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APPLICATION OF B-SPLINE REPRESENTATION IN NANOMETROLOGY

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Abstract. In nanometrology, the common tools of metrological software are not sufficient to obtain the correct shape characteristic of measured details because the shape deviations can be comparable with the dimensions of measured detail. Therefore, the new methods for measured data processing are developed to obtain more detailed description of the measured shape. In this contribution, the method based on B-spline representation of measured data is described. This method has been developed and applied in the cooperation with Czech Metrology Institute.

Introduction

In metrological software, the discrete least squares method (DLSM) is commonly used to fit the known shape through the measured points. When measuring the details of common dimensions, this approach is satisfactory. In nanometrology, the shape of real object can be very different from the nominal shape given by technical documentation and the DLSM is often insufficient. In nanometry, there is necessary to obtain more detailed information of the shape of measured object.

In this contribution, the method for data processing measured by optical measured probe on an axis aligned square is described. In case of real square measuring, the measured object is ideally considered to be a quadrilateral and the required output of measured data processing is represented by coordinates of reference point (the centre of the square, in ideal case) and coordinates of quadrilateral vertices. The result of DLSM used in implemented metrological software is a square in general position with respect to the coordinate system only [2]. To obtain more detailed description of the measured square by means of implemented metrological software it is necessary to use the knowledge of nominal dimensions such as coordinates of square centre and length of square edge. However, the nominal dimensions do not be necessarily have to be known.

The main advantage of the suggested method for measured data processing is evaluation of required characteristic dimensions without knowledge of nominal dimensions. Additionally, this method allows each square edge take into account individually. Therefore, the measured object can be described as a quadrilateral and deviation of real shape can be evaluated with respect the edges of this quadrilateral.

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1. Measured data processing

The suggested method for measured data processing based on B-spline representation of measured data will be demonstrated on a sample set of input data in graphical form. The set of points measured on an axis aligned square which is shown in Fig. 1 will be used as the sample of input data.

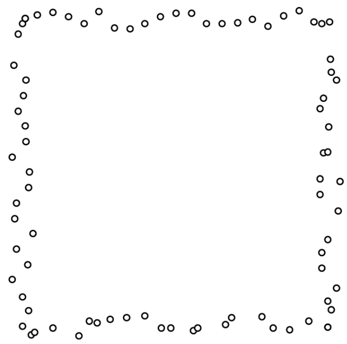

Figure 1. Input data

The suggested method consists of the following steps

1.1. Continuous representation of discrete measured data. The sequence of measured points is considered to be a set of control vertices for closed uniform cubic B-spline curve which is approximated through the all input data, see red curve in Fig. 2.

1.2. Reference point of the measured square. The reference point **S** of the measured object is given by centroid of the area bounded by the closed uniform cubic B-spline curve, see point **S** in Fig. 2.

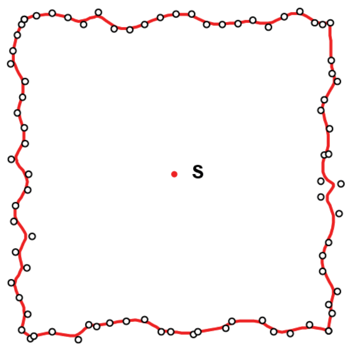

Figure 2. B-spline curve and reference point

1.3. **Discrete and continuous input data of quadrilateral edges.** As the reference point is determined, the sequence of measured points is divided into four subsequences [1]. Each subsequence represents input data for the corresponding edge of quadrilateral. Consequentially, the closed B-spline curve is divided into four parts. Each part of B-spline curve is supposed to be a continuous representation of measured points corresponding to one edge of parallelogram.

1.4. **Direction of quadrilateral edges.** The average tangent vector **t** of each part of B-spline curve is determined, see Fig. 3 where the upper subsequence, corresponding B-spline curve and its average tangent vector **t** are depicted. The direction of average tangent vector determines the direction of corresponding quadrilateral edge.

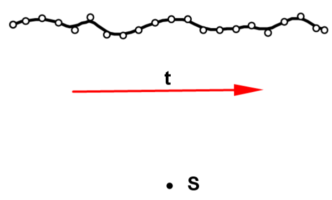

FIGURE 3. Average tangent vector

1.5. **Position of quadrilateral edges.** To obtain the position of quadrilateral edge, the surface area of the region R is calculated, first. Region R is bounded by part of B-spline curve, straight line p parallel to average tangent vector and passing through reference point **S** and straight lines m, n perpendicular to p and passing through end points of B-spline curve, see grey region R in Fig. 4.

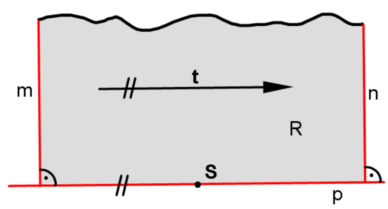

FIGURE 4. Surface area of region

Then, the surface area of region R is recalculated on the rectangle of the same surface area, see Fig. 5.

After that, the length of this rectangle is equal to the distance between m and n straight lines, altitude of this rectangle determines the position of straight line which is the representation of quadrilateral edge, see Fig. 6.

1.6. **Vertices of quadrilateral.** The above described procedure is repeated for each part of B-spline curve, see Fig. 7 where the obtained quadrilateral is shown. Vertices of this quadrilateral are considered to be the intersections of quadrilateral edges, see Fig. 8.

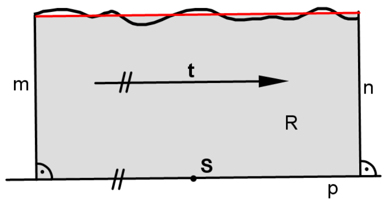

FIGURE 5. Rectangle

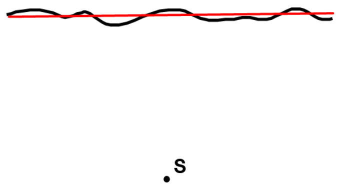

FIGURE 6. Quadrilateral edge

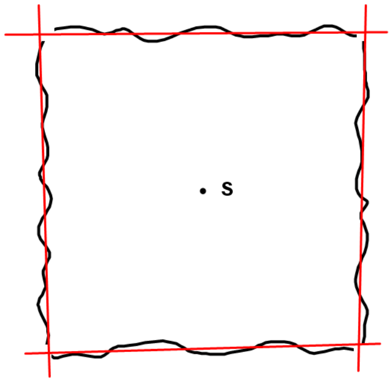

FIGURE 7. Quadrilateral edges

1.7. **Shape deviation.** Finally, the shape deviations of the real measured shape are evaluated as the normal distance of the point on closed B-spline curve from corresponding edge of quadrilateral.

The continuous behavior of deviation function is depicted in Fig. 9 (the scale of deviations is drawn ten times the real size), where $\pm 10\%$ tolerance limits are considered and expressed in colours. Deviation of points in tolerance zone (deviations of which are less than 10 % with respect to the biggest length of quadrilateral edge) are depicted in blue, deviation of points outside of tolerance zone (deviations of which are bigger than 10 % with respect to the biggest length of quadrilateral edge) are depicted in red.

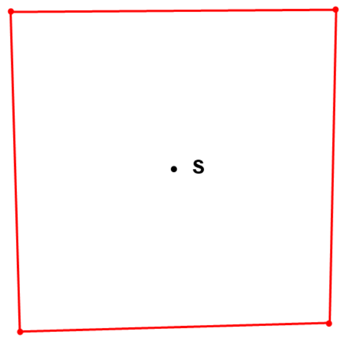

FIGURE 8. Quadrilateral

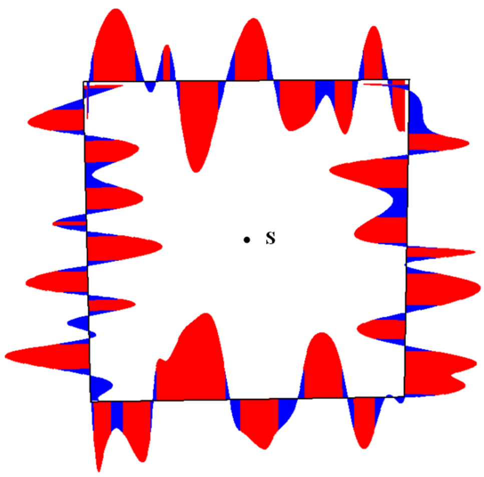

FIGURE 9. Measured shape deviations

CONCLUSION

The new approach to measured data processing is described in this paper. This approach is based on B-spline representation of measured data and determination of characteristic dimensions of the measured object from this B spline curve. The suggested method is suitable for nanometrology where the detailed information about the shape of measured object is important.

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REFERENCES

[1] Bütefisch, S. – Zelený, V.: Novel calibration artefacts and tactile probes for micro/nano-CMMs, 257th PTB-Seminar, Braunschweig, Germany, 2010.

[2] Zelený, V. – Skalník, P.: Redundant Measurements Using Laser.In: IMEKO - International Measurement Confederation. Genova: IMEKO, 2004, vol. 1, p. 101-108. ISBN 88-901344-0-2.

[3] I. Linkeová: Application of B-spline in nanometrology, Proceedings of 30th Conference on Geometry and Graphics, 2010, 171–174, (available on http://www.csgg.cz/30zlenice/sbornik2010.pdf).

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