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HOW TO USE DYNAMIC GEOMETRY IN A TRANSFORMATIVE WAY

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ABSTRACT. In 1980s, with the uptake of digital technology in education, the use of computers was contrasted in two metaphors: in the amplifier metaphor, technology allows performing tasks faster, more efficiently and accurately, whereas in the reorganizer metaphor, technology qualitatively changes the content and the cognitive processes engaged in problem solving. In this paper, we take dynamic geometry as an example of digital technology to illustrate various ways in which it can be used, referring to the SAMR model. Focusing on the dragging functionality, the pivotal feature of dynamic geometry system, we highlight a variety of uses and the corresponding mathematical conceptualizations. We conclude with some implications bringing to light challenges that mathematics teachers face with the use of digital technologies.

INTRODUCTION: ROLE OF DIGITAL TECHNOLOGY IN EDUCATION

Whether to use or not digital technology in mathematics classrooms is not an issue anymore nowadays, the question rather shifted to how to use it more efficiently and how to benefit the best from its affordances [5]. Since 1980s, researchers question the role technology should play in education. Two distinct roles have been highlighted by Pea [18] and described in terms of amplifier and reorganizer metaphors. The amplifier metaphor suggests that technology changes "how effectively we do traditional tasks, amplifying or extending our capabilities, with the assumption that these tasks stay fundamentally the same" (p. 168), while the reorganizer metaphor posits that technology changes "the tasks we do by reorganizing our mental functioning, and not only by amplifying it" (ibid.). A simplified vision of the two metaphors leads to considering the use of digital technology either to do traditional tasks although in a different way, or to do new tasks that cannot be done without this technology [22]. Likewise, Thomas and Lin [24] point out that key affordances of technology emanate from the tasks that are used with it. However, designing tasks incorporating technology and having an epistemic value [7] is not trivial for mathematics teachers.

In this paper, we aim at highlighting that a given (mathematical) digital tool can be mobilized in manifold ways with different learning potential. We illustrate these considerations on the example of dynamic geometry (DG). The choice of dynamic geometry is motivated by a discrepancy between its potential to support students' learning evidenced by numerous research (e.g., [14], [2], [1]) on the one hand, and its limited use in mathematics classrooms (e.g., [17], [8], [3]).

Jones [6] claims that "carefully designed tasks" with their appropriate enactment by the teacher are necessary for an efficient use of DG fostering students' learning:

> Overall, research in this area [use of DG software] indicates that successful access to geometrical theory does not happen without carefully designed tasks, professional teacher input, and opportunities for students to conjecture, to make mistakes, to reflect,

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to interpret relationships among objects, and to offer tentative mathematical explanations. (p. 29)

This paper therefore proposes an analysis of selected DG tasks aiming at highlighting their differences in terms of student's cognitive activity, thus showing the range of potential use of DG.

The tasks, analyzed in Section 2, are categorized according to the SAMR framework presented in Section 1. The concluding section discusses implications of the analyses for the teaching and learning mathematics with technology.

1. SAMR MODEL

The model proposed by Puentedura [19] identifies four different levels of classroom technology integration. The acronym "SAMR" stands for Substitution, Augmentation, Modification and Redefinition (Figure 1), the four levels that allow for questioning how technology is integrated into teaching and learning processes.

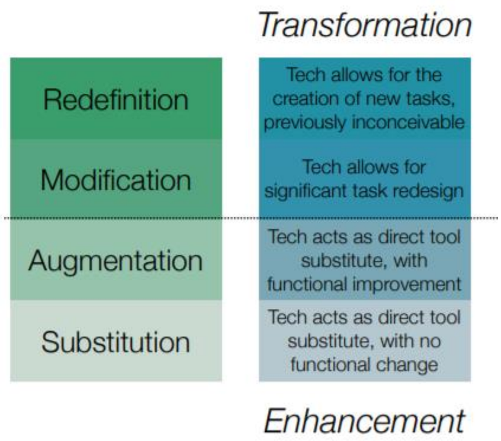


FIGURE 1. SAMR model1 [19]

Let us briefly present the four levels of technology use from substitution to redefinition and illustrate them on the example of a quiz.

At the *substitution* level, technology is simply substituted to traditional teaching tools or methods. An online version of a traditional paper-based quiz, where the student checks what she considers as correct answers, does not offer her any functional change. Technology in this case is thus used as a *substitute* of a paper quiz, although it can facilitate administration of the quiz (via a url instead of paper copies for example) and collection of students' responses.

At the *augmentation* level, technology substitutes traditional tools or methods, but with significant enhancements to the student experience. If the environment within which the quiz is implemented can provide feedback about the correctness of the answer, this functional improvement fosters student learning. Indeed numerous studies suggest that feedback is most effective when it is provided immediately, rather than days or weeks later, and seems to positively impact both students' achievement (e.g. [20]) and engagement (e.g. [23]).

At the *modification* level, technology deeply modifies tasks assigned to students offering them a richer learning experience. If the environment within which a quiz is designed provides not only true-false feedback but a more elaborated feedback such as hints (e.g., link to lessons)

1 http://hippasus.com/resources/tte/puentedura_tte.pdf

in case of incorrect answers, the student's learning experience is significantly changed: such feedback supports learning by orienting the student toward appropriate remedial activities.

At the *redefinition* level, technology allows for designing learning experiences that are not possible without it. A quiz that personalizes student's path through the items according to her answers can only be developed with technology.

As we show in the following section, the four levels of the SAMR model align with the four roles of dynamic geometry identified by Laborde [10]. For this reason, we refer to this framework when considering various uses of dynamic geometry.

2. VARIOUS USES OF DYNAMIC GEOMETRY

Laborde [10] identified four different roles of dynamic geometry in the tasks:

- DG is used "mainly as facilitating material aspects of the task while not changing it conceptually" (p. 293). These are for example construction tasks in which the only difference "lies in the drawing facilities offered" by dynamic geometry (ibid.). Dynamic geometry can be seen as a substitute of traditional tool.
- DG "is supposed to facilitate the mathematical task that is considered as unchanged". In this case, "DG is used as a visual amplifier […] in the task of identifying properties" (ibid.). Indeed, geometric properties of a figure being preserved while dragging its free elements, their visual recognition is facilitated. Dynamic geometry substitutes traditional tools, but brings certain functional improvement (augmentation).
- DG "is supposed to modify the solving strategies of the task due to the use of some of its tools and to the possibility that the task might be rendered more difficult" (ibid.). Whereas a construction of a geometric figure with traditional tools can result in a visually correct drawing although controlled by perception, the same task in DG environment requires using geometric properties to obtain a figure that resists while dragging its free elements. Solving strategies in DG environment are thus deeply modified.
- The task only exists in DG environment. Laborde [10] refers to the so-called "black box" tasks in which students are asked to reconstruct a dynamic figure provided in a DG environment that preserves geometric relations when its free elements are dragged (redefinition).

In the following sections (2.1-2.4), organized according to the levels of the SAMR model, we discuss various possible uses of DG. Following Lopez-Real and Leung [15], who consider that

> dragging in DGE can open up some kind of semantic space (meaning potential) for mathematical concept formation in which dragging modalities (strategies) are temporal-dynamic semiotic mediation instruments that can create mathematical meanings (p. 666),

we focus on the dragging functionality of dynamic geometry to highlight its contribution to mathematical conceptualization.

2.1. Substitution Level Tasks

Inviting students to make a free drawing using DG tools, without paying attention to geometric relations is perhaps the “simplest” task (Figure 2). Such a task can offer an opportunity to the students to get acquainted with DG menus and tools when they are introduced to this technology. Such a task can also be an occasion to exploit the semiotic potential of DG tools by comparing and contrasting them with traditional tools. For example, the fact that, in order to draw a straight line, the user needs to click to two different spots on the screen, which results in creating two distinct points and subsequently a line passing through these points, conveys the idea that a straight line passes through two distinct points. This is not necessarily the case with using a ruler to draw a straight line, which rather emphasizes the straightness of the line.

FIGURE 2. Example of a freely drawn man

Another example of a task at the substitution level is constructing a geometric figure following a construction program (i.e., a series of instructions). In the example shown in Figure 3, the task is proposed to primary school pupils. The use of DG presents several advantages. The figures pupils construct can be quite complex, not only usual ones, since the task is facilitated within the DG environment. Pupils with motor difficulties of drawing with traditional instruments can succeed the task. Self-evaluation is also easier as drawings are more accurate and pupils can modify elements of the figure without deleting the correct steps.

Construction steps	Self-evaluation
Draw a segment [AB] called *a*.	
Create C, the midpoint of [AB].	
Draw a line *b* perpendicular to [AB] and passing through C.	
Create a point D wherever you want, but neither on [AB], on *a* nor on *b*.	
Draw a line *c* parallel to *b* passing through D.	
Draw a line parallel to [AB] passing through D.	
Create E the point of intersection of *b* and *d*.	
Draw a circle *e* with center E and radius EB.	
Draw a line *f* passing through A and D.	
Create F the point of intersection of [EC] and (AD).	
Draw a circle *g* with center F and radius FB.	
Draw a line perpendicular to (AD) and passing through B. It cuts (AF) in G.	
Draw a line parallel to (ED) passing through G.	

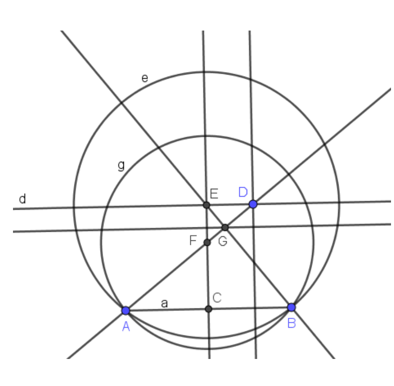
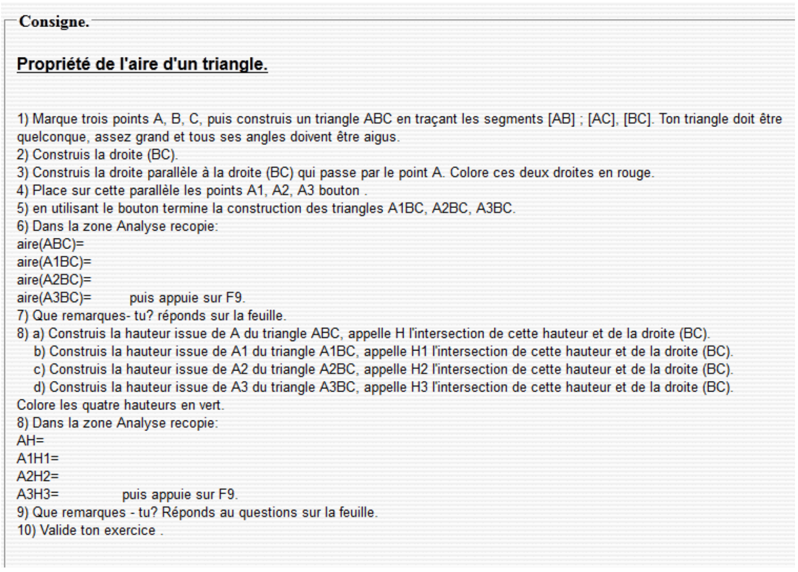


FIGURE 3. Construction program (left) yielding a geometric figure (right)[2]

Drag mode in these tasks is used to a limited extent, if at all: to adjust elements of a drawing either for the purposes of perceptive satisfaction (free drawing) or to separate elements of a figure to ease its construction (for example, when two points are too close to each other that they may be confused when selecting one of them). Restrepo [21] classifies this dragging modality as *dragging without mathematical purpose*.

It is not rare to find resources in which DG is used as a mere substitute of traditional tools although its potential could have been exploited to a greater extent. An example is given in Figure 4 showing a task aiming at discovering that the area of a triangle ABC does not change when one of its vertices, say A, belongs to a line parallel to the opposite side [BC]. Instead of dragging the vertex A on the line, the task invites to construct three distinct points A1, A2 and A3, construct four triangles ABC, A1BC, A2BC, A3BC, display their areas and observe the property.

2 Task retrieved from http://www.ac-grenoble.fr/ien.st-gervais/spip.php?article1420


Consigne.

Propriété de l'aire d'un triangle.

1) Marque trois points A, B, C, puis construis un triangle ABC en traçant les segments [AB] ; [AC], [BC]. Ton triangle doit être quelconque, assez grand et tous ses angles doivent être aigus.
2) Construis la droite (BC).
3) Construis la droite parallèle à la droite (BC) qui passe par le point A. Colore ces deux droites en rouge.
4) Place sur cette parallèle les points A1, A2, A3 bouton .
5) en utilisant le bouton termine la construction des triangles A1BC, A2BC, A3BC.
6) Dans la zone Analyse recopie:
aire(ABC)=
aire(A1BC)=
aire(A2BC)=
aire(A3BC)= puis appuie sur F9.
7) Que remarques- tu? réponds sur la feuille.
8) a) Construis la hauteur issue de A du triangle ABC, appelle H l'intersection de cette hauteur et de la droite (BC).
b) Construis la hauteur issue de A1 du triangle A1BC, appelle H1 l'intersection de cette hauteur et de la droite (BC).
c) Construis la hauteur issue de A2 du triangle A2BC, appelle H2 l'intersection de cette hauteur et de la droite (BC).
d) Construis la hauteur issue de A3 du triangle A3BC, appelle H3 l'intersection de cette hauteur et de la droite (BC).
Colore les quatre hauteurs en vert.
8) Dans la zone Analyse recopie:
AH=
A1H1=
A2H2=
A3H3= puis appuie sur F9.
9) Que remarques - tu? Réponds au questions sur la feuille.
10) Valide ton exercice .

FIGURE 4. Task aiming at discovering a property of the area of a triangle

In this task, the drag mode is not exploited at all. The only contribution of dynamic geometry is the accuracy of measures of lengths of the segments and of the areas of the triangles. Such tasks could benefit from the drag mode and DG would then be used at the augmentation level, as we discuss it in the following section.

2.2. Augmentation Level Tasks

Tasks analyzed in this section fall under the *robust construction* paradigm. Laborde [9] characterizes robust constructions as those “for which the drag mode preserves their properties”. The author provides the example of an angle AMB inscribed in a circle (Figure 6). When the point M is dragged along the circle, one can easily observe that the angle AMB remains right. This robust construction shows that “for any point of the circle (except A and B) angle AMB is a right angle” [9].

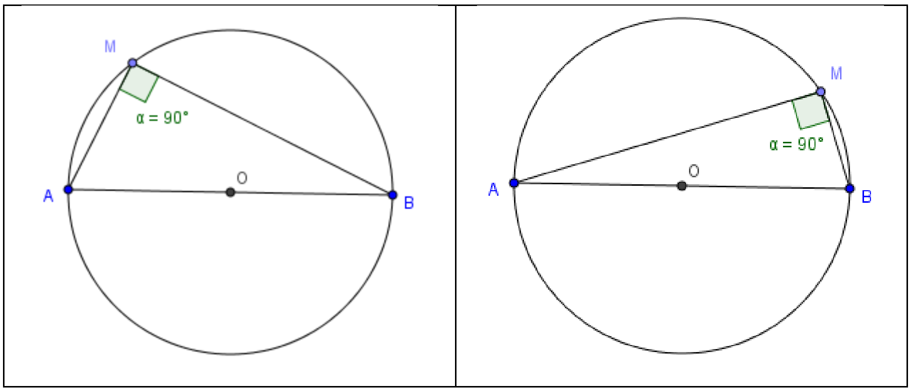


FIGURE 5. Angle AMB inscribed in a semi-circle

Typical tasks within this paradigm consist in exploring robust constructions. These may be either constructed by a teacher or by students who follow detailed instructions. The students are then invited to vary elements of the figure (point M in the above-mentioned example) in order to recognize or to discover a geometric property based on the observation of the figure: the property at stake remains invariant (the measure of the angle AMB). Referring to the Marton et al.'s [16] framework of variation, described in [13], the epistemic function of variation enabled by dragging in a robust construction is to allow *separation* of aspects of a figure that vary from other aspects that remain invariant. In the above-mentioned example, dragging points A and B allows observing for example that the segment [AB] is always a diameter of the circle, but its horizontal direction is not a necessary condition for the angle AMB to be right. Laborde [9] sums up the contribution of this robust construction to the learning of the associated geometric theorem as follows:

> The robust construction contributes to a better identification in action of the elements of [the theorem] for several reasons:
>
> - The construction requires to take into account two conditions to get a right angle: AB must be a diameter and M a point on a circle […].
> - It allows contrasting the invariance of the angle and the varying nature of point M.
> - It exteriorizes the variable nature of point M and the set in which it varies [9].

Dragging elements of a robust construction allows producing quickly a number of different drawings sharing the same geometric property, which helps students "extend their visual images of a property […] and reject some spatio-graphical properties" that they can attach to the figure. Thus, "the drag mode is used as tool for distinguishing between contingence and necessity" [9], which constitutes a clear functional improvement comparing to traditional tools. From the instrumental perspective, Restrepo [21] ranges this modality of dragging among *exploratory dragging modalities*: its purpose is to look for invariants in a given figure, which facilitates identification of its geometric properties.

2.3. Modification Level Tasks

Robust construction tasks are another kind of tasks falling under robust construction paradigm. Students are asked to construct geometric figures that satisfy given conditions even when their elements are varied by dragging, for example, construct a square given its side or given its diagonal. As Laborde [9] specifies,

> Eye ball constructions are invalidated by the drag mode since it becomes visible that some of the conditions are not satisfied. The drag mode is a critical factor in robust construction tasks that makes the difference with a paper and pencil environment. In such construction tasks in dynamic geometry, the drag mode provides a visual feedback from the fact that the construction does not meet all the required conditions. The strength of DGE lies in this possibility of showing at the spatio-graphical level the theoretical weakness of the construction.

The necessity to resort to geometric properties when constructing a figure (for example a square given its side or its diagonal) modifies deeply the construction task in comparison to the same task realized in paper and pencil environment, where the students "very often stay at a graphical level and try only to satisfy the visual constraints" [9]. The drag mode provides students with a visual feedback about the correctness of their construction; it is therefore used as an *instrument for validating* constructions (also called *dragging test*, e.g. by Arzarello et al., [1]) and helps students gaining awareness of the distinction between a *drawing* (material diagram representing a geometric object) and a *figure* (theoretical object defined with its properties) [11].

Less common tasks are those in which students are asked to look for conditions under which certain configurations are obtained. In the example taken from [9], a circle with segment [AB] as a diameter and a point M not belonging to the circle are given. Students are asked to find a position of M outside the disk such that the angle AMB is obtuse (Figure 6). The purpose of this task is to let students explore the situation and notice that the angle AMB is acute when M is outside the disc and obtuse when it is inside and eventually discover the relationship between the measure of AMB and the position of the point M.

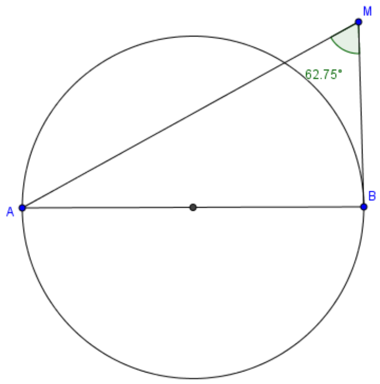


FIGURE 6. Searching for particular position of the point M

This construction is coined soft because, as the point M is not constructed as a point on the circle, the targeted geometric property, namely the fact that the angle AMB is right when M belongs to the circle, is not directly visible, as it is the case in the robust construction (see Fig. 5). Rather, this property is inferred from observing that "the circle is the border between two regions, one in which angle AMB is obtuse and one in which angle AMB is acute" [9], hence AMB must be right when M in on the circle.

Soft constructions present several features that offer interesting learning opportunities. First, tasks that exploit soft constructions are more engaging than their robust versions. Indeed, they offer genuine problems to be solved and dynamic geometry is a support for exploring given situations. Pea [18] evokes

> dynamic what-if capacities of such systems [that] make it possible to display immediately the consequences of different approaches to a problem (p. 171).

We claim that soft construction dynamic geometry task is significantly modified as it offers support for generating and testing various conjectures given different hypothetical conditions. Dragging plays a crucial role in this exploration. Moreover, students' exploration of a soft construction leads to putting more emphasis on the link between the condition (in our case, M is on the circle) and the consequence (the angle AMB is right), which facilitates grasping the meaning of a geometric property as an implication, which makes it particularly relevant in proof oriented tasks.

2.4. Redefinition Level Tasks

Among the tasks that cannot exist but within a dynamic geometry are the so called "black box" tasks. Clerc [4] describes a black box in dynamic geometry as a geometric figure made up of initial objects and final objects the construction and displacement of which are linked to the initial objects. The construction of these final objects is hidden. A mathematical task that can be set up with a black box consists in asking the students to solve them, that is to say to find out how to construct the final objects from the initial ones, the construction must of course resist when the free objects in the figure are dragged. Figure 7 shows a black box where the initial objects are three distinct points A, B, and C (or a triangle ABC), and the final objects are points X and Y.

The student is expected to explore the figure, make conjectures, verify them experimentally and eventually reconstruct the points X and Y. While dragging, the student can observe for example that the point X remains inside the triangle whereas the point Y can get outside (Figure 7, middle). She can draw lines, circles, midpoints… to enrich the figure; she can measure distances or angles (Figure 7, right). Dragging clearly play a critical role in searching for the hidden relationships. It is used both for exploring the figure and verifying conjectured geometric properties by highlighting their invariance.

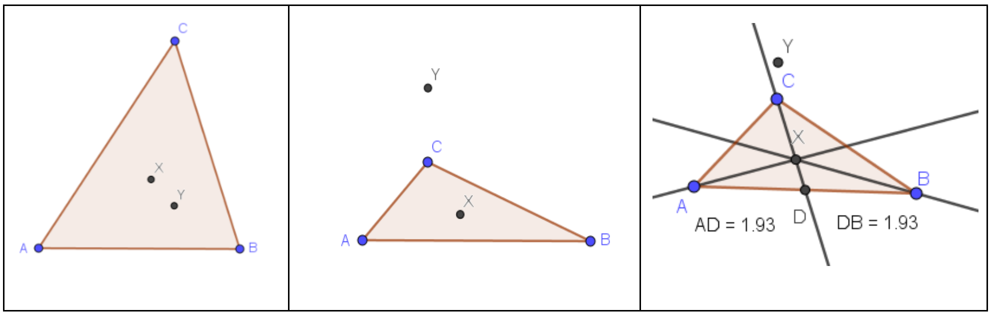


FIGURE 7. A black box task

CONCLUDING REMARKS

The purpose of this paper was to show that a given digital educational technology can be used in many different ways, ranging from a mere substitute of traditional tools to offering unique learning opportunities in novel tasks. Dynamic geometry has been taken as an emblematic example and the tasks analyzed have been taken from past research of from available curricular resources.

The various uses of digital technology illustrated on the example of dynamic geometry can apply to other tools. Let us consider spreadsheets [12]. At the substitution level, spreadsheet can be used as a traditional double entry table, to organize data. The use of formulas and their dragging adds a functional improvement to performing calculations (augmentation level). Tasks mobilizing spreadsheet functions are deeply modified compared to traditional approaches as they require modeling and generalization. Finally, tasks mobilizing advanced functionalities, such as conditional formatting, charts or programming macros fall under redefinition level.

Our analyses highlight that tasks at the transformation levels (modification and redefinition) show a greater potential for a student-centered approach, engaging students in inquiry-based problem-solving activity, compared to the tasks at the enhancement levels (substitution and augmentation), which are rather teacher-centered, requiring less important cognitive activity from students.

Therefore, digital technology itself is not transformative; it is the way how it is used that can be transformative. As the teachers' role in technology-based education is crucial, it is urgent to provide them with support aimed at raising their awareness of the manifold uses of technology and to help them develop practices at the transformation levels.

REFERENCES

[1] Arzarello, F., Olivero, F., Paola, D. & Robutti, O. (2002). A cognitive analysis of dragging practices in Cabri environments. *ZDM – Mathematics education* 34(3), 66-72.

[2] Baccaglini-Frank, A., & Mariotti, M. A. (2010). Generating conjectures in dynamic geometry: The maintaining-dragging model. *International Journal of Computers for Mathematical Learning*, 15(3), 225–253.

[3] Bretscher, N. (2010). Dynamic geometry software: the teacher's role in facilitating instrumental genesis. In V. Durrand-Guerrier et al. (Eds.), *Proceedings of CERME 6* (pp. 1340-1348). Lyon: INRP.

[4] Clerc, B. (2006). Boîte noire en géométrie dynamique. *MathémaTICE* 2. http://revue.sesamath.net/spip.php?article13

[5] Gibson, J. J. (1977). The theory of affordances. In Shaw, R., & Bransford, J. (Eds.), *Perceiving, acting, and knowing: Toward an ecological psychology* (pp. 67-82). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

[6] Jones, K. (2005), Research on the Use of Dynamic Geometry Software: implications for the classroom. In J. Edwards & D. Wright (Eds.), *Integrating ICT into the Mathematics Classroom* (pp. 27-29). Derby: Association of Teachers of Mathematics.

[7] Kieran, C., & Drijvers, P. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning*, 11, 205-263.

[8] Kriek, J., & Stols, G. (2011). Why don't all maths teachers use dynamic geometry software in their classrooms? *Australasian Journal of Educational Technology*, 27(1), 137-151.

[9] Laborde, C. (2005). Robust and soft constructions: two sides of the use of dynamic geometry environments. In S.-C. Chu et al. (Eds.), *Proceedings of the 10th Asian Technology Conference in Mathematics (ATCM)* (pp. 22-35). Korea National University of Education.

[10] Laborde, C. (2001). Integration of technology in the design of Geometry tasks with Cabri-Geometry. *International Journal of Computers for Mathematical Learning,* 6, 283–317.

[11] Laborde, C., & Capponi, B. (1994). Cabri-géomètre constituant d'un milieu pour l'apprentissage de la notion de figure géométrique. *Recherches En Didactique Des Mathématiques*, 14(1.2), 165–210.

[12] Lagrange, J.B., & Erdogan, E. (2009). Teacher's emergent goals in spreadsheet based lessons: Analysing the complexity of technology integration. *Educational Studies in Mathematics*, 71(1), 65- 84.

[13] Leung, A. (2017). Variation in tool-based mathematics pedagogy: the case of dynamic virtual tool. In R. Huang & Y. Li (Eds.), *Teaching and Learning Mathematics through Variation – Confucian Heritage Meets Western Theories* (pp.69-84). Rotterdam, Sense Publishers.

[14] Leung, A. (2015). Discernment and reasoning in dynamic geometry environments. In S. Cho (Ed.), *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 451–469). Cham: Springer.

[15] Lopez-Real, F., & Leung, A. (2006). Dragging as a conceptual tool in dynamic geometry environments. *International Journal of Mathematical Education in Science and Technology*, 37(6), 665-679.

[16] Marton, F., Runesson, U., & Tsui, A. B. M. (2004). The space of learning. In F. Marton & A. B. M. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3–40). New Jersey: Lawrence Erlbaum Associates, INC Publishers.

[17] Molnár, P., & Lukáč, S. (2015). Dynamic Geometry Systems in Mathematics Education: Attitudes of Teachers. *International Journal of Information and Communication Technologies in Education* 4(4), 19-33.

[18] Pea, R. D. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist*, 20(4), 167-182.

[19] Puentedura, R. R. (2006). *Transformation, Technology, and Education*. Screencast of the presentation delivered August 18, 2006. Retrieved from http://hippasus.com/resources/tte/

[20] Razzaq, R., Ostrow, K. S., & Heffernan, N.T. (2020). Effect of Immediate Feedback on Math Achievement at the High School Level. In I. Bittencourt., M. Cukurova, K. Muldner, R. Luckin, & E. Millán (Eds.), *Artificial Intelligence in Education. AIED 2020. Lecture Notes in Computer Science* (Vol. 12164, pp. 263-267). Cham: Springer.

[21] Restrepo, A. M. (2008), Genèse instrumentale du déplacement en géométrie dynamique chez des élèves de 6ème. Doctoral thesis, Joseph Fourier University, Grenoble.

[22] Ripley, M. (2009). Transformational computer-based testing. In F. Scheuermann & J. Björnsson (Eds.), *The transition to computer-based assessment* (pp. 92-98). Luxemburg: Office for Official Publications of the European Communities.

[23] Sancho-Vinuesa, T., Escudero-Viladoms, N., & Masia, R. (2013). Continuous activity with immediate feedback: A good strategy to guarantee student engagement with the course. *Open Learning*, 28(1), 51-66.

[24] Thomas, M. O. J., & Lin, C. (2013). Designing tasks for use with digital technology. In C. Margolinas (Ed.), *Task design in mathematics education. Proceedings of ICMI Study 22* (Vol. 1, pp. 111-119). Oxford.

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