ON INTRODUCTION OF QUADRATIC FUNCTION BY COMPUTER AT SCHOOL

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ABSTRACT. In the paper we document possible utilization of computers at elementary school we describe the propaedeutic of the concepts of a quadratic function and parabola using dynamic geometry software. In the Year 7 these concepts are, in our textbooks, introduced intuitively at the discussion of the properties of the orthocentrum of a triangle; in the Year 9 these findings are used at the teaching of the properties of a function. The results of the experiment show that the utilization of a computer becomes an important tool at the motivation and building-up of concepts, and at the exploration of mathematical concepts. The computer can thus serve not only as a demonstration instrument and a tool for simplifying computations, but also for discovering and stabilizing concepts.

INTRODUCTION

The educational system has been changing systematically both in the contentual and formal aspect. An important feature of the transformation of the teaching that we particularly have in mind is an onset of new forms that emphasize more the active participation of pupils in the teaching process. The consequence of these gradual changes is - among others - a partial retreat from the so-called frontal teaching which, as we at least hope, cannot disappear from schools completely but loses its absolutely dominant position that it used to have some time ago.

Passive transmission (from a pupil's point of view) creates a knowledge apparatus which the pupils can often reproduce but are not able to use for solving tasks (in more detail and expertly, see e.g. [4]). The teaching, and not only in mathematics, should proceed on the basis of a gradual creation of individual concepts and their incorporation to the knowledge structures (about their creation in more detail, see e.g. [9]), not in the form of a handover of complete knowledge and its inanimate algorithmisation.

However, teachers are often put into an uneasy situation because classic textbooks are mostly created for the frontal method of teaching, and when creating teaching aids, teachers are almost exclusively dependent on a do-it-yourself production.

When writing textbooks for children of 11-15 (see e.g. [3]), we therefore purposely tried to put before teachers, and students in particular, textbooks in spirit of the ideas we regard as right. Should we characterize the style of our textbooks in one sentence, the well-known quotation by G. Polya (1887-1985) would appear the most apt: *The best way to learn something is - to discover it.*

In the textbooks, we prepare for the pupils a whole range of situations at which concepts are gradually created, usually by the method of discovering. The piece of knowledge thus, with a correct procedure, remains more permanently stored in memory, and the pupils can incorporate it into their knowledge structures well.

At the beginning of our work we asked ourselves a question inspired by the work of Balacheff and others (see [2], p. 12):

How do we learn from and reconcile the rich networked learning that children and young adults are engaging in as technology-fluent, powerful multimedia communicators outside of school, when they are forbidden to use them for school?

In this paper we show how we tried to take into account the above mentioned principles, for instance at the teaching of quadratic functions. The described approach, in our opinion, corresponds to the opinions described by Juang, Hiu and Chan ([5]):

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The affordance of digital technology can enhance learner abilities, enable learners to engage with activities, and change the way in which they deal with tasks ([5], p. 4) and Those researches demonstrated that the affordances of digital technologies not only provide teachers with a means of performance support in professional development but also provide additional opportunities for learning, debate and collaboration. However, most supporting models and provided tools are designed for individual teachers or groups of teachers. Fewer tools are available for other school members to use to help teachers understand the school curriculum and the learning background of each class, or to design appropriate lesson plans for their students. ([5], p. 5).

1. Utilisation of computers in the teaching process

One thing which we have to approach systematically is the utilisation of computers in the teaching process. The invasion of computers into the teaching process is an unarguable fact to which education has to react. At the same time, we have to be aware that there has not yet been a situation that every pupil has a computer at home and at school and can use it for everyday work.

We implement work with geometric programs in classes of geometry from Year 6. We work with geometric program Cabri II+ which is available in many schools in the Czech Republic. For schools for which Cabri is not available, we can alternatively use free accessible geometric notepads of Geonext and GeoGebra which are, at the elementary school level, a fully comparable substitute of Cabri. Problems which can be solved by pupils using a computer are denoted with a symbol and if it is possible, pupils solve them with the assistance of their teacher. It is, of course, possible that pupils can solve them on their own at home. The textbook and its electronic version present prepared problems in such a way that pupils can investigate given problems without paying attention to the assignment construction instead of the particular geometric problem. Pupils get used to working with computers from Year 6.

The textbooks are designed so that a computer is considered as a helping tool in the teaching process so as not to handicap those pupils who do not have a computer at home.

The utilisation of computers in the teaching of mathematics has undergone difficulties in development recently. Computers are usually used as "demonstrational means" enabling students to visualise processes which cannot be performed with "chalk on the board" in the standard lessons. An array of programs, including freeware, offers teachers a relatively wide utilisation range.

More complicated is the utilisation of computers when pupils work on their own. How to reach the situation when computers help pupils to understand the presented subject matter, when they help to remove worthless demands of calculations, constructions etc., which can distract attention from real understanding of the subject matter and not to cut down necessary (and very often under-estimated) "craft skills". This all was and has been solved in scientific literature at various levels. We can name, for example, among others, the work of Kutzler [7] or of Healy and Sutherland [4].

Recently, attention has been paid to more sophisticated utilisation of computers: is it possible that computers and information technologies generally help pupils in unsupported discovery and contribute to the development of their creative invention? And can computers help effectively to create the factual structure of knowledge? (See for example [1], [2], [5] and [8].)

One of the possibilities of utilising a computer in gradual concept creation is demonstrated in an example of the propedeutic of the concepts of a quadratic function and parabola. We have to emphasize that the presented example is not the main or the only propedeutic of the given concepts. On the other hand, we assume that the connection of the quadratic function with geometric background, which is not at first sight obvious, can be surprising and stimulating.

2. Teaching in Year 7

In the textbook of geometry for Year 7, after introducing concepts of height in a triangle, intersection of heights and after introducing the basic characteristics, we present the following problem (See [3], p. 28): What mutual characteristics do line segments ZP, XP, and YP' have? (See Fig. 1)

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In this form, it is a "classic" school exercise helping pupils to verify that the given line segments are the heights and point V is their intersection. The exercise is supplemented with another exercise which is hard to solve without the use of a computer. Using the computer enables pupils to acquire a new dynamic image of the presented concepts. We create a pre-concept of the concept of quadratic function.

Look at the picture on the computer. What is the curve on which point V moves if we move point Z along a line parallel to line XY? What is point V? Try to estimate what the shape of the curve is.

In our interactive textbooks, which are issued at the same time as their paper version, the situation is prepared. If a teacher does not have this interactive textbook, the situation can be prepared using a suitable program - for example Cabri. Without the interactive presentation, pupils would be deprived of a proper insight into the problem.



FIGURE 1

In the Teacher's Guide, which is prepared for each school year textbook, teachers are told how to use the given situation when the point moves along a parabola, although pupils do not know the concept of a parabola.

3. Teaching in Year 9

In Year 9, we return to the described problem again. In the prepared problem, pupils go back to the characteristics of triangles. Working individually, they can discover some of the characteristics of quadratic functions at the same time. During the computer experiment with the quadratic function, they can - besides other things - observe the role of parameters and the sense of the constant term in the standard form of the quadratic function notation. This is, of course, one of many supportive motivations of the learning process of quadratic functions. On the other hand, the described geometrical link can be beneficial to a new view of this mathematical matter.

Pupils are given the following assignment:

1. Using any of the geometric programs (e.g. Cabri II+), draw a horizontal line p and mark a line segment AB on the line. Draw a line q parallel to the line p. Choose a point C on the line q. Construct a triangle ABC and draw intersection V of its heights. Move a point C along the line q and observe the curve formed by a point V. Use tool Trail yes/no.

Using the button Set in the program Cabri, draw a curve indicating the movement of a point V when the position of a point C is changed. Display axis and grid points in the monitor pad. Using