

2. cvičení

Určete parciální derivace $\frac{\partial F}{\partial x}$ a $\frac{\partial F}{\partial y}$. Nezapomeňte na definiční obory.

1 $F(x, y) = x^2 + 4y^2$

2 $F(x, y) = 3x^4y - 5xy^2 + 2y$

3 $F(x, y) = x^2y + 2x - 5y$

4 $F(x, y) = (3x - y)^5$

5 $F(x, y) = \frac{x^3}{y}$

6 $F(x, y) = \frac{y}{\sqrt{x}}$

7 $F(x, y) = \frac{y}{e^x}$

8 $F(x, y) = y - \arcsin x$

9 $F(x, y) = \ln(x - y) + x$

10 $F(x, y) = \ln(xy + y)$

11 $F(x, y) = \sqrt{x - y}$

12 $F(x, y) = \sqrt{x^2 + y^2 - 9}$

13 $F(x, y) = \ln(x + \ln y)$

14 $F(x, y) = \ln\left(\frac{4x^2 + y^2 - 36}{x^2 + y^2 - 25}\right)$

15 $F(x, y) = xy^3 - ye^{x+y^2}$

16 $F(x, y) = \operatorname{arctg} \frac{x}{y}$

17 $F(x, y) = e^{xy} \cdot (x + y)$

18 $F(x, y) = \sin \frac{x - y}{3}$

19 $F(x, y) = \cos \frac{x^2}{y}$

20 $F(x, y) = x \cdot \sqrt[3]{y} + \frac{y^2}{\sqrt{x}}$

21 $F(x, y) = \sqrt{x \cdot \cos y}$

22 $F(x, y) = \ln(x \cdot \ln(x + y))$

23 $F(x, y) = y^x$

24 $F(x, y) = \left(2x^2y + \frac{1}{y}\right) \cdot e^{-3x+1}$

25 $F(x, y) = x^y$

26 $F(x, y) = \left(3xy^2 + \frac{1}{x^2}\right) \cdot e^{x-y}$

Výsledky — definiční obory: Zkuste si sami.

Výsledky — derivace: **1** $\frac{\partial F}{\partial x} = 2x, \frac{\partial F}{\partial y} = 8y$; **2** $\frac{\partial F}{\partial x} = 12x^3y - 5y^2, \frac{\partial F}{\partial y} = 3x^4 - 10xy + 2$;

3 $\frac{\partial F}{\partial x} = 2xy + 2, \frac{\partial F}{\partial y} = x^2 - 5$; **4** $\frac{\partial F}{\partial x} = 15(3x - y)^4, \frac{\partial F}{\partial y} = -5(3x - y)^4$;

5 $\frac{\partial F}{\partial x} = \frac{3x^2}{y}, \frac{\partial F}{\partial y} = -\frac{x^3}{y^2}$; **6** $\frac{\partial F}{\partial x} = -\frac{y}{2\sqrt{x^3}}, \frac{\partial F}{\partial y} = \frac{1}{\sqrt{x}}$; **7** $\frac{\partial F}{\partial x} = -y \cdot e^{-x}, \frac{\partial F}{\partial y} = e^{-x}$;

8 $\frac{\partial F}{\partial x} = -\frac{1}{\sqrt{1-x^2}}, \frac{\partial F}{\partial y} = 1$; **9** $\frac{\partial F}{\partial x} = \frac{x-y+1}{x-y}, \frac{\partial F}{\partial y} = -\frac{1}{x-y}$; **10** $\frac{\partial F}{\partial x} = \frac{1}{x+1}, \frac{\partial F}{\partial y} = \frac{1}{y}$;

11 $\frac{\partial F}{\partial x} = \frac{1}{2\sqrt{x-y}}, \frac{\partial F}{\partial y} = -\frac{1}{2\sqrt{x-y}}$; **12** $\frac{\partial F}{\partial x} = \frac{x}{\sqrt{x^2+y^2-9}}, \frac{\partial F}{\partial y} = \frac{y}{\sqrt{x^2+y^2-9}}$;

13 $\frac{\partial F}{\partial x} = \frac{1}{x+\ln y}, \frac{\partial F}{\partial y} = \frac{1}{y \cdot (x+\ln y)}$; **14** $\frac{\partial F}{\partial x} = \frac{2x \cdot (3y^2 - 64)}{(4x^2 + y^2 - 36)(x^2 + y^2 - 25)}$;

$\frac{\partial F}{\partial y} = -\frac{2y \cdot (3x^2 - 11)}{(4x^2 + y^2 - 36)(x^2 + y^2 - 25)}$; **15** $\frac{\partial F}{\partial x} = y^3 - y \cdot e^{x+y^2}, \frac{\partial F}{\partial y} = 3xy^2 - (2y^2 + 1) \cdot e^{x+y^2}$;

16 $\frac{\partial F}{\partial x} = \frac{y}{y^2+x^2}, \frac{\partial F}{\partial y} = -\frac{x}{y^2+x^2}$; **17** $\frac{\partial F}{\partial x} = e^{xy}(xy + y^2 + 1), \frac{\partial F}{\partial y} = e^{xy}(x^2 + xy + 1)$;

$$\begin{aligned}
\boxed{18} \quad \frac{\partial F}{\partial x} &= \frac{1}{3} \cos \frac{x-y}{3}, \quad \frac{\partial F}{\partial y} = -\frac{1}{3} \cos \frac{x-y}{3}; & \boxed{19} \quad \frac{\partial F}{\partial x} &= -\frac{2x}{y} \cdot \sin \frac{x^2}{y}, \\
\frac{\partial F}{\partial y} &= \frac{x^2}{y^2} \cdot \sin \frac{x^2}{y}; & \boxed{20} \quad \frac{\partial F}{\partial x} &= \sqrt[3]{y} - \frac{y^2}{2\sqrt{x^3}}, \quad \frac{\partial F}{\partial y} = \frac{x}{3\sqrt[3]{y^2}} + \frac{2y}{\sqrt{x}}; & \boxed{21} \quad \frac{\partial F}{\partial x} &= \frac{\cos y}{2\sqrt{x \cos y}}, \\
\frac{\partial F}{\partial y} &= -\frac{x \sin y}{2\sqrt{x \cos y}}; & \boxed{22} \quad \frac{\partial F}{\partial x} &= \frac{(x+y) \cdot \ln(x+y) + x}{x \cdot (x+y) \cdot \ln(x+y)}, \quad \frac{\partial F}{\partial y} = \frac{1}{(x+y) \cdot \ln(x+y)}; & \boxed{23} \quad \frac{\partial F}{\partial x} &= y^x \cdot \ln y, \\
\frac{\partial F}{\partial y} &= x \cdot y^{x-1}; & \boxed{24} \quad \frac{\partial F}{\partial x} &= (4xy - 6x^2y - \frac{3}{y})e^{-3x+1}, \quad \frac{\partial F}{\partial y} = (2x^2 - \frac{1}{y^2})e^{-3x+1}; \\
\boxed{25} \quad \frac{\partial F}{\partial x} &= y \cdot x^{y-1}, \quad \frac{\partial F}{\partial y} = x^y \cdot \ln x; & \boxed{26} \quad \frac{\partial F}{\partial x} &= (3y^2 - \frac{2}{x^3} + 3xy^2 + \frac{1}{x^2})e^{x-y}, \\
\frac{\partial F}{\partial y} &= (6xy - 3xy^2 - \frac{1}{x^2})e^{x-y}.
\end{aligned}$$