyle\lim_{[x,y]\to[2,1]} \sqrt{1 + \frac{x}{y}}$

3 $\displaystyle\lim_{[x,y]\to[0,1]} \sqrt{1 + \frac{x}{y}}$

4 $\displaystyle\lim_{\substack{[x,y]\to[1,0]\\ y>0}} \sqrt{1 + \frac{x}{y}}$

5 $\displaystyle\lim_{[x,y]\to[2,1]} \frac{x+y}{x-y}$

6 $\displaystyle\lim_{[x,y]\to[0,0]} \frac{x-y}{x+y}$

7 $\displaystyle\lim_{[x,y]\to[1,1]} \frac{x^2-y^2}{x-y}$

8 $\displaystyle\lim_{\substack{[x,y]\to[1,1]\\ x\neq y}} \frac{x^2-y^2}{x-y}$

9 $\displaystyle\lim_{[x,y]\to[0,0]} \frac{x^4+x^2y^4+y^4}{x^4+y^4}$

10 $\displaystyle\lim_{[x,y]\to[1,0]} \frac{\ln(1+e^{xy})}{x+y}$

11 $\displaystyle\lim_{[x,y]\to[1,0]} \frac{\ln(e^{x^2y^2}-1)}{x+y}$

12 $\displaystyle\lim_{\substack{[x,y]\to[1,0]\\ y\neq 0}} \frac{\ln(e^{x^2y^2}-1)}{x+y}$

13 $\displaystyle\lim_{[x,y]\to[0,0]} \frac{1}{x^4+y^4}$

14 $\displaystyle\lim_{[x,y]\to[0,0]} \frac{1}{\sqrt{x^4+y^4}}$

Výsledky: **1** 0; **2** $\sqrt{3}$; **3** 1; **4** ∞; **5** 3; **6** limita neexistuje; **7** limita neexistuje; **8** 2; **9** neumíme; **10** $\ln 2$; **11** limita neexistuje; **12** $-\infty$; **13** ∞; **14** ∞.