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THE TYPOLOGY OF ARITHMETICAL CONCEPT CARTOONS

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ABSTRACT. The paper introduces an educational tool called Concept Cartoons that might be used as a tool in investigating and developing future teachers mathematical knowledge (subject matter knowledge as well as pedagogical content knowledge), and a long-term qualitative empirical study focusing in general on the content and inner structure of Concept Cartoons. The results of the study have a form of a structured typology that can be also used as a step-by-step guide when creating new Concept Cartoons.

INTRODUCTION

Recently, an educational tool called Concept Cartoons has been introduced as a powerful tool in professional preparation of mathematics teachers [14, 15, 17], especially in investigating and developing mathematical subject matter knowledge and pedagogical content knowledge [19, 20] of future primary and secondary school teachers. From the general perspective, Concept Cartoons can be considered as an example of a partial representation of school practice [5] as well as an example of an educational vignette [22, 4, 3, 7]. In the current pandemic situation when personal presence of future teachers in training schools is not always possible, the role of various representations of school practice increases. These representations are able to bring the classroom reality closer to the future teachers and provide it independently of the actual possibility or impossibility to attend the classrooms in training schools. In mathematics education, the representations of classroom practice acquire varied appearances [1] since they might be based on video-recordings, pictures (illustrations, photos, comics), texts, and their combinations. At our Faculty of Education, we have been using Concept Cartoons in professional preparation of primary and secondary school teachers for several years. Our systematic work with them led us to the need to sort Concept Cartoons generally and organize them with respect to their content and inner structure, i.e. to establish their typology.

The typology of Concept Cartoons that is presented in this paper is based on a long-term study with a qualitative exploratory design that took place between years 2012 and 2019. During this period, many smaller as well as larger qualitative research studies with future teachers as participants were conducted, in which Concept Cartoons figured as a tool for data collection (e.g. [14]) or as a teaching aid (e.g. [15]). Data from all these previous studies became the source of data for the present research study. The study addresses two research questions:

- What are the key aspects for selecting and creating Concept Cartoons when teaching arithmetic and training future teachers to teach arithmetic?
- What is the order of the key aspects that complies the process of creating new Concept Cartoons?

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1. CONCEPT CARTOONS

An educational tool called Concept Cartoons was established ca. 30 years ago in the Great Britain [8] as a tool to enhance motivation and engagement of pupils during school science lessons at primary and secondary school levels. In later years, the tool also spread to other school subjects including mathematics [2].

Concept Cartoons are individual pictures that show a sketch of a situation more or less related to the curriculum and a group of several children in a bubble dialog. In bubbles, the children discuss the pictured situation by presenting various alternative opinions. Similarly, as in reality, the wording of the opinions in bubbles is rather short and informal and the correctness and clarity of the presented opinions may vary. As for the order of bubbles, the top left bubble is usually considered the first, it may narrow the discussed topic or offer a further explanation of the pictured situation. One of the bubbles might be blank (with just a question mark in it), to indicate that there are other alternative opinions not mentioned in the bubbles yet. For a sample of a Concept Cartoon see Figure 1.

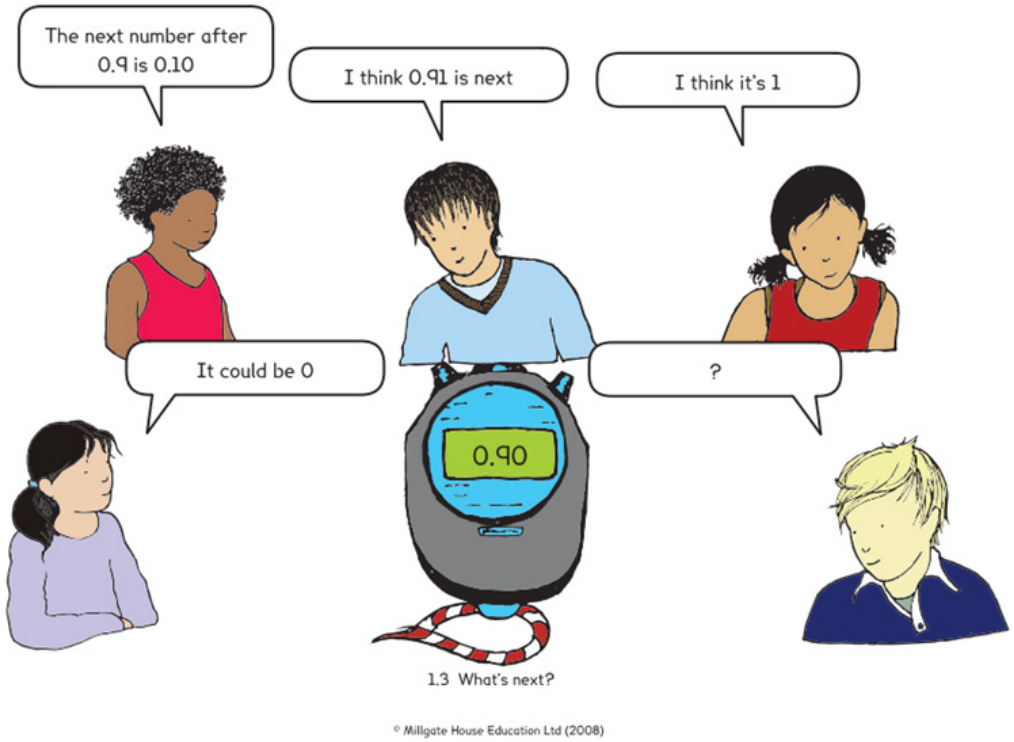

FIGURE 1. Concept Cartoon on decimal numbers on stopwatches;
(source of the picture: [2]: no. 1.3)

When employed in primary and secondary classrooms, Concept Cartoons are often introduced to pupils accompanied by questions such as “What do you think about it?”, “Who in the picture is right?”, “Why?”, and the pupils discuss about answers. For our use in professional preparation of primary and secondary teachers, we use two diverse sets of accompanying

questions. The shorter one consists of three questions that aim primarily at subject matter knowledge: “Which children in the picture are right?”, “Which are wrong?”, “Why?”. The longer one consists of two questions and four instructions that aim also at pedagogical content knowledge: “Which child do you strongly agree with?”, “Which child do you strongly disagree with?”, “Decide which ideas are right and which are wrong. Give reasons for your decision.”, “Try to discover the cause of the mistakes.”, “Advise the children who made the mistakes how to correct them.”, “Propose two texts that could be filled in the blank bubble – one of them correct, and the other one incorrect.” ([14]: 65).

The research conducted with Concept Cartoons in primary and secondary classrooms showed increase in pupils’ motivations and engagement as well as in argumentation skills [9, 11]. In professional preparation of teachers, the research showed the nature of teacher knowledge that the tool can reveal [14] and help develop [15]. It also showed that Concept Cartoons can reveal inconsistency in knowledge that standard written tests cannot do [13].

1.1. Methodology

The participants of this study were 319 future teachers, students of the professional teacher preparation university programmes designated for training primary school generalist teachers or secondary school mathematics teachers.

As already mentioned in the introduction, the initial stage of data collection took place in years 2012 to 2019, within the framework of several research studies that used Concept Cartoons as a tool for data collection or as a teaching aid. We obtained data in the form of written records of responses to various Concept Cartoons on arithmetic topics and to various sets of accompanying questions. To these written records, we added our own field notes from the process of creation new Concept Cartoons and from the process of sorting and selecting already created Concept Cartoons. In particular, we got use of written records associated to eight original arithmetic Concept Cartoons created by Dabell et al. [2] and to sixteen arithmetic Concept Cartoons which we have newly created themselves. We also got use of Concept Cartoons without associated data – these Concept Cartoons served to illustrate selected phenomena or as an inspiration in the process of creation additional illustrative Concept Cartoons. In particular, we employed one hundred and twenty original Concept Cartoons (i.e., the rest of the original mathematical set created by Dabell et al. [2]) and thirteen arithmetic Concept Cartoons which we have newly created themselves.

Data analysis aimed at finding a structured list of qualitative aspects and categories according to which it would be possible to appropriately select, sort and create Concept Cartoons. Due to the nature of the research questions, we proceeded from a qualitative research design of the exploratory type. We processed data using open coding and constant comparison [10]. During data analysis, we monitored the frequency and quality of data related to individual Concept Cartoons images, the relationships between individual Concept Cartoons and the mathematical content of their data; all from the perspective of individual participants as well as various groups of participants.

1.2. Findings – key aspects and relevant categories

Detailed data analysis highlighted the importance of distinguishing between aspect that might be considered objective and aspect that might be considered subjective. In this particular context, we perceive as *objective* aspects that can classify Concept Cartoons regardless of current knowledge and as *subjective* aspects that classify tasks with regard to respondents' current knowledge (for instance, when for different pupils and different periods of schooling, one and the same Concept Cartoon can be included in different categories). It showed that objective aspects are better structured and more suitable for proper grasping.

Subsequently, two key factors for objective aspects were identified that should be thoroughly distinguished: *the task in the background* and *the content of the bubbles*. As the task in the

background of a given Concept Cartoon we perceive the task that is illustrated by the picture in the background of the Concept Cartoon, together with the part of the text in the top left bubble that clarifies the pictured situation or explains what in particular the children discuss in other bubbles.

From the perspective of the task in the background, two relevant objective factors were revealed in data: *mathematical focus of the task* and *openness of the task* (in the sense of the open approach, see e.g. [12, 18]). From the perspective of the content of the bubbles, three relevant objective factors were revealed in data: *mathematical correctness of individual bubbles*, *mathematical format of the texts* and *polyvalence of bubbles* (polyvalence in the sense of polyvalent tasks, see e.g. [6, 18]). In this context, the polyvalence means that the decision about correctness is more difficult for some of the bubbles than for the others. In order to stay objective, the polyvalence of bubbles is considered only in cases when the more difficult bubbles cannot be assessed without knowledge that is needed for assessing the less difficult bubbles, i.e. from the perspective of knowledge, the more difficult bubbles can be considered an extension of the less difficult ones.

For each identified objective key aspect, a structure of its respective categories was created on the basis of our data. Type abbreviations were assigned to all the resulting relevant qualitative categories. A structured summary of all the relevant categories, arranged chronologically according to their importance in the process of the creation of new Concept Cartoons, is shown in Figures 2 and 3.

The task in the background

- Mathematical focus of the task
  - purely mathematical calculation task (code **MC**)
  - purely mathematical propositional task (code **MP**)
  - application task externally unaffected (code **AU**)
  - application task externally affected (code **AA**)
- Openness of the task
  - non-open task (code **OX**)
  - task with open starting situation (code **OS**)
  - task with open solution procedure (code **OP**)
  - task with open end situation (code **OE**)
  - task with multiple-step solution procedure (code **OM**)

FIGURE 2. Relevant qualitative categories for the key aspect *the task in the background*

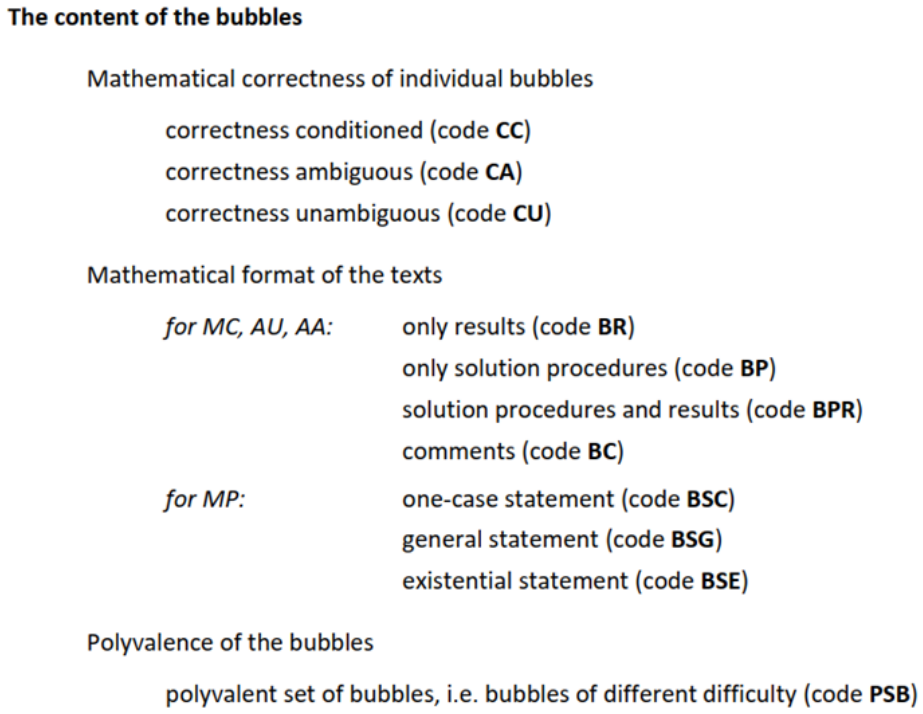

FIGURE 3. Relevant qualitative categories for the key aspect *the content of the bubbles*

1.3. Findings – illustrative examples

To illustrate the meaning of the key aspects and relevant categories, we would analyse several Concept Cartoons with respect to them. We would start with the Concept Cartoon from Figure 1 which discusses decimal numbers on stopwatches. As a task in the background of this Concept Cartoon, we may consider the task "Which number is next after the number 0.9 on stopwatches?". Such a task is an externally affected application task (code **AA**), since it addresses a practically based problem (numbers appearing on the stopwatches) and the answer to the problem depends on the accuracy of the particular stopwatches – it may be 0.01 as well as 0.05, thus the correct answer might be 0.91 as well as 0.95. The starting situation and the end situation are open, both due to the missing piece of information on the accuracy (codes **OS**, **OE**). As for the correctness of the bubbles, it is conditioned – based on the particular accuracy (code **CC**): for the accuracy 0.01, the top middle bubble is correct and the others are incorrect, for the accuracy 0.05, none of the bubbles is correct. As for the mathematical format of the bubbles, all of them show just results (code **BR**).

An externally unaffected application task (code **AU**) might be easily created from the Concept Cartoon in Figure 1 by adding the information about the accuracy to the picture, for instance, by adding a plate with "accuracy 0.01" beside the stopwatches. In that case, the task in the background would modify to "Which number is next after the number 0.9 on stopwatches with the accuracy 0.01?", and the task would not be open (code **OX**). The correctness of all the bubbles would be unambiguous (code **CU**).

In Figure 4, the task in the background may be assigned as "Decide whether numbers 74 and 144 are divisible by number 4." which means that the task is propositional (discusses the validity of a mathematical proposition; code **MP**). This task has a multiple-step solution procedure

which is open (there are more ways of how to decide about the divisibility and the decisions consist in more steps to complete; codes **OP**, **OM**). The top bubble shows an opinion that is ambiguous since the wording is inaccurate and one cannot see whether the child considers the decimals and units as a two-digit number or as two individual one-digit numbers (code **CA**), the other three bubbles show opinions that are clearly incorrect (code **CU**). All of the bubbles provide statements that are not general nor existential, their conclusions refer to individual cases only (code **BSC**).

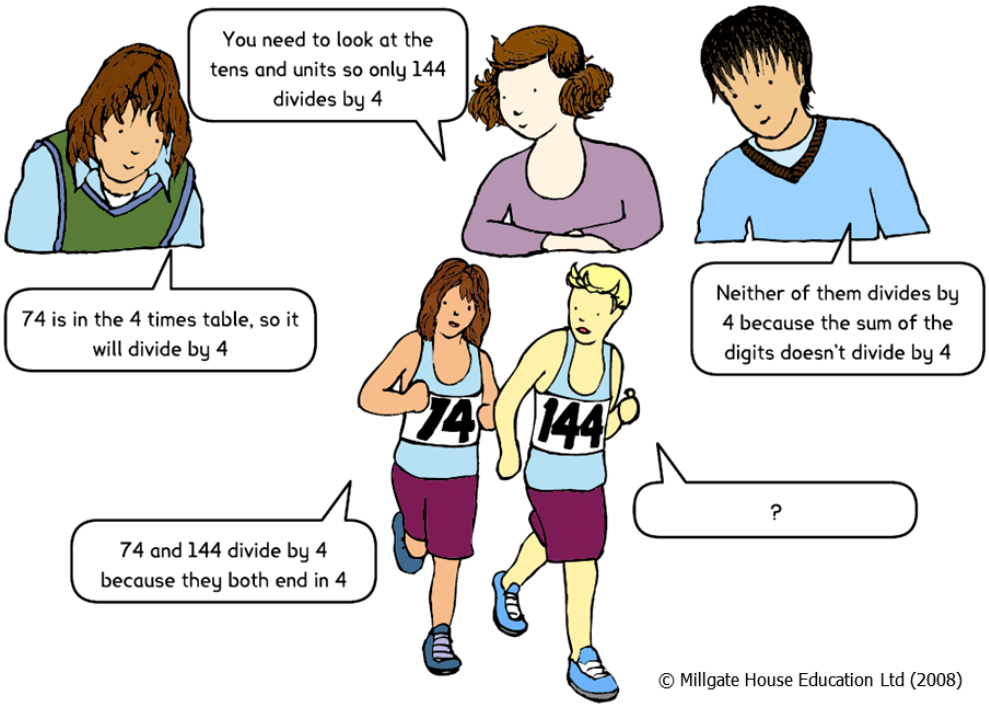

FIGURE 4. Concept Cartoon on divisibility by number 4;
(source of the picture: [2]: no. 1.9)

In Figure 5, the task in the background may be assigned as "Calculate 25 – 73." which means that it is a calculation task (code **MC**). This task has a multiple-step solution procedure which is open (there are more ways of how to calculate and the calculations consist in more steps to complete; codes **OP**, **OM**). The middle bubble shows a commentary on feasibility of the calculation (code **BC**) and its correctness depends on the number domain in which the calculation is executed – natural numbers or integers (code **CC**). The other three bubbles present solution procedures without results (code **BP**), two of them clearly incorrect and one clearly correct (code **CU**). A Concept Cartoon with polyvalent set of bubbles (code **PSB**) might be created from Figure 5, for instance, by modifying the middle bubble to "Take 78 off 30." Such a procedure requires not only knowledge of the process of subtraction itself but also knowledge of how compensating works in subtractions.

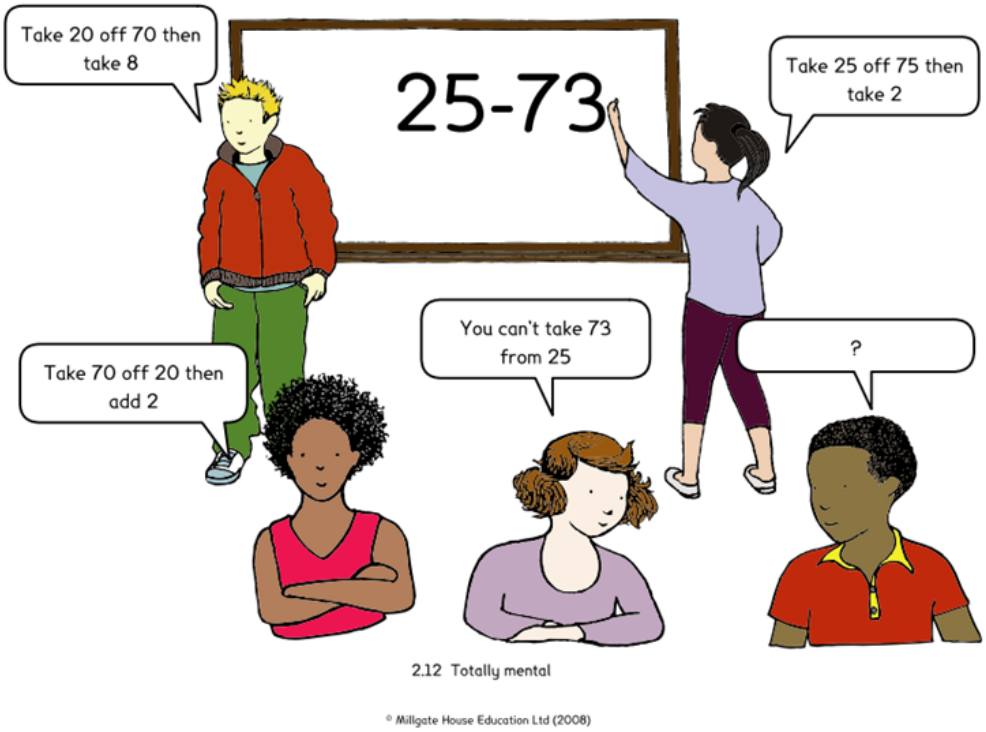

FIGURE 5: Concept Cartoon on subtraction;
(source of the picture: [2]: no. 2.12)

CONCLUSION

Representations of learning situations that are based on pictures, including Concept Cartoons, have only recently appeared to a greater extent in our educational environment. The typology of arithmetical Concept Cartoons presented in this paper was initially created as a support for our own systematic research and educational activities using the Concept Cartoons environment. However, its possible use exceeds our own needs. The typology can serve, for example, as a support for systematic research or educational activities of other teacher educators, or as an inspiration for establishing a typology of Concept Cartoons in geometry or in other school subjects.

Professional preparation of teachers of mathematics often includes problem posing activities that serve as a means to develop and investigate subject matter knowledge and pedagogical content knowledge of future teachers [21]. Our previous attempts to include Concept Cartoons to the problem posing activities usually failed due to excessive data fragmentation (see [16]). Since the order of relevant categories in the typology refers to their importance in the process of the creation of new Concept Cartoons and thus the comprehensive overview of the typology given in Figures 2 and 3 can be used as a step-by-step guide for creating new arithmetical Concept Cartoons, we hope that the fragmentation of data from problem posing might be avoided by letting future teachers work and familiarize with the typology before starting the problem posing process.

The typology of arithmetic Concept Cartoons is elaborated in a monography called *The Concept Cartoons Method* [17], where 32 Concept Cartoons having various combinations of relevant categories are presented and analysed.

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