

5. cvičení

Určete extrémy funkce F na dané množině:

- 1** $F(x, y) = \frac{x^3}{3} + y^2, x^2 + y^2 \leq 9$
- 2** $F(x, y) = x^2 + 3y^2, x^2 + y^2 = 2$
- 3** $F(x, y) = 2x^2 - 4xy + y^4, 0 \leq y \leq \sqrt{2x}, x \in \langle 0, 2 \rangle$
- 4** $F(x, y) = \sqrt{x} + \sqrt{y}, 0 \leq y \leq \sqrt{x}, x \in \langle 0, 4 \rangle$
- 5** $F(x, y) = x^3 - 3xy + 3y, x^3 \leq y \leq 8, x \in \langle 0, 2 \rangle$
- 6** $F(x, y) = x^3 - 3xy + 3y, \langle 0, 2 \rangle \times \langle 0, 2 \rangle$
- 7** $F(x, y) = \arctg(x^2 - y^2), x^2 + y^2 \leq 1$
- 8** $F(x, y) = x^2 + 3y^2 + 5, x^2 + y^2 \leq 2$
- 9** $F(x, y) = x^2 + 4y^2 - x + 2y, x^2 + 4y^2 \leq 1$
- 10** $F(x, y) = x^2 + 2xy + 2y + 3, y = x^2, x \in \langle -1, 1 \rangle$
- 11** $F(x, y) = x^2 + 2xy + \frac{2}{3}y^3, x + 2y = 1, x \in \langle -5, 5 \rangle$
- 12** $F(x, y) = \frac{2}{3}x^3 - y^2$, uzavřený trojúhelník s vrcholy $[0, 0]$, $[2, 0]$, $[2, 2]$
- 13** $F(x, y) = 5 + 4x - 2x^2 + 3y - y^2, |x| \leq y \leq 2$
- 14** $F(x, y) = x^2 - 4xy + 3y^2 + y$, uzavřený čtyřúhelník
s vrcholy $[0, 0]$, $[4, 0]$, $[3, 3]$, $[7, 3]$
- 15** $F(x, y) = \arctg\left(\frac{x^2 - xy + 6y - 13}{9}\right), x + y = 2, x \in \langle -1, 5 \rangle$
- 16** $F(x, y) = (x^2 - 4y) \cdot e^{x-2y}, -4 \leq y \leq -x^2$

Nápověda — podezřelé body: **1** $[-3, 0]$, $[0, 3]$, $[0, -3]$, $[2, \sqrt{5}]$, $[2, -\sqrt{5}]$, $[3, 0]$, $[0, 0]$;
2 $[0, \sqrt{2}]$, $[0, -\sqrt{2}]$, $[\sqrt{2}, 0]$, $[-\sqrt{2}, 0]$; **3** $[0, 0]$, $[\frac{1}{2}, 1]$, $[2, 2]$, $[2, 0]$, $[2, \sqrt[3]{2}]$, $[1, 1]$;
4 $[0, 0]$, $[4, 0]$, $[4, 2]$; **5** $[0, 0]$, $[0, 8]$, $[2, 8]$, $[1, 1]$; **6** $[0, 0]$, $[0, 2]$, $[2, 0]$, $[2, 2]$, $[\sqrt{2}, 2]$,
 $[1, 1]$; **7** $[0, 1]$, $[0, -1]$, $[1, 0]$, $[-1, 0]$, $[0, 0]$; **8** $[0, \sqrt{2}]$, $[0, -\sqrt{2}]$, $[\sqrt{2}, 0]$, $[-\sqrt{2}, 0]$, $[0, 0]$;
9 $[\sqrt{\frac{1}{2}}, -\sqrt{\frac{1}{8}}]$, $[-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{8}}]$, $[0, -\frac{1}{2}]$, $[0, \frac{1}{2}]$, $[\frac{1}{2}, -\frac{1}{4}]$; **10** $[0, 0]$, $[-1, 1]$, $[1, 1]$; **11** $[-1, 1]$,
 $[3, -1]$, $[5, -2]$, $[-5, 3]$; **12** $[0, 0]$, $[2, 0]$, $[2, 2]$, $[1, 1]$; **13** $[0, 0]$, $[2, 2]$, $[\frac{7}{6}, \frac{7}{6}]$, $[-2, 2]$, $[1, 2]$,
 $[1, \frac{3}{2}]$; **14** $[0, 0]$, $[4, 0]$, $[3, 3]$, $[7, 3]$, $[1, \frac{1}{2}]$, $[6, 3]$; **15** $[-1, 3]$, $[2, 0]$, $[5, -3]$; **16** $[-2, -4]$,
 $[2, -4]$, $[0, 0]$, $[1, \frac{3}{4}]$.

Výsledky: **1** $\min = F(-3, 0) = -9$, $\max = F(0, \pm 3) = F(3, 0) = 9$;
2 $\min = F(\pm\sqrt{2}, 0) = 2$, $\max = F(0, \pm\sqrt{2}) = 6$; **3** $\min = F(1, 1) = -1$,
 $\max = F(2, 2) = F(2, 0) = 8$; **4** $\min = F(0, 0) = 0$, $\max = F(4, 2) = 2 + \sqrt{2}$;
5 $\min = F(2, 8) = -16$, $\max = F(0, 8) = 24$; **6** $\min = F(0, 0) = 0$,
 $\max = F(2, 0) = 8$; **7** $\min = F(0, \pm 1) = -\frac{\pi}{4}$, $\max = F(\pm 1, 0) = \frac{\pi}{4}$;
8 $\min = F(0, 0) = 5$, $\max = F(0, \pm\sqrt{2}) = 11$; **9** $\min = F(\frac{1}{2}, -\frac{1}{4}) = -\frac{1}{2}$,
 $\max = F(-\sqrt{\frac{1}{2}}, \sqrt{\frac{1}{8}}) = 1 + \sqrt{2}$; **10** $\min = F(0, 0) = 3$, $\max = F(1, 1) = 8$;
11 $\min = F(-1, 1) = F(5, -2) = -\frac{1}{3}$, $\max = F(-5, 3) = 13$;
12 $\min = F(1, 1) = -\frac{1}{3}$, $\max = F(2, 0) = \frac{16}{3}$; **13** $\min = F(-2, 2) = -9$,
 $\max = F(1, \frac{3}{2}) = \frac{37}{4}$; **14** $\min = F(6, 3) = -6$, $\max = F(4, 0) = 16$;
15 $\min = F(2, 0) = -\frac{\pi}{4}$, $\max = F(-1, 3) = F(5, -3) = \frac{\pi}{4}$;
16 $\min = F(1, \frac{3}{4}) = -\frac{2}{\sqrt{e}}$, $\max = F(2, -4) = 20 \cdot e^{10}$.