

6. cvičení

Spočtěte:

1 $\int e^{xy} dy$	2 $\int e^{xy} dx$	3 $\int \frac{x^3}{y} dx$
4 $\int \frac{x^3}{y} dy$	5 $\int \frac{y}{e^x} dx$	6 $\int \frac{y}{e^x} dy$
7 $\int \sqrt{x-y} dx$	8 $\int \sqrt{x-y} dy$	9 $\int (3x^4y - 5xy^2 + 2y) dx$
10 $\int x \cdot \cos(y) dy$	11 $\int x \cdot \cos(y) dx$	12 $\int (3x^4y - 5xy^2 + 2y) dy$
13 $\int \sin(xy) dx$	14 $\int \sin(x^2y) dy$	15 $\int \frac{1}{x^2y} dx$
16 $\int \frac{1}{x^2y} dy$	17 $\int \frac{1}{\sqrt[3]{xy^2}} dy$	18 $\int \frac{1}{\sqrt[3]{xy^2}} dx$
19 $\int \frac{y}{x^2+y^2} dy$	20 $\int \frac{y}{x^2+y^2} dx$	21 $\int \frac{1}{x^2+y^2} dx$

Výsledky: **1** $\frac{1}{x}e^{xy} + c(x)$, $x \neq 0$; **2** $\frac{1}{y}e^{xy} + c(y)$, $y \neq 0$; **3** $\frac{x^4}{4y} + c(y)$, $y \neq 0$;
4 $x^3 \cdot \ln|y| + c(x)$, $y \neq 0$; **5** $-\frac{y}{e^x} + c(y)$; **6** $\frac{y^2}{2e^x} + c(x)$; **7** $\frac{2}{3}\sqrt{(x-y)^3} + c(y)$, $x > y$;
8 $-\frac{2}{3}\sqrt{(x-y)^3} + c(x)$, $x > y$; **9** $\frac{3}{5}x^5y - \frac{5}{2}x^2y^2 + 2xy + c(y)$; **10** $x \sin(y) + c(x)$;
11 $\frac{x^2}{2} \cos(y) + c(y)$; **12** $\frac{3}{2}x^4y^2 - \frac{5}{3}xy^3 + y^2 + c(x)$; **13** $-\frac{\cos(xy)}{y} + c(y)$, $y \neq 0$;
14 $-\frac{\cos(x^2y)}{x^2} + c(x)$, $x \neq 0$; **15** $-\frac{1}{xy} + c(y)$, $x \neq 0$, $y \neq 0$; **16** $\frac{1}{x^2} \ln|y| + c(x)$,
 $x \neq 0$, $y \neq 0$; **17** $\frac{3\sqrt[3]{y}}{\sqrt[3]{x}} + c(x)$, $x \neq 0$, $y \neq 0$; **18** $\frac{3\sqrt[3]{x^2}}{2\sqrt[3]{y^2}} + c(y)$, $x \neq 0$, $y \neq 0$;
19 $\frac{1}{2} \ln(x^2+y^2) + c(x)$, $[x, y] \neq [0, 0]$; **20** $\operatorname{arctg}\left(\frac{x}{y}\right) + c(y)$, $y \neq 0$; **21** $\frac{1}{y} \operatorname{arctg}\left(\frac{x}{y}\right) + c(y)$,
 $y \neq 0$.

Spočtěte:

1 $\int (x + y + z) dy$	2 $\int \cos(x + 2y + 3z) dz$	3 $\int \frac{x+z}{\sqrt{y}} dz$
4 $\int \frac{x+z}{\sqrt{y}} dy$	5 $\int (xy - z)^5 dz$	6 $\int (xy - z)^5 dx$

Výsledky: **1** $xy + \frac{y^2}{2} + zy + c(x, z)$; **2** $\frac{1}{3} \sin(x + 2y + 3z) + c(x, y)$; **3** $\frac{2xz+z^2}{2\sqrt{y}} + c(x, y)$,
 $y > 0$; **4** $2(x+z)\sqrt{y} + c(x, z)$, $y > 0$; **5** $-\frac{(xy-z)^6}{6} + c(x, y)$; **6** $\frac{(xy-z)^6}{6y} + c(y, z)$, $y \neq 0$.

Spočtěte:

$$\begin{array}{lll} \boxed{1} & \int_0^1 (x^2 + y^2) dx & \boxed{2} & \int_{-1}^1 (x^2 + y^2) dy & \boxed{3} & \int_1^e \frac{1}{xy^3} dx \\ \boxed{4} & \int_0^4 \sqrt{x+y} dx & \boxed{5} & \int_{-1}^1 \frac{1}{\sqrt{x-y}} dy & \boxed{6} & \int_0^\pi \cos(x+y^2) dx \\ \boxed{7} & \int_0^{\frac{\pi}{2}} \sin(x+y) dy & \boxed{8} & \int_0^1 y \cdot \ln x dy & \boxed{9} & \int_0^1 y \cdot \ln x dx \end{array}$$

Výsledky: $\boxed{1}$ $\frac{1}{3} + y^2$, $y \in \mathbb{R}$; $\boxed{2}$ $2x^2 + \frac{2}{3}$, $x \in \mathbb{R}$; $\boxed{3}$ $\frac{1}{y^3}$, $y \neq 0$; $\boxed{4}$ $\frac{2}{3}\sqrt{(4+y)^3} - \frac{2}{3}\sqrt{y^3}$, $y \geq 0$; $\boxed{5}$ $2\sqrt{x+1} - 2\sqrt{x-1}$, $x > 1$; $\boxed{6}$ $-2\sin(y^2)$, $y \in \mathbb{R}$; $\boxed{7}$ $\sin x + \cos x$, $x \in \mathbb{R}$; $\boxed{8}$ $\frac{1}{2} \ln x$, $x > 0$; $\boxed{9}$ $-y$, $y \in \mathbb{R}$.

Spočtěte:

$$\begin{array}{ll} \boxed{1} & \int_0^1 \int_{-1}^2 \frac{3x^2}{y+2} dx dy & \boxed{2} & \int_{-1}^1 \int_0^3 (x^2 + y^2) dx dy \\ \boxed{3} & \int_0^{\frac{\pi}{2}} \int_0^\pi \sin(x+2y) dx dy & \boxed{4} & \int_1^2 \int_0^1 (2y + 3xy^2 + x^3) dy dx \\ \boxed{5} & \int_0^2 \int_0^1 \frac{x}{(x+y)^2} dy dx & \boxed{6} & \int_1^2 \int_0^1 x^y dx dy \\ \boxed{7} & \int_0^2 \int_{-1}^1 (3x^2y^2 + x^4y - x + y) dx dy & \boxed{8} & \int_0^{\sqrt{5}} \int_0^2 \frac{xy}{\sqrt{y^2+1}} dx dy \end{array}$$

Výsledky: $\boxed{1}$ $9 \ln \frac{3}{2}$; $\boxed{2}$ 20; $\boxed{3}$ 0; $\boxed{4}$ $\frac{25}{4}$; $\boxed{5}$ $\ln 3$; $\boxed{6}$ $\ln \frac{3}{2}$; $\boxed{7}$ $\frac{152}{15}$; $\boxed{8}$ $2\sqrt{6} - 2$.