3. cvičení

Určete extrémy funkce F na dané množině.

1 $F(x,y) = 10 - x^2 - y^2,\ \langle -1,1\rangle \times \langle -2,2\rangle$

2 $F(x,y) = 10 - x^2 - y^2,\ (-1,1) \times (-2,2)$

3 $F(x,y) = 10 - x^2 - y^2,\ (0,1) \times (0,2)$

4 $F(x,y) = x^2 - 2xy + y,\ (0,3) \times \langle 0,2\rangle$

5 $F(x,y) = x^2 - 2xy + y,\ \langle 0,3\rangle \times (0,2)$

6 $F(x,y) = x^2 - 2xy + y,\ \langle 0,1\rangle \times \langle 0,1\rangle$

7 $F(x,y) = x^2 - 2xy + y,\ \langle -1,1\rangle \times \langle -1,1\rangle$

8 $F(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y},\ (0,4) \times (0,4)$

9 $F(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y},\ \langle 0,4\rangle \times \langle 0,4\rangle$

10 $F(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y},\ (0,\dfrac{1}{4}) \times (0,\dfrac{1}{4})$

11 $F(x,y) = \dfrac{1}{x} + xy + \dfrac{1}{y},\ \langle 1,\infty) \times \langle 1,\infty)$

12 $F(x,y) = \ln(xy) + x + y,\ (0,1) \times (0,1)$

13 $F(x,y) = \ln(xy) - 2x - y,\ (0,1\rangle \times (0,1\rangle$

14 $F(x,y) = \ln(xy) - 2x - y,\ \langle 1,\infty) \times \langle 1,\infty)$

15 $F(x,y) = \cos(x + \pi y),\ (0,\dfrac{\pi}{4}) \times (-\dfrac{1}{2},\dfrac{1}{4})$

16 $F(x,y) = \sin(x + \pi y),\ (0,\dfrac{\pi}{4}) \times (-\dfrac{1}{2},\dfrac{1}{4})$

Výsledky: **1** min $= 5 = F(-1,-2) = F(-1,2) = F(1,-2) = F(1,2)$, max $= 10 = F(0,0)$; **2** $\inf = 5$, max $= 10 = F(0,0)$; **3** $\inf = 5$, $\sup = 10$; **4** min $= -2 = F(2,2)$, $\sup = 9$; **5** $\inf = -2$, $\sup = 9$; **6** min $= 0 = F(0,0) = F(1,1)$, max $= 1 = F(0,1) = F(1,0)$; **7** min $= -2 = F(-1,-1)$, max $= 4 = F(-1,1)$; **8** min $= 3 = F(1,1)$, $\sup = \infty$; **9** nelze, funkce není definovaná na celé množině; **10** $\inf = 8\frac{1}{16}$, $\sup = \infty$; **11** min $= 3 = F(1,1)$, $\sup = \infty$; **12** $\inf = -\infty$, $\sup = 2$; **13** $\inf = -\infty$, max $= -\ln 2 - 2 = F(\frac{1}{2},1)$; **14** $\inf = -\infty$, max $= -3 = F(1,1)$; **15** $\inf = 0$, max $= 1 = F(x,-\frac{x}{\pi})$; **16** $\inf = -1$, $\sup = 1$.