1. cvičení

Ověřte, zda je řada geometrická. Pokud ano, určete její součet.

$\boxed{1}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{n}$ $\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{3}\right)^{n}$ $\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{5}{7}\right)^{n+1}$

$\boxed{4}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{7}{5}\right)^{n}$ $\boxed{5}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{1}{4}\right)^{n}$ $\boxed{6}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{3}{5}\right)^{n}$

$\boxed{7}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{5}{3}\right)^{n}$ $\boxed{8}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n}\left(\frac{3}{5}\right)^{n}$ $\boxed{9}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{2n+1}\left(\frac{1}{3}\right)^{n}$

$\boxed{10}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{4}\right)^{2n}$ $\boxed{11}$ $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^{3n+1}$ $\boxed{12}$ $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\left(\frac{1}{3}\right)^{2n+1}$

$\boxed{13}$ $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{2}{3}\right)^{n}$ $\boxed{14}$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^{n}}$ $\boxed{15}$ $\displaystyle\sum_{n=1}^{\infty}3\cdot\left(\frac{1}{5}\right)^{n}$

$\boxed{16}$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{2^{n}\cdot 3^{n-1}}$ $\boxed{17}$ $\displaystyle\sum_{n=1}^{\infty}\frac{5^{n}}{3^{n}}$ $\boxed{18}$ $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n}}{2^{3n}}$

$\boxed{19}$ $\displaystyle\sum_{n=1}^{\infty}\frac{5^{n+1}}{3\cdot 5^{n}}$ $\boxed{20}$ $\displaystyle\sum_{n=1}^{\infty}5\cdot\frac{2^{n+1}}{3^{n}}$ $\boxed{21}$ $\displaystyle\sum_{n=1}^{\infty}3\cdot\left(-\frac{2}{3}\right)^{n}$

$\boxed{22}$ $\displaystyle\sum_{n=1}^{\infty}\frac{3^{n}}{5\cdot 4^{n+1}}$ $\boxed{23}$ $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n}}{n\cdot 3^{n}}$ $\boxed{24}$ $\displaystyle\sum_{n=1}^{\infty}\frac{n}{2^{n}\cdot\ln(e^{n})}$

$\boxed{25}$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{(2n-1)^{n}}$ $\boxed{26}$ $\displaystyle\sum_{n=1}^{\infty}e^{-n}$ $\boxed{27}$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{\sqrt{e^{n}}}$

$\boxed{28}$ $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt[3]{e^{n}}}$ $\boxed{29}$ $\displaystyle\sum_{n=1}^{\infty}\sqrt{\left(\frac{2}{3}\right)^{n}}$ $\boxed{30}$ $\displaystyle\sum_{n=1}^{\infty}\frac{1}{3^{\frac{n}{3}}}$

Výsledky: $\boxed{1}$ $\frac{1}{3}$; $\boxed{2}$ 2; $\boxed{3}$ $\frac{25}{14}$; $\boxed{4}$ diverguje do $+\infty$; $\boxed{5}$ $\frac{1}{5}$; $\boxed{6}$ $-\frac{3}{8}$; $\boxed{7}$ diverguje; $\boxed{8}$ $\frac{3}{2}$; $\boxed{9}$ $-\frac{1}{2}$; $\boxed{10}$ $\frac{1}{15}$; $\boxed{11}$ $\frac{1}{14}$; $\boxed{12}$ $-\frac{1}{30}$; $\boxed{13}$ $-\frac{2}{5}$; $\boxed{14}$ $\frac{1}{2}$; $\boxed{15}$ $\frac{3}{4}$; $\boxed{16}$ $\frac{3}{5}$; $\boxed{17}$ diverguje do $+\infty$; $\boxed{18}$ $\frac{3}{5}$; $\boxed{19}$ diverguje do $+\infty$; $\boxed{20}$ 20; $\boxed{21}$ $-\frac{6}{5}$; $\boxed{22}$ $\frac{3}{20}$; $\boxed{23}$ není geometrická; $\boxed{24}$ 1; $\boxed{25}$ není geometrická; $\boxed{26}$ $\frac{1}{e-1}$; $\boxed{27}$ $\frac{1}{\sqrt{e}-1}$; $\boxed{28}$ $\frac{1}{\sqrt[3]{e}+1}$; $\boxed{29}$ $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}$; $\boxed{30}$ $\frac{1}{\sqrt[3]{3}-1}$.

Vyjádřete řadu jako součet nebo rozdíl geometrických řad, určete její součet.

$\boxed{1}$ $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n}+1}{3^{n}}$ $\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty}\frac{2^{n}-1}{5^{n}}$

$\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 5^n}{7 \cdot 10^n}$ $\boxed{4}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + 1}{3^{n+2}}$

$\boxed{5}$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n + 3^n}{6^n}$ $\boxed{6}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2} - 5 \cdot 4^{n+1}}{2 \cdot 12^n}$

$\boxed{7}$ $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+2} + 3 \cdot 5^n}{7 \cdot 20^{n-1}}$ $\boxed{8}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1 - 2^n}{2^{n+3}}$

Výsledky: $\boxed{1}$ $2 + \frac{1}{2} = \frac{5}{2}$; $\boxed{2}$ $\frac{2}{3} - \frac{1}{4} = \frac{5}{12}$; $\boxed{3}$ $\frac{1}{28} + \frac{1}{7} = \frac{5}{28}$; $\boxed{4}$ diverguje do $+\infty$; $\boxed{5}$ $\frac{1}{2} + 1 = \frac{3}{2}$; $\boxed{6}$ $\frac{3}{2} - 5 = -\frac{7}{2}$; $\boxed{7}$ $\frac{80}{7} + \frac{20}{7} = \frac{100}{7}$; $\boxed{8}$ diverguje do $-\infty$.

S využitím podílového kritéria rozhodněte o konvergenci řady:

$\boxed{1}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$ $\boxed{2}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^n}$ $\boxed{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n!}$

$\boxed{4}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+1)!}$ $\boxed{5}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n+1}$ $\boxed{6}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n!}$

$\boxed{7}$ $\displaystyle\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot \ldots \cdot (2n-1)}{5^n \cdot n!}$ $\boxed{8}$ $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$ $\boxed{9}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

$\boxed{10}$ $\displaystyle\sum_{n=1}^{\infty} \frac{5 \cdot 10 \cdot 15 \cdot \ldots \cdot 5n}{5^n \cdot n!}$ $\boxed{11}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!}$ $\boxed{12}$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n \cdot n!}{n^n}$

$\boxed{13}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n \cdot n!}{n^n}$ $\boxed{14}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(n!)^3}$ $\boxed{15}$ $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{\sqrt{n^n}}$

Mezivýsledky — hodnota $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$: $\boxed{1}$ $\frac{1}{2}$; $\boxed{2}$ $\frac{1}{3}$; $\boxed{3}$ 0; $\boxed{4}$ 0; $\boxed{5}$ 3; $\boxed{6}$ 0; $\boxed{7}$ $\frac{2}{5}$; $\boxed{8}$ $\frac{1}{2}$; $\boxed{9}$ $\frac{1}{4}$; $\boxed{10}$ 1; $\boxed{11}$ $\frac{1}{27}$; $\boxed{12}$ $\frac{2}{e}$; $\boxed{13}$ $\frac{3}{e}$; $\boxed{14}$ 0; $\boxed{15}$ ∞.

Výsledky: (K = konverguje, D = diverguje) $\boxed{1}$ K; $\boxed{2}$ K; $\boxed{3}$ K; $\boxed{4}$ K; $\boxed{5}$ D; $\boxed{6}$ K; $\boxed{7}$ K; $\boxed{8}$ K; $\boxed{9}$ K; $\boxed{10}$ D (je to geometrická řada, $q = 1$); $\boxed{11}$ K; $\boxed{12}$ K; $\boxed{13}$ D; $\boxed{14}$ K; $\boxed{15}$ D.

Pozn.: Všechny divergující řady divergují do $+\infty$, protože mají nezáporné členy.