

Solve First – Insight Later: a new way of understanding geometry using Symbolic Geometry and CAS

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Abstract

The thing which makes geometry so appealing is that sometimes cleverness can yield simple solutions to complicated problems. This can also make geometry frustrating on the occasions when the cleverness eludes us.

Symbolic geometry software, such as Geometry Expressions facilitates the creation of mathematical models and the expression of the geometry as algebra (or trigonometry). The algebraic form may be copied into a CAS and standard operations applied to work to a solution. The solution may in turn be copied back into the symbolic geometry system and analyzed. Examination of the solution obtained in this manner can yield geometric insight which can lead to more succinct, simpler proofs. This yields a fundamentally new approach to solving geometry problems, which allows the insight to come after the problem is solved. In this way, progress can be made, even without the insight.

In this talk we present a number of geometry problems whose analysis yielded geometric insight in just this way. Many of these problems arose during summer mathematics projects undertaken by students at the Portland Saturday Academy over the course of the last 10 years.

The more engaging student projects involved the analysis of real-world problems, the formulation of tractable mathematical models, and solution of the mathematical problem posed by the model. We give examples of mathematics motivated by application domains from central pivot irrigation, design of solar cookers, wind power and telescope aberration. We highlight the formulation of mathematical problems from application domains as a highly creative mathematical activity and one to which the student is traditionally not exposed.

WORKSHOP

From Inductive Reasoning to Mathematical Induction with Geometry Expressions

Dynamic Geometry systems facilitate inductive reasoning, as the student makes conjectures from the interactive diagram. Such conjectures are either left unproved or proved by conventional means. Symbolic Geometry systems such as Geometry Expressions allow the student both to make more elaborate conjectures and to prove them.

In this beginner level workshop we will learn how to use constraints in Geometry Expressions to specify a model symbolically and how to extract algebra to make and prove a conjecture about Pappus Chains.