8. cvičení

Určete parciální derivace $\frac{\partial F}{\partial x}$ a $\frac{\partial F}{\partial y}$. Nezapomeňte na definiční obory.

1 $F(x,y) = x^2 + 4y^2$

2 $F(x,y) = 3x^4y - 5xy^2 + 2y$

3 $F(x,y) = x^2y + 2x - 5y$

4 $F(x,y) = (3x-y)^5$

5 $F(x,y) = \dfrac{x^3}{y}$

6 $F(x,y) = \dfrac{y}{\sqrt{x}}$

7 $F(x,y) = \dfrac{y}{e^x}$

8 $F(x,y) = xy^3 - ye^{x+y^2}$

9 $F(x,y) = \ln(x-y) + x$

10 $F(x,y) = \ln(xy+y)$

11 $F(x,y) = \sqrt{x-y}$

12 $F(x,y) = \sqrt{x^2+y^2-9}$

13 $F(x,y) = \ln(x + \ln y)$

14 $F(x,y) = \operatorname{arctg} \dfrac{x}{y}$

15 $F(x,y) = e^{xy} \cdot (x+y)$

16 $F(x,y) = \sin \dfrac{x-y}{3}$

17 $F(x,y) = \cos \dfrac{x^2}{y}$

18 $F(x,y) = x \cdot \sqrt[3]{y} + \dfrac{y^2}{\sqrt{x}}$

Výsledky — definiční obory: Zkuste si sami.

Výsledky — derivace: **1** $\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = 8y$; **2** $\frac{\partial F}{\partial x} = 12x^3y - 5y^2$, $\frac{\partial F}{\partial y} = 3x^4 - 10xy + 2$; **3** $\frac{\partial F}{\partial x} = 2xy + 2$, $\frac{\partial F}{\partial y} = x^2 - 5$; **4** $\frac{\partial F}{\partial x} = 15(3x-y)^4$, $\frac{\partial F}{\partial y} = -5(3x-y)^4$; **5** $\frac{\partial F}{\partial x} = \frac{3x^2}{y}$, $\frac{\partial F}{\partial y} = -\frac{x^3}{y^2}$; **6** $\frac{\partial F}{\partial x} = -\frac{y}{2\sqrt{x^3}}$, $\frac{\partial F}{\partial y} = \frac{1}{\sqrt{x}}$; **7** $\frac{\partial F}{\partial x} = -y \cdot e^{-x}$, $\frac{\partial F}{\partial y} = e^{-x}$; **8** $\frac{\partial F}{\partial x} = y^3 - y \cdot e^{x+y^2}$, $\frac{\partial F}{\partial y} = 3xy^2 - (2y^2+1) \cdot e^{x+y^2}$; **9** $\frac{\partial F}{\partial x} = \frac{x-y+1}{x-y}$, $\frac{\partial F}{\partial y} = -\frac{1}{x-y}$; **10** $\frac{\partial F}{\partial x} = \frac{1}{x+1}$, $\frac{\partial F}{\partial y} = \frac{1}{y}$; **11** $\frac{\partial F}{\partial x} = \frac{1}{2\sqrt{x-y}}$, $\frac{\partial F}{\partial y} = -\frac{1}{2\sqrt{x-y}}$; **12** $\frac{\partial F}{\partial x} = \frac{x}{\sqrt{x^2+y^2-9}}$, $\frac{\partial F}{\partial y} = \frac{y}{\sqrt{x^2+y^2-9}}$; **13** $\frac{\partial F}{\partial x} = \frac{1}{x+\ln y}$, $\frac{\partial F}{\partial y} = \frac{1}{y\cdot(x+\ln y)}$; **14** $\frac{\partial F}{\partial x} = \frac{y}{y^2+x^2}$, $\frac{\partial F}{\partial y} = -\frac{x}{y^2+x^2}$; **15** $\frac{\partial F}{\partial x} = e^{xy}(xy + y^2 + 1)$, $\frac{\partial F}{\partial y} = e^{xy}(x^2 + xy + 1)$; **16** $\frac{\partial F}{\partial x} = \frac{1}{3}\cos\frac{x-y}{3}$, $\frac{\partial F}{\partial y} = -\frac{1}{3}\cos\frac{x-y}{3}$; **17** $\frac{\partial F}{\partial x} = -\frac{2x}{y} \cdot \sin\frac{x^2}{y}$, $\frac{\partial F}{\partial y} = \frac{x^2}{y^2} \cdot \sin\frac{x^2}{y}$; **18** $\frac{\partial F}{\partial x} = \sqrt[3]{y} - \frac{y^2}{2\sqrt{x^3}}$, $\frac{\partial F}{\partial y} = \frac{x}{3\sqrt[3]{y^2}} + \frac{2y}{\sqrt{x}}$.

Určete rovnici tečné roviny ke grafu funkce F v bodě $[x_0, y_0, F(x_0,y_0)]$:

1 $F(x,y) = x^2y + 2x - 5y,\ x_0 = 1,\ y_0 = 3$

2 $F(x,y) = x^3y^2 - 2xy + 5,\ x_0 = 1,\ y_0 = -1$

3 $F(x,y) = \ln(x-y) + x,\ x_0 = 3,\ y_0 = 2$

4 $F(x,y) = e^{xy},\ x_0 = -1,\ y_0 = 0$

5 $F(x,y) = \sqrt{x-y},\ x_0 = 3,\ y_0 = -1$

6 $F(x,y) = \operatorname{arctg}(xy^2),\ x_0 = 0,\ y_0 = 2$

7 $F(x,y) = e^{2x-y} + 1,\ x_0 = 1,\ y_0 = 2$

8 $F(x,y) = \cos(xy+\pi),\ x_0 = \dfrac{\pi}{2},\ y_0 = 1$

9 $F(x,y) = \dfrac{xy}{x-2y},\ x_0 = 3,\ y_0 = 1$

10 $F(x,y) = e^{2y} \cdot \cos x,\ x_0 = \pi,\ y_0 = 1$

Výsledky: **1** $z = 8x - 4y - 6$; **2** $z = 5x - 4y - 1$; **3** $z = 2x - y - 1$; **4** $z = 1 - y$; **5** $z = \frac{x-y+4}{4}$; **6** $z = 4x$; **7** $z = 2x - y + 2$; **8** $z = x + \frac{\pi}{2}y - \pi$; **9** $z = -2x + 9y$; **10** $z = -2e^2y + e^2$.