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SKETCHOMETRY AND JSXGRAPH – DYNAMIC GEOMETRY FOR MOBILE DEVICES

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Abstract. Tablet computers seem to be a possible solution to many long-standing problems for technology based mathematics education. But a traditional user interface may not be the best way to interact with dynamic geometry software on a touch device. The new software *sketchometry* enables students to create mathematical constructions by drawing sketches with fingers or pens. This direct approach to Geometry invites students to experiment and thus stimulates active discovery learning. *sketchometry* is based on the software library *JSXGraph* and the programming language *JessieCode*, developed by the authors. *JSXGraph* and *JessieCode* offer a broad range of possibilities for mathematical visualization. For the ambitious user there is an innovative programming environment *SketchBin* to fine-tune constructions.

1. Introduction

Using computer technology in mathematics education has been a very active research area at least since the availability of the personal computer in the 1980s, see e.g. [8, 7].

Arguably, for many years one of the biggest obstacle for the integration of computer technology in mathematics education has been the necessity of a computer laboratory. Usually, the class has to stay the whole lesson in the laboratory and not in classroom. But most computer laboratories are filled with big computer screens, the tables are covered by the keyboards. Regular paper-and-pencil work is limited in such an environment. This may have been one of the reasons why many teachers restricted the use of computer technology to presentations of dynamic content in front of the class [12].

With the presentation of the Apple iPhone in 2007 and the Apple iPad in 2010, followed by a plethora of mobile devices with constantly dropping prices, the situation changed entirely. This new generation of mobile devices is well suited for the classroom. Envisioned already 40 years ago by Alan Kay, one of the pioneers for proposing computers in education [9], with an astonishing precision, tablets are now available for everyone. In contrast to usage of desktop computers in a computer laboratory, working with tablets blends well with paper-and-pencil work. There is no need anymore to leave the classroom in order to use a computer.

"Because mobile technology ... [are] is becoming more and more relevant for mathematics education" [2]. This movement is amplified by the efforts of several countries to replace printed textbooks by electronic ones. Among the countries which are preparing to switch to electronic material are Slovenia, South Korea, and

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some states in the United States [10]. As a consequence for the future, educational software will have to blend well with electronic textbooks.

This article is an overview about software projects developed at the University of Bayreuth, Germany, with involvement of the author. *sketchometry*, *SketchBin*, and *JSXGraph*, *JessieCode* are all based on HTML5 technology and therefore can be used on practically all mobile computers as well as on desktop computers.

2. Mathematics visualization in the web browser

The availability of powerful handheld devices provokes new challenges for interactive mathematics visualization software [3, 15]. Todays dynamic mathematics software is expected to be usable on desktop computers as well as on mobile devices. Software developers therefore have not much choice but to use a platform independent design. At the time of writing the only software standard which is able to run on desktop computers and simultaneously on the vast variety of mobile platforms is the HTML5[1] quasi-standard. This is the common denominator for software which is available on Android, iOS and desktop systems (Windows, Mac OS X and Linux). In technical terms, this means the user interface has to be realized with HTML and CSS, while graphics is realized with SVG (scalable vector graphics) and/or the canvas DOM-element. The programming language which glues everything together is JavaScript.

An advantage of using HTML5 is that eBooks are based essentially on the same technology. Many HTML5 applications can therefore be included in eBooks and enable an unprecedented interactive reading experience.

Being forced to develop new software for mobile devices can be taken as chance to rethink the way of interaction. A user interface is good if students do not experience high cognitive load which disrupts their problem solving performance [13]. With the project *sketchometry* we present our first humble experiences with a so called "natural user interface" (NUI), [16, 17, 11].

3. Sketchometry

The free to use dynamic geometry system (DGS) *sketchometry*[2] comes with a radically new interface which is the first step towards a natural user interface. The user constructs geometric objects and defines their dependencies by simply drawing sketches of lines, circles, and points on the screen.

Figure 1. *sketchometry* in action.

[1] http://www.w3.org/TR/html51
[2] http://www.sketchometry.org

First of all, *sketchometry* is intended to be used by students in the classroom. On purpose it does not have the amount of functionality easily available as offered by other dynamic geometry systems. In many cases these other programs are solely used by the teacher to prepare interactive demonstrations. Together with interactive whiteboards they "encourage teachers to do presentations rather than teach" [12]. Rarely, students are expected to start constructing from scratch with these systems.

On the contrary, *sketchometry* tries to revitalize an active participation of the students. Just as a surplus, it is well suited for classroom demonstrations on an interactive whiteboard, too.

sketchometry consists of a minimal user interface in connection with a novel mixture [3] of algorithms to interpret the user input and convert it into exact mathematical objects. Sketches and gestures are used to input geometric objects and their dependencies directly without the preselection of a specific construction tool from a toolbar. *sketchometry* interprets the paths generated by sketches and gestures and enables constructing by natural interactions.

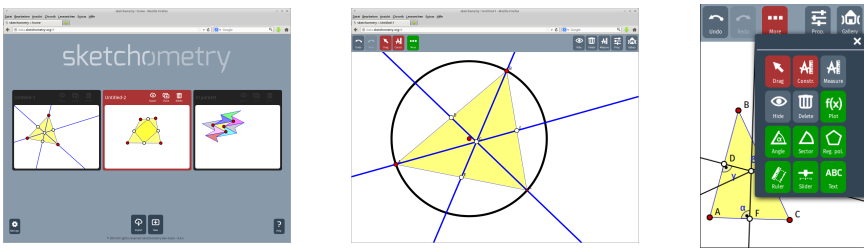

FIGURE 2. *sketchometry* screenshots: "gallery" of all locally stored constructions (left), construction board (middle), running on a smartphone (right).

In most other DGS for constructing geometric objects, the user constantly has to use toolbar buttons. This may interrupt the users work cycle. In contrast, *sketchometry* tries to break this behavior and the user rarely does have to select a tool. For most constructions just a few simple pre-defined figures have to be memorized, and after a short learning period the cognitive load is reduced.

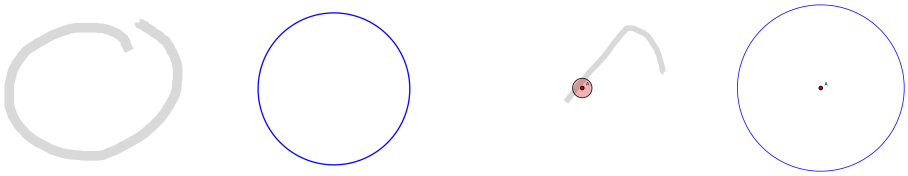

FIGURE 3. Circle sketch (left), circle sketch using a point as center (right).

sketchometry runs on all modern HTML5 browsers on nearly all display sizes. Constructions are saved transparent to the user – there is no save button anymore.

```
// JessieCode
p = point(0,1);
q = point(-1,-2);
li = line(p,q);
```

```
// JSXGraph (JavaScript)
var board = JXG.JSXGraph.initBoard('
    jxgbox'),
  p = board.create('point', [0,1]),
  q = board.create('point', [1,-2]),
  li = board.create('line', [p,q]);
```

FIGURE 4. Comparison of the same construction with *JessieCode* (left) and *JSXGraph* (right).

For external storage several cloud storage services can be used.[3] Meanwhile, there are translation for German, English, Italian, Polish, Portugese, and Spanish, and *sketchometry* is also available as stand-alone app for iOS and firefox OS.

4. JSXGRAPH AND JESSIECODE

JSXGraph[4] [5, 6, 3] is a software library for dynamic mathematics. It is not intended to be used by students, its user group consists of software developers, developers of mathematics visualizations and authors of eBooks. As the most prominent example, *sketchometry* is built upon *JSXGraph*.

JSXGraph is a cross-browser JavaScript library, for interactive geometry, function plotting, and data visualization. First presented in 2008 [4], *JSXGraph* meanwhile provides a mighty and extensive API consisting of several hundred classes and methods.[5] *JSXGraph* constructions can also be included in eBooks based on the file format epub3 [14]. *JSXGraph* is available under the LGPL and the MIT license.

JessieCode[6] is a special purpose scripting language developed at the University of Bayreuth to describe *JSXGraph* constructions. It uses datatypes and a syntax very similar to JavaScript and integrates the *JSXGraph* API in the language.

While *JSXGraph* offers the full power and flexibility of a software library, *JessieCode* can be regarded as a middle layer between the *JSXGraph* library and applications like *sketchometry*. It allows to use a mathematically oriented language to create constructions, see Fig. 4 for an example. Additionally, it provides the necessary security measures to be used as a communication language for mathematics in public web forums or wikis.

The intended use is in environments where JavaScript poses a security risk in the form of Cross Site Scripting (XSS) attacks, e.g. a website with user contributed content like a discussion board or a wiki. To allow advanced constructions in such a site, some kind of scripting is desired which easily becomes a security risk. Since filtering dangerous JavaScript commands is not feasible [1], *JessieCode* is used to control access to the DOM of the browser. This is done by preventing access to

[3] At the time of writing, export to and import from Dropbox, Google Drive, Microsoft Skydrive and Ubuntu One are supported.

[4] `http://www.jsxgraph.org`

[5] A comprehensive documentation is available at `http://jsxgraph.uni-bayreuth.de/docs`, The *JSXGraph* wiki at `http://jsxgraph.uni-bayreuth.de/wiki` contains more than 200 programming examples with source code.

[6] `http://github.com/jsxgraph/JessieCode`

the document and window object, reducing the properties that can be accessed on objects, and filtering HTML from text elements.

5. SketchBin

Beside an easy to use software like *sketchometry* and a programming language like *JessieCode* or an API like *JSXGraph* there is also demand for a tool for the more ambitious user who wants to fine-tune constructions or script certain actions on the board.

For this we developed the authoring tool *SketchBin*.[7] It is an innovative programming editor to develop *JessieCode* contructions, inspired by B. Victor, "Inventing on Principle"[8].

First, the *SketchBin* interface, see Fig. 5, contains an area where so called markdown text including TEX syntax can be typed in, which is converted into good-looking HTML. But the most interesting part of the editor is the Sketch part. Here, the user builds a geometric construction by sketching in the right window. The corresponding *JessieCode* code is instantly displayed in the left window. Conversely, if the user changes the *JessieCode* code in the left window, the construction in the right window will update itself according to the code the left window. Dragging with the right mouse key pressed changes the coordinates given in the *JessieCode* window. Already existing *sketchometry* files can be imported by simple drag-and-drop.

6. Interactive eBooks

In 2011, the eBook file format standard epub3 has been published [14]. It turned out that this new format version is based on large parts of HTML5. Among other HTML5 features, epub3 allows the use of SVG, JavaScript, and MathML.

It is easy to embed *JSXGraph*, *JessieCode* or *sketchometry* files in epub3 books, see Fig. 6. Together with MathJax[9] for typesetting of mathematical formulae this paves the road to truly interactive mathematical e-textbooks.

In 2012 the free software *Apple iBooks Author*[10] was published by Apple. The file format of eBooks produced by this tool is closely related to epub3. It is an easy manner to write so called widgets consisting of *JSXGraph*, *JessieCode* or *sketchometry* files, which can be included in books written with iBooks Author.

7. Conclusion

The projects *sketchometry*, *SketchBin*, *JSXGraph*, and *JessieCode* offer tools of varying complexity to enable dynamic mathematics on a great variety of devices from desktop computers to mobile devices. Small handheld devices are well suited for use in classroom and allow to use electronic material side by side with printed textbooks.

JSXGraph and JessieCode are intended to be used by software developers and eBook authors to create interactive content for web pages and eBooks. *sketchometry* is expected to be used by students in the classroom. It is the first step towards a natural user interface which allows a "hands-on" exploration of geometry.

[7] http://bin.sketchometry.org
[8] http://www.youtube.com/watch?v=PUv66718DII
[9] http://mathjax.org
[10] http://www.apple.com/ibooks-author/

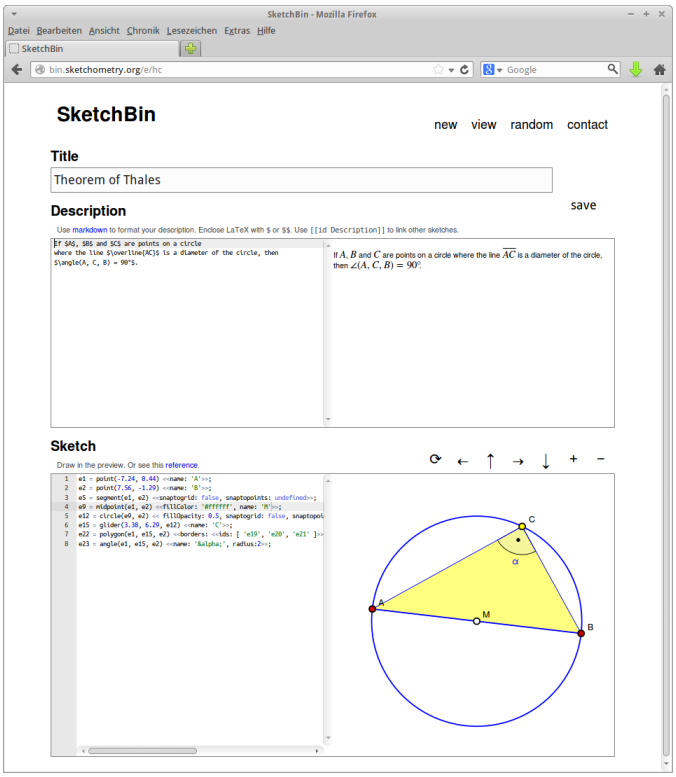

FIGURE 5. SketchBin in action. The description is in markdown format and may contain TEX syntax. The construction can be built by sketching (right) or programming (left).

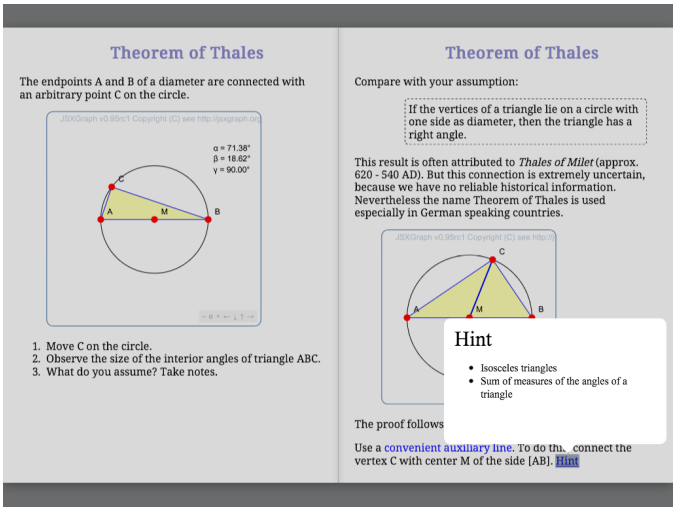

FIGURE 6. *JSXGraph* as part of an epub3 textbook displayed in iOS iBooks.

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