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GESTURE-BASED GEOMETRY LEARNING WITH SKETCHOMETRY

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ABSTRACT. Touchscreen devices offer numerous possibilities for interactive mathematics learning. This article presents a didactic framework for integrating the gesture-based dynamic geometry software sketchometry into classroom practice. Unlike traditional dynamic geometry systems, sketchometry allows geometric constructions to be created directly through finger sketches and gestures without explicit tool selection. Building on this interaction paradigm, structured teaching modules have been developed that combine digital exploration with worksheets, documentation tasks, and classroom discussion. The modules follow a four-phase instructional model consisting of construction, exploration, discussion, and instruction. In addition to the instructional framework, the article discusses the theoretical background of gesture-based interaction, principles of gesture recognition, and implications for mathematics education. Special attention is given to the role of embodied interaction, inquiry-based learning, and the integration of digital and analog learning environments.

INTRODUCTION

The rapid spread of tablets and smartphones with touch-sensitive displays has opened a wide range of possibilities for digital learning environments. In mathematics education, these devices allow students to interact directly with mathematical objects through gestures such as drawing, dragging, and tracing. Geometry in particular benefits from interactive technologies because geometric reasoning is closely connected to visual representation and manipulation of figures.

The close relationship between geometry and drawing has a long historical tradition. In ancient mathematics, geometric figures were often sketched directly in sand or on tablets. Even today, drawing remains a central activity in learning geometry, whether on paper or on digital devices. The act of sketching supports spatial reasoning and helps learners connect abstract concepts with visual representations.

Traditional dynamic geometry software enabled interactive exploration of geometric constructions. However, these systems typically require users to select tools from menus and follow predefined sequences of commands. While this approach is powerful, it sometimes shifts the learner’s attention from mathematical ideas toward software procedures.

The dynamic geometry environment *sketchometry* [10] was developed to provide a more natural interaction paradigm. Instead of selecting tools, users draw geometric objects directly with their finger or stylus. Freehand sketches are automatically interpreted as precise geometric objects. Additional constructions are created through gestures that encode mathematical relationships. This gesture-based interaction reflects the close connection between drawing and geometric thinking.

The aim of this article is to present the didactic principles underlying gesture-based geometry interaction and to illustrate how sketchometry can be integrated into classroom practice through structured teaching modules. In particular, the article discusses the role of gestures in

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mathematical thinking, the design principles of gesture-based tools, and the use of inquiry-oriented activities in geometry learning.

1. Sketches and Gestures as Interaction Paradigms

A central feature of sketchometry is the distinction between sketches and gestures. Sketches represent freehand drawings of basic geometric objects such as lines, circles, or triangles, as illustrated in Figure 1. When the user draws such a shape on the screen, the software analyzes the drawing and converts it into a mathematically precise object.

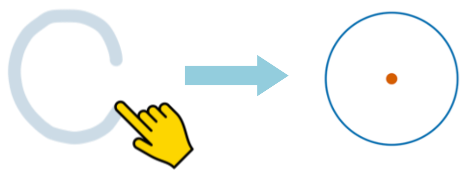


FIGURE 1. A freehand circle is recognised and interpreted as a geometric circle.

Gestures, on the other hand, act as tools. They are movements that create new geometric objects from existing constructions. For example, an L-shaped gesture applied to a line constructs a perpendicular line (see Figure 2). The gesture itself expresses the geometric relationship between the new object and the existing one.

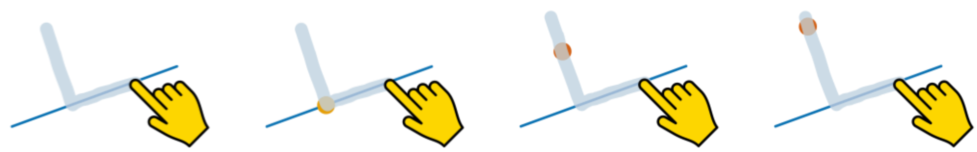


FIGURE 2. The “L-gesture” is used to construct perpendicular lines.

This interaction model differs significantly from the traditional tool-selection paradigm. Instead of choosing tools from a toolbar, the user communicates the intended construction directly through movement. As a result, gestures carry mathematical meaning and are not merely technical commands.

2. Embodied Interaction and Mathematical Thinking

The gesture-based interaction paradigm can be interpreted through the concept of embodied cognition. According to this perspective, cognitive processes are closely connected to bodily actions and sensory experiences.

In mathematics classrooms, gestures frequently accompany explanations and reasoning. Teachers often illustrate angles, directions, or transformations with hand movements. Such gestures support spatial reasoning and help learners connect abstract ideas with visual representations.

Gesture-based software extends this principle by integrating gestures directly into the construction process. Students do not merely observe geometric relationships; they actively create and manipulate them through physical movements. Empirical studies have shown that tracing mathematical representations with the finger can improve understanding and reduce cognitive load during learning [7].

3. Design Principles of Gesture-Based Tools

For gesture-based interaction with sketchometry to work reliably in educational contexts, several design principles must be considered. From a technical perspective, gestures must be recognizable regardless of their position, size, or orientation on the screen. They should be drawn in a single continuous stroke and must reference the geometric objects relevant to the construction.

From a pedagogical perspective, gestures should reflect the characteristic properties of the constructed object. For example, the gesture for constructing a tangent resembles the letter T, indicating the perpendicular relationship between the radius and the tangent line. Gestures must also be easy to remember and should correspond clearly to a single mathematical construction.

4. Gesture Recognition Process

The gesture recognition in sketchometry follows a structured multi-step process, as illustrated in Figure 3. In the first stage (acquisition), the system records the path drawn by the user on the touchscreen and interprets it as a sequence of geometric elements. This curve is then compared with stored gesture templates in a pattern matching step that is invariant with respect to position, size, and orientation.

FIGURE 3. The gesture recognition process gives immediate feedback.

In the next stage (object identification), the system determines which geometric objects in the construction are referenced by the gesture. Finally, a validation step checks whether the recognized gesture and the detected objects form a meaningful geometric construction before the corresponding object is created.

A particularly interesting feature of sketchometry is its early recognition mechanism. During the acquisition and pattern matching process, the software already displays a preliminary interpretation of the gesture while it is still being drawn. This immediate feedback enhances interactivity, allows for a high tolerance of drawing inaccuracies, and supports intuitive experimentation with geometric constructions.

5. Comparison with Traditional Dynamic Geometry Software

Traditional dynamic geometry systems rely on explicit tool selection. Users must first activate a construction tool and then apply it to specific objects by clicking on them. In practice, this leads to a repeated interaction pattern for many different tools: first selecting the tool, then clicking on the relevant objects in the construction. While this approach provides a clear procedural structure, it introduces additional steps that may distract from the underlying mathematical relationships.

In sketchometry, the tool is embedded directly in the movement. This integration reduces the distance between mathematical intention and digital action. Students can focus more strongly on exploring geometric ideas rather than learning software procedures.

Another advantage of gesture-based interaction lies in its contextual flexibility. A gesture does not arbitrarily generate different objects; instead, it responds to the geometric situation in which it is performed. The construction that results from the gesture depends on the objects referenced during the movement. For example, the perpendicular gesture can be used in several situations: it may create a perpendicular to a given line, a perpendicular through a point on the line, or a perpendicular through a point outside the line. In all these cases the gesture itself remains unchanged, while the resulting construction adapts to the geometric context. This context-sensitive behaviour supports exploratory work with geometric configurations.

6. Sketchometry Teaching Modules

Technology alone does not guarantee meaningful learning. To integrate sketchometry effectively into classroom teaching, structured teaching modules have been developed. Each module typically consists of a student worksheet, a teacher information sheet, a documentation format such as a study journal or result sheet, and an additional exercise sheet for further practice.

The worksheets guide students through construction and exploration tasks, while the teacher information sheet describes prerequisites and learning objectives. This structure helps teachers integrate digital tools into their lessons without abandoning familiar teaching practices.

The teaching modules follow a four-phase instructional model consisting of construction, exploration, discussion, and instruction [2], as illustrated in Figure 4.

FIGURE 4. Sketchometry teaching modules follow a four-phase instructional model.

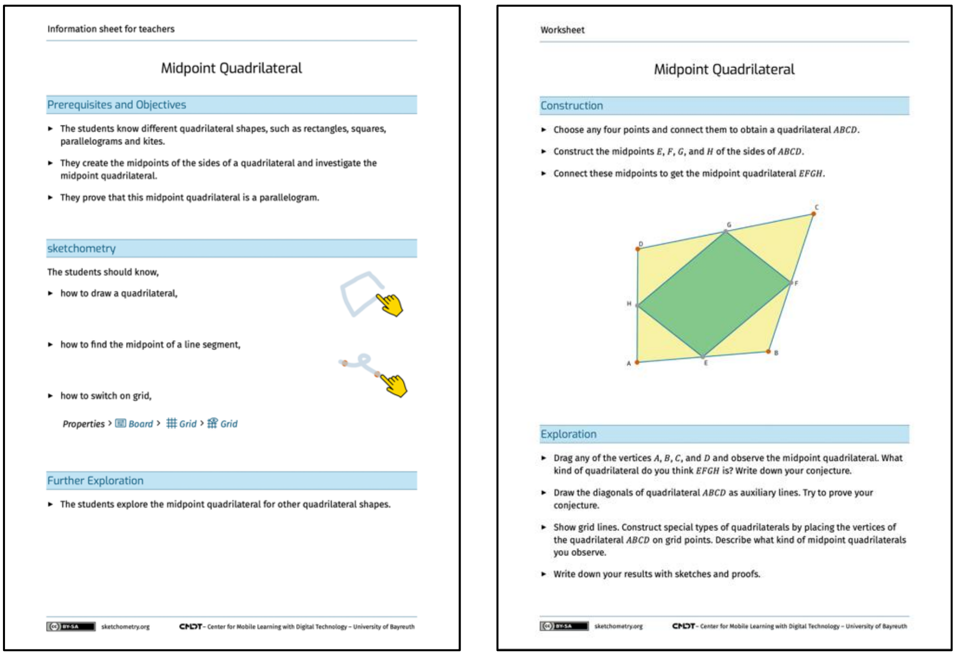
During the construction phase students create geometric configurations using sketchometry. In the exploration phase they manipulate these constructions and observe patterns or invariants. Students record their observations in study journals or result sheets.

The discussion phase allows learners to compare their findings with peers and articulate their reasoning. Finally, in the instruction phase the teacher summarizes the results and connects them to formal mathematical concepts.

7. Classroom Example: Exploring the Midpoint Quadrilateral

An illustrative classroom activity within a sketchometry teaching module is the investigation of the midpoint quadrilateral [11]. Students begin by constructing an arbitrary quadrilateral by choosing four points and connecting them. They then construct the midpoints of the four sides and connect these points to obtain a new quadrilateral, the so-called midpoint quadrilateral.

Once the construction has been completed, students explore the configuration dynamically by dragging the vertices of the original quadrilateral (see Figure 5). Through this manipulation they observe how the midpoint quadrilateral changes while maintaining certain geometric properties. This dynamic experimentation encourages students to formulate conjectures about the nature of the midpoint quadrilateral. In many cases they conjecture that the midpoint quadrilateral appears to be a parallelogram.


Information sheet for teachers

Midpoint Quadrilateral

Prerequisites and Objectives

- The students know different quadrilateral shapes, such as rectangles, squares, parallelograms and kites.
- They create the midpoints of the sides of a quadrilateral and investigate the midpoint quadrilateral.
- They prove that this midpoint quadrilateral is a parallelogram.

sketchometry

The students should know,

- how to draw a quadrilateral,
- how to find the midpoint of a line segment,
- how to switch on grid,

Properties > *Board* > *Grid* > *Grid*

Further Exploration

- The students explore the midpoint quadrilateral for other quadrilateral shapes.

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Worksheet

Midpoint Quadrilateral

Construction

- Choose any four points and connect them to obtain a quadrilateral $ABCD$.
- Construct the midpoints E, F, G, and H of the sides of $ABCD$.
- Connect these midpoints to get the midpoint quadrilateral $EFGH$.

Exploration

- Drag any of the vertices A, B, C, and D and observe the midpoint quadrilateral. What kind of quadrilateral do you think $EFGH$ is? Write down your conjecture.
- Draw the diagonals of quadrilateral $ABCD$ as auxiliary lines. Try to prove your conjecture.
- Show grid lines. Construct special types of quadrilaterals by placing the vertices of the quadrilateral $ABCD$ on grid points. Describe what kind of midpoint quadrilaterals you observe.
- Write down your results with sketches and proofs.

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FIGURE 5. The information sheet guides teachers in using the student worksheet.

The activity then moves from experimental observation toward mathematical reasoning. Students are asked to construct auxiliary elements, such as the diagonals of the original quadrilateral, and analyze the resulting geometric relationships. By comparing pairs of triangles that arise in the construction, they can identify parallel segments and develop arguments explaining why the midpoint quadrilateral must always be a parallelogram. In this way, the activity connects exploratory work in a dynamic geometry environment with deductive reasoning.

Further exploration tasks invite students to investigate special cases (see Figure 6). Using the grid, they construct particular quadrilateral types such as squares, rectangles, rhombi, or parallelograms and examine the corresponding midpoint quadrilaterals. Comparing these cases helps students recognize invariant geometric relationships and strengthens their understanding of the underlying theorem.

From a didactic perspective, this example illustrates how gesture-based dynamic geometry can support inquiry-based learning processes. Students begin with construction, engage in experimentation, formulate conjectures, and finally develop arguments that justify their observations. The dynamic environment allows them to test many configurations efficiently, while the structured worksheet guides the transition from empirical observation to mathematical proof.

8. IMPLICATIONS FOR TEACHER EDUCATION

Gesture-based geometry environments such as sketchometry are also relevant for teacher education. Future teachers must learn not only how to operate digital tools but also how to design meaningful learning activities around them.

Teacher education programs can use sketchometry modules to illustrate how digital technology can support inquiry-based learning. At the same time, students can reflect on the didactic principles underlying such teaching approaches.

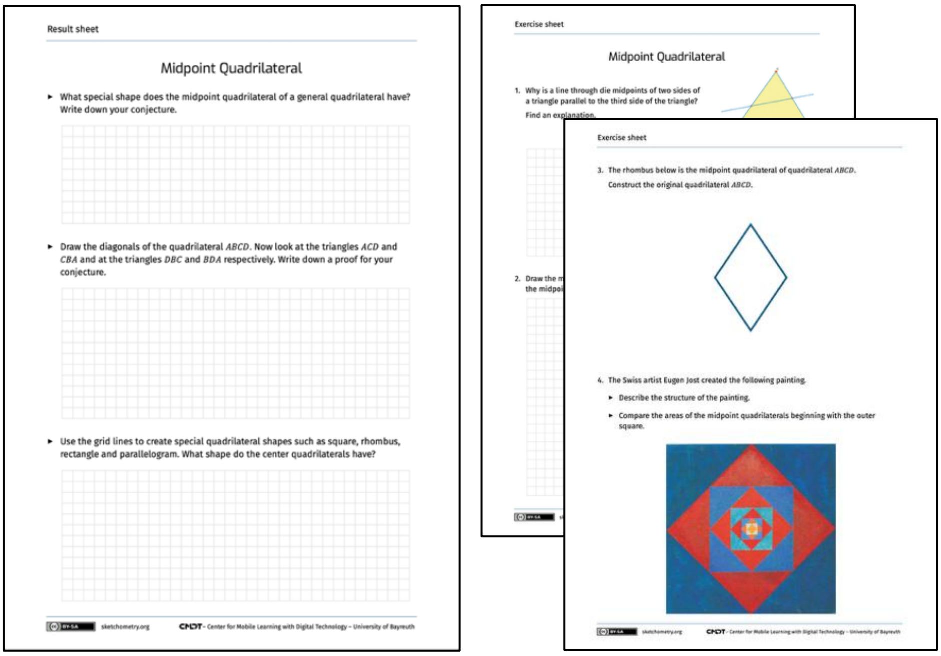

FIGURE 6. Result sheet used by students to record observations, conjectures, and proofs.

9. CONCLUSION

Gesture-based dynamic geometry environments represent an important step in the evolution of digital mathematics tools. By integrating gestures directly into the construction process, they provide an intuitive and conceptually meaningful form of interaction.

When embedded in structured teaching modules, gesture-based interaction supports inquiry-based learning, mathematical communication, and conceptual understanding. For mathematics educators, this approach offers a promising way to combine digital innovation with well-established didactic principles.

10. ACKNOWLEDGEMENT

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