



evropský
sociální
fond v ČR



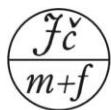
EVROPSKÁ UNIE



MINISTERSTVO ŠKOLSTVÍ,
MLÁDEŽE A TĚLOVÝCHOVY



OP Vzdělávání
pro konkurenčeschopnost



Jednota českých
matematiků a fyziků

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

URČITÝ INTEGRÁL - ŘEŠENÍ

Řešení jednotlivých integrálů:

$$1. \int_0^2 (3 \cdot x^2 + 2 \cdot x - 4) dx = \left[3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 4 \cdot x \right]_0^2 = 8 + 4 - 8 = 4$$

$$2. \int_2^4 \left(x + \frac{1}{x} \right) dx = \left[\frac{x^2}{2} + \ln|x| \right]_2^4 = 8 + \ln 4 - 2 - \ln 2 = 6 + \ln 2$$

$$3. \int_0^{\frac{\pi}{2}} (\sin x + \cos x) dx = \left[-\cos x + \sin x \right]_0^{\frac{\pi}{2}} = 0 + 1 + 1 - 0 = 2$$

$$4. \int_0^{\frac{\pi}{4}} \operatorname{tg} x dx = \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos x} dx = \begin{vmatrix} t = \cos x & t_1 = 1 \\ dt = -\sin x dx & t_2 = \frac{\sqrt{2}}{2} \end{vmatrix} = - \int_1^{\frac{\sqrt{2}}{2}} \frac{dt}{t} = \int_{\frac{\sqrt{2}}{2}}^1 \frac{dt}{t} = \ln 1 - \ln \frac{\sqrt{2}}{2} = \ln \sqrt{2} = \frac{1}{2} \cdot \ln 2$$

$$5. \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x} dx = \begin{vmatrix} t = \cos x & t_1 = 1 \\ dt = -\sin x dx & t_2 = \frac{\sqrt{2}}{2} \end{vmatrix} = - \int_1^{\frac{\sqrt{2}}{2}} \frac{dt}{t^2} = \int_{\frac{\sqrt{2}}{2}}^1 \frac{dt}{t^2} = \left[-\frac{1}{t} \right]_{\frac{\sqrt{2}}{2}}^1 = -1 + \frac{2}{\sqrt{2}} = \sqrt{2} - 1$$

$$6. \int_0^{\frac{\pi}{2}} 3 \cdot \sin x \cdot \cos x dx = \begin{vmatrix} t = \sin x & t_1 = 0 \\ dt = \cos x dx & t_2 = 1 \end{vmatrix} = 3 \cdot \int_0^1 t dt = 3 \cdot \left[\frac{t^2}{2} \right]_0^1 = \frac{3}{2}$$

$$7. \int_1^5 \sqrt{x-1} dx = \begin{vmatrix} t = x-1 & t_1 = 0 \\ dt = dx & t_2 = 4 \end{vmatrix} = \int_0^4 \frac{1}{t^2} dt = \left[\frac{\frac{3}{2}}{3} \right]_0^4 = \frac{8}{3} = \frac{16}{3}$$

$$8. \int_1^e \frac{\ln x}{x} dx = \begin{vmatrix} t = \ln x & t_1 = 0 \\ dt = \frac{1}{x} dx & t_2 = 1 \end{vmatrix} = \int_0^1 t dt = \left[\frac{t^2}{2} \right]_0^1 = \frac{1}{2}$$

$$9. \int_0^{\sqrt{3}} \frac{x dx}{\sqrt{4-x^2}} = \begin{vmatrix} t = 4-x^2 & t_1 = 4 \\ dt = -2x dx & t_2 = 1 \end{vmatrix} = -\frac{1}{2} \cdot \int_4^1 \frac{dt}{\sqrt{t}} = \frac{1}{2} \cdot \int_1^4 t^{-\frac{1}{2}} dt = \frac{1}{2} \cdot \left[\frac{\frac{1}{2}}{2} \right]_1^4 = 2 - 1 = 1$$

$$10. \int_0^{\frac{\pi}{4}} x \cdot \cos x dx = \begin{vmatrix} u = x & v' = \cos x \\ u' = 1 & v = \sin x \end{vmatrix} = \left[x \cdot \sin x \right]_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \sin x dx = \left[x \cdot \sin x + \cos x \right]_0^{\frac{\pi}{4}} = \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 0 - 1 = \frac{\sqrt{2}}{8} \cdot (\pi + 8) - 1$$

Správné řešení tedy je: 1 – c, 2 – h, 3 – a, 4 – b, 5 – d, 6 – j, 7 – i, 8 e, 9 – g, 10 – f.