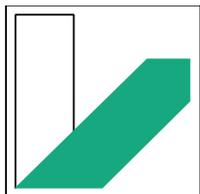


Supporting Mathematically Gifted Students in School



UNIVERSITÄT
BAYREUTH

Prof. Dr. Volker Ulm

Structure

1. **Models for Giftedness from Psychology**
2. **Model for Mathematical Giftedness**
3. **Diagnosis of Mathematical Giftedness**
4. **Supporting Gifted Pupils**
5. **Natural Differentiation in Regular Lessons**
6. **Additional Offers in Regular Lessons**

1. Models for Giftedness from Psychology

1.1 Intelligence Quotient IQ

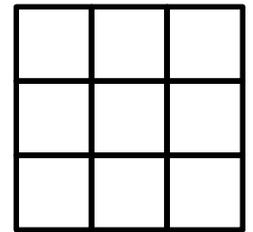
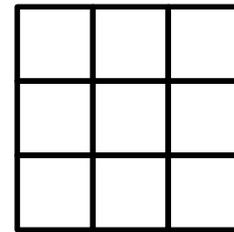
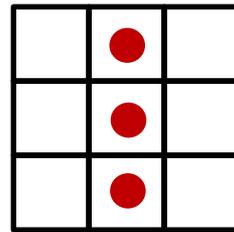
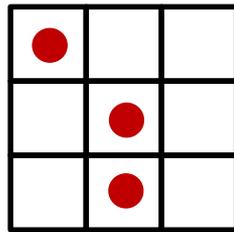
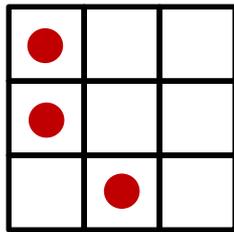
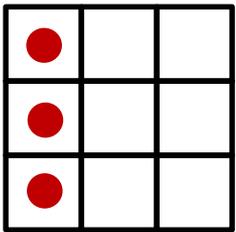
Giftedness

:= Intelligence

**:= that, what intelligence tests
measure by the IQ**

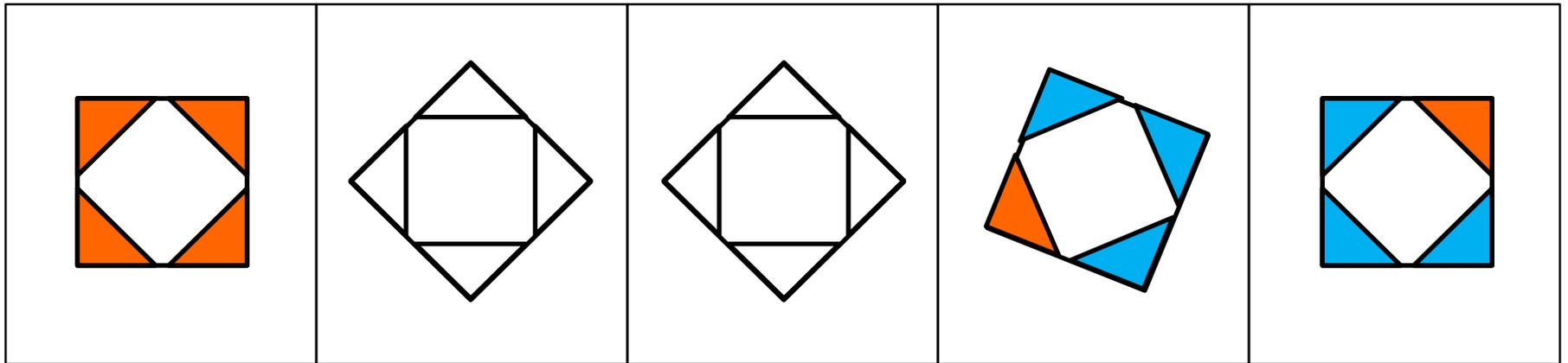
Tasks from intelligence tests

Continue patterns



Tasks from intelligence tests

Which figure does not fit to the others?



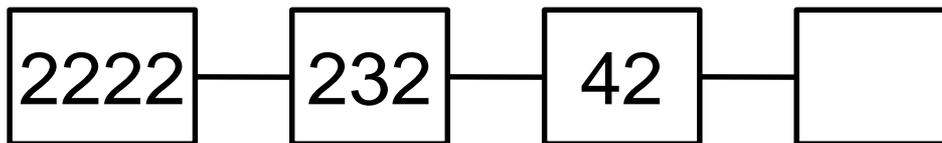
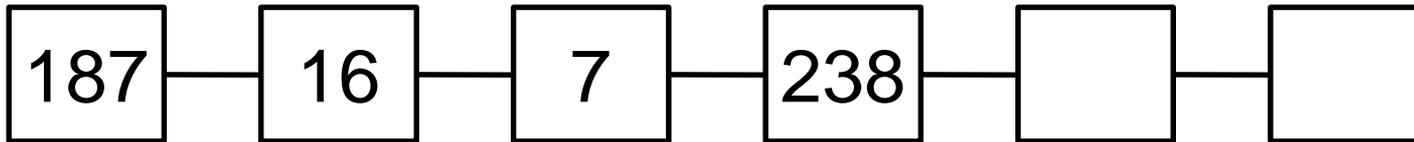
Tasks from intelligence tests

Which word does not fit to the others?

Train	Ship	Bus	Bike	Plane
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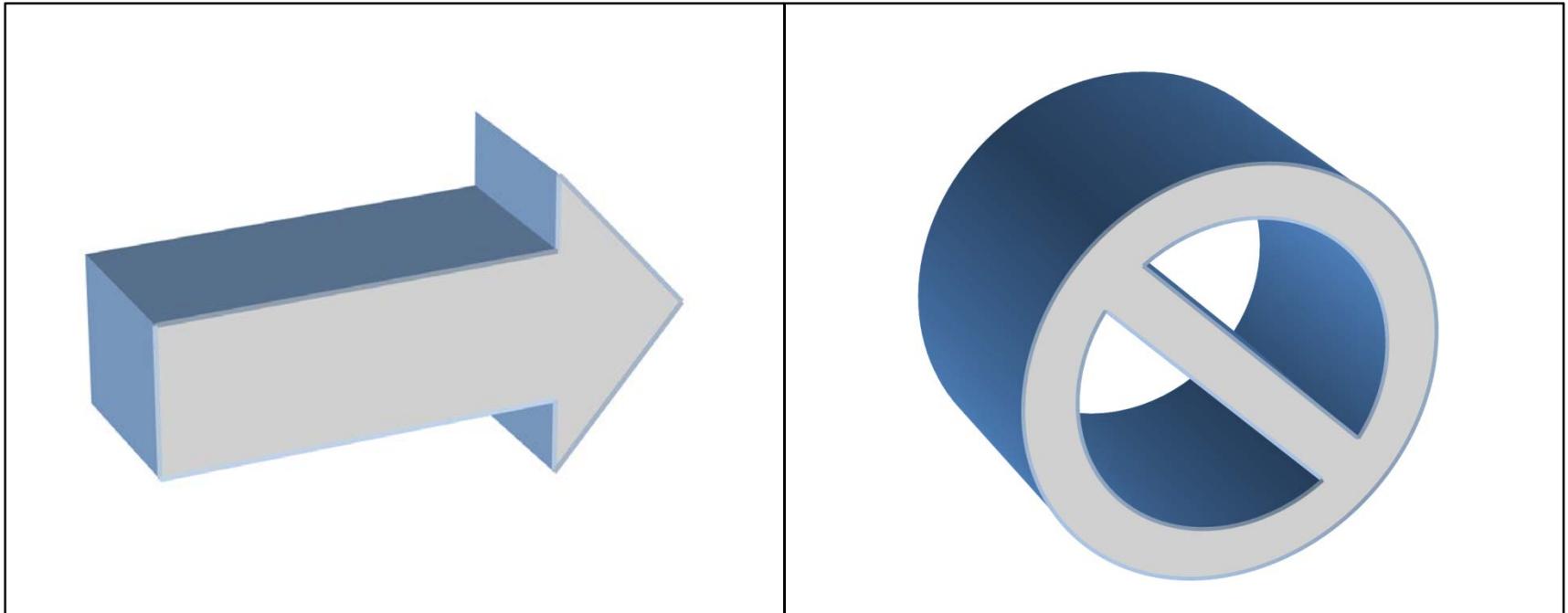
Tasks from intelligence tests

Continue the series of numbers



Tasks from intelligence tests

How many sides do the solids have?



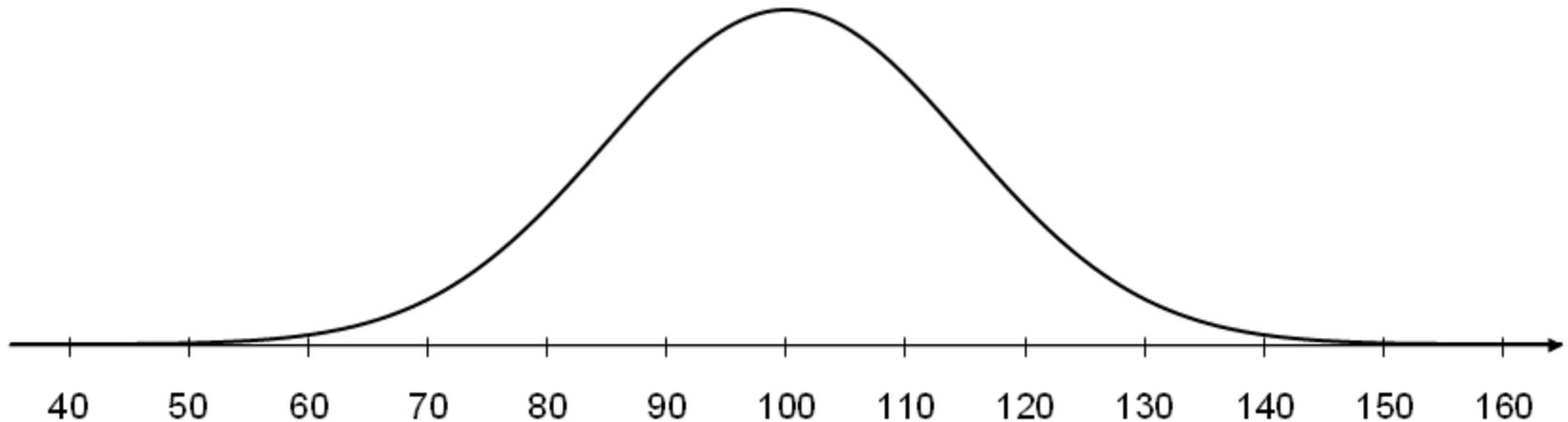
Assessment of intelligence tests

- For each task only one answer is accepted to be right.
- For right answers one gets points.
- The total number of points is converted into the IQ by a simple table.

Distribution of the IQ

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

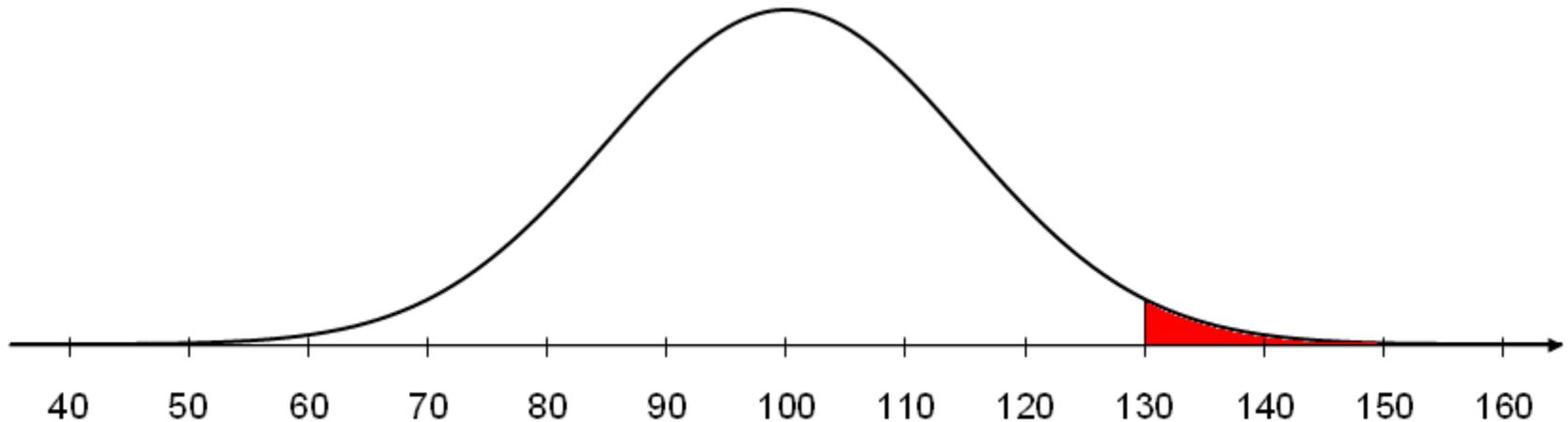
$$\mu = 100, \quad \sigma = 15$$



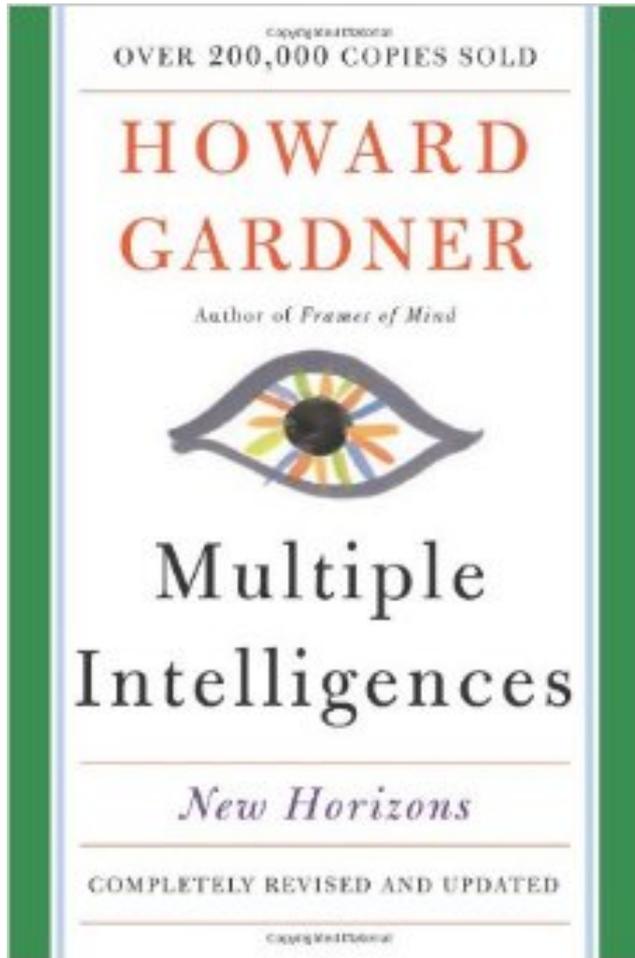
Highly gifted persons

$$IQ - \mu \geq 2\sigma \quad \Leftrightarrow \quad IQ \geq 130$$

2% of the population



1.2 Theory of multiple intelligences



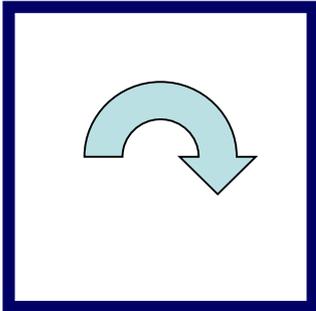
Multiple Intelligences

1. Verbal-linguistic intelligence



Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence



{0; 1; 2; 3}



Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence



Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence



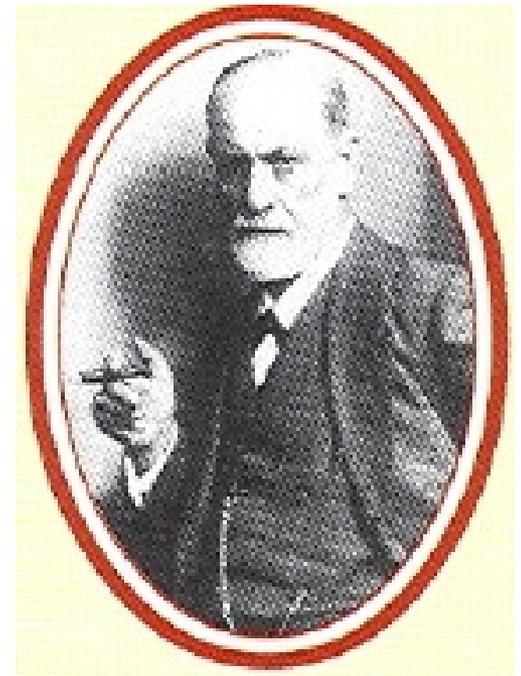
Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence
5. **Bodily-kinesthetic intelligence**



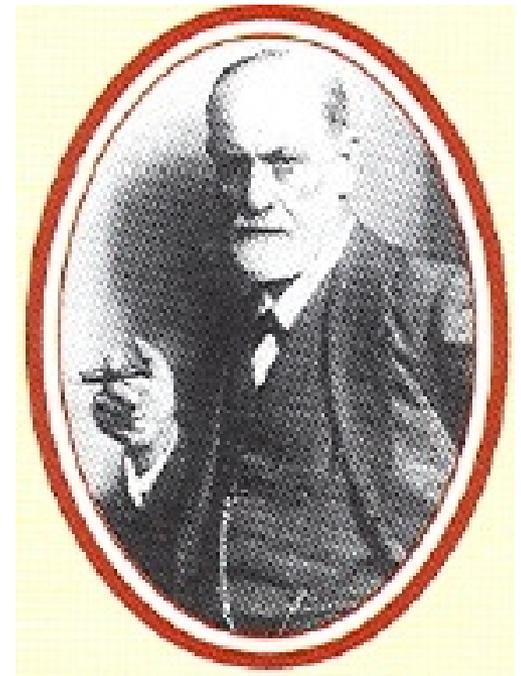
Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence
5. Bodily-kinesthetic intelligence
6. Interpersonal intelligence



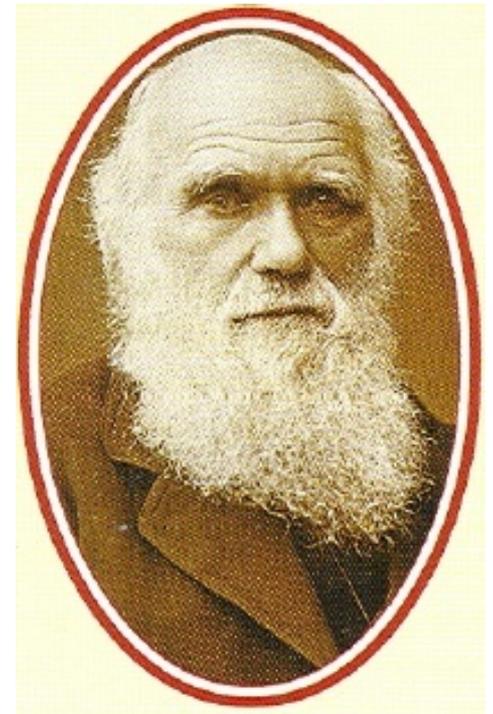
Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence
5. Bodily-kinesthetic intelligence
6. Interpersonal intelligence
7. Intrapersonal intelligence



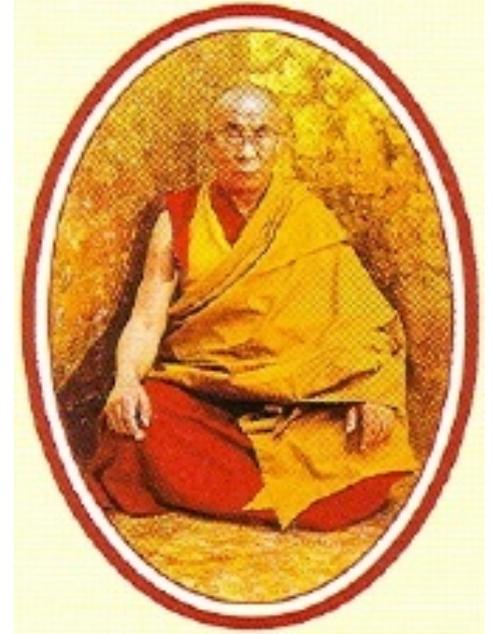
Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence
5. Bodily-kinesthetic intelligence
6. Interpersonal intelligence
7. Intrapersonal intelligence
8. **Naturalistic intelligence**



Multiple Intelligences

1. Verbal-linguistic intelligence
2. Logical-abstract intelligence
3. Visual-spatial intelligence
4. Musical-rhythmic and harmonic intelligence
5. Bodily-kinesthetic intelligence
6. Interpersonal intelligence
7. Intrapersonal intelligence
8. Naturalistic intelligence
9. **Existential intelligence**

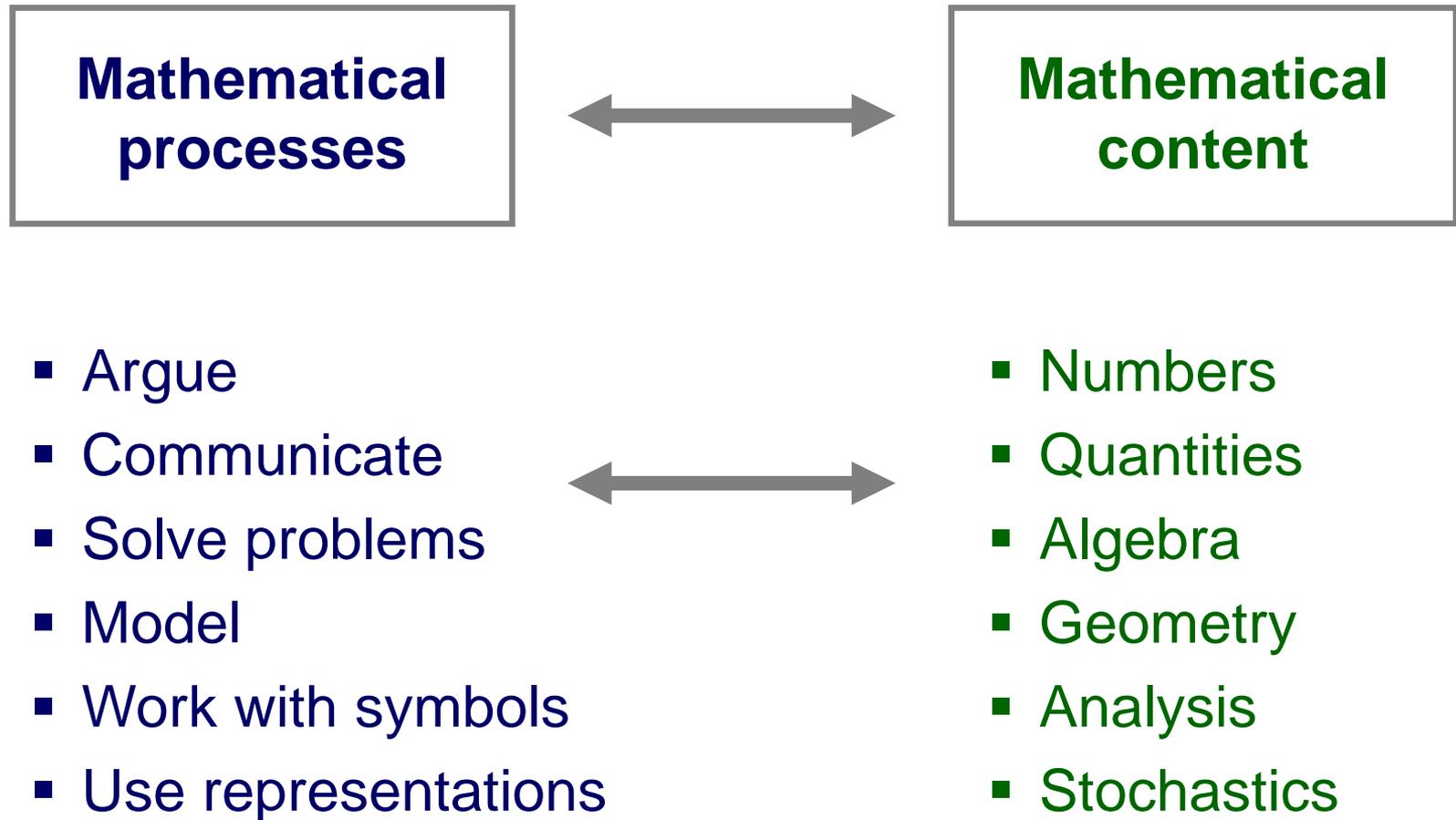


Multiple Intelligences

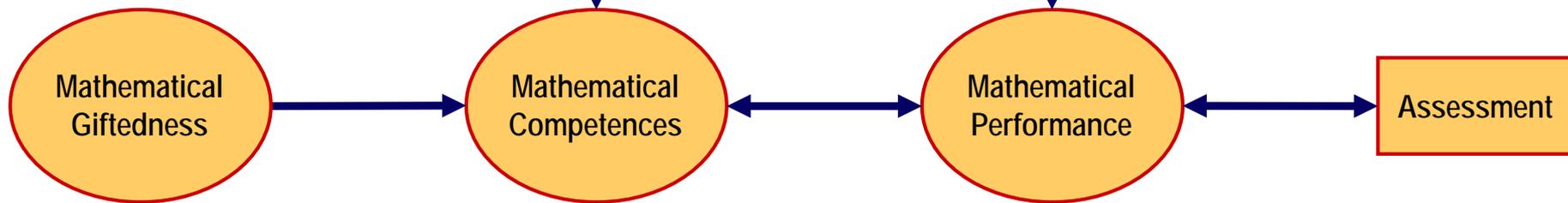
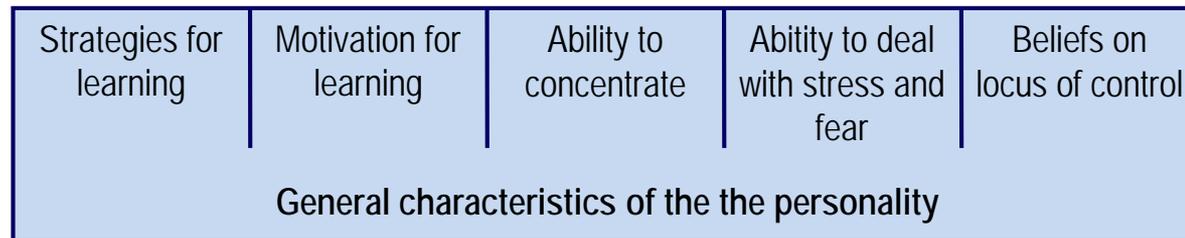
1. **Verbal-linguistic intelligence**
2. **Logical-abstract intelligence**
3. **Visual-spatial intelligence**
4. **Musical-rhythmic and harmonic intelligence**
5. **Bodily-kinesthetic intelligence**
6. **Interpersonal intelligence**
7. **Intrapersonal intelligence**
8. **Naturalistic intelligence**
9. **Existential intelligence**

2. Model for Mathematical Giftedness

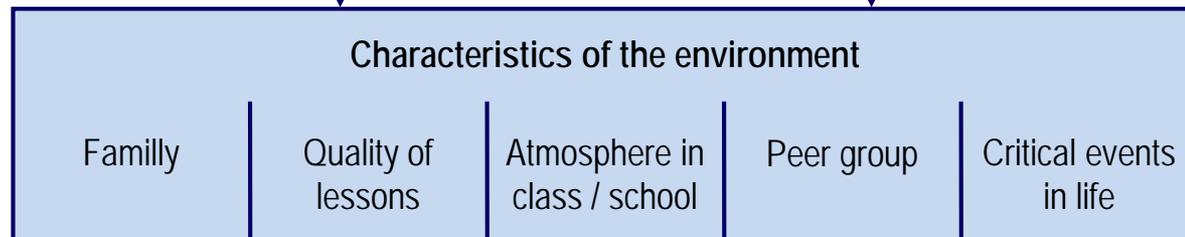
2.1 Mathematical Competences



2.2 Giftedness, Competence, Performance



= Potential for the development of mathematical competences



3. Diagnosis of Mathematical Giftedness

Memorise numbers

Memorise numbers

1	19	18	2
9	11	12	8
7	13	14	6
3	17	16	4

Memorise numbers

Memorise numbers

1	19	18	2
9	11	12	8
7	13	14	6
3	17	16	4

Examples with pupils

Memorise numbers:

Helena

Nina

Add numbers:

Leon

Helena

3. Diagnosis of Mathematical Giftedness

- **Create substantial situations that make pupils think mathematically**
- **Observe pupils when they work**
- **Ask pupils to explain considerations**
- **Analyse what pupils have written**

4. Supporting Gifted Pupils

Acceleration	Enrichment	Support in regular lessons
<ul style="list-style-type: none">▪ Skip a grade▪ Visit lesson in higher grades	<ul style="list-style-type: none">▪ Optional courses in the afternoon▪ Competitions▪ Seminars during holidays▪ Studies at university	<ul style="list-style-type: none">▪ Natural differentiation▪ Additional offers in regular lessons

5. Natural Differentiation in Regular Lessons



Formulate questions about the picture and try to answer them.

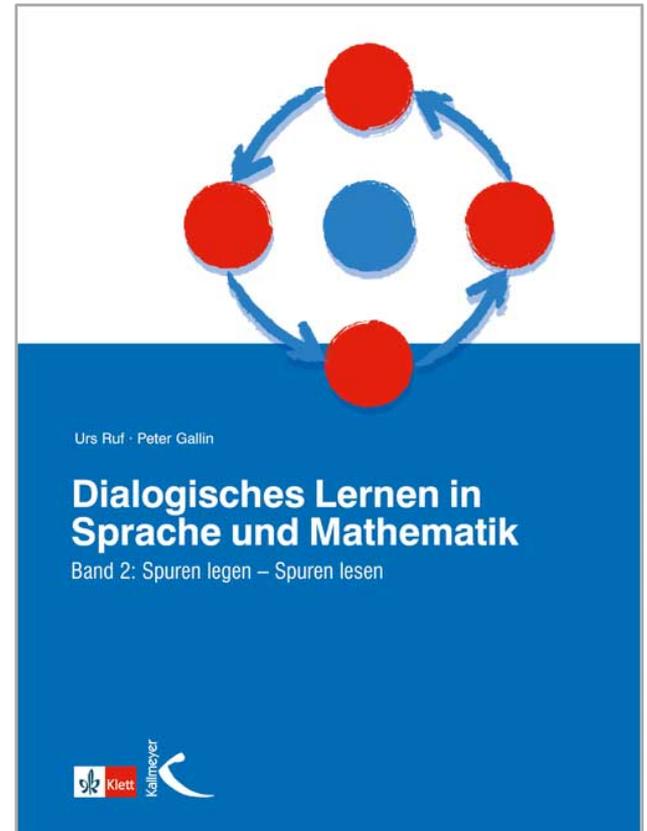
I – You – We

I: Individual Working

You: Cooperation with partners

We: Communication in Class

by P. Gallin & U. Ruf



I: Individual Working

Since learning is an individual process, students initially work on their own. They are faced with the necessity to explore the content, activate their prior knowledge, develop ideas and make discoveries.

You: Cooperation with Partners

It is very natural for students to discuss their ideas with partners in small groups and work on problems cooperatively.

This communication helps to order thoughts and to get more ideas. Meanwhile, the teacher can stay in the background or turn his attention to individuals.

We: Presentation of Ideas

After having worked individually and in groups the students present their ideas and discuss them in class. The different contributions reveal multiple aspects of the topic so it can be viewed from different perspectives. Moreover, students develop presentation, communication and argumentation skills.

We: Summary of Results

Finally, the students' results are summarized and possibly expanded by the teacher. It is the teacher's task to introduce subject-related conventions and to consider curricular regulations.

However, since the students have already explored the new content on their own paths, they are more likely able to integrate the teacher's explanations into their individual cognitive structures.

Method for Natural Differentiation

Individual Working

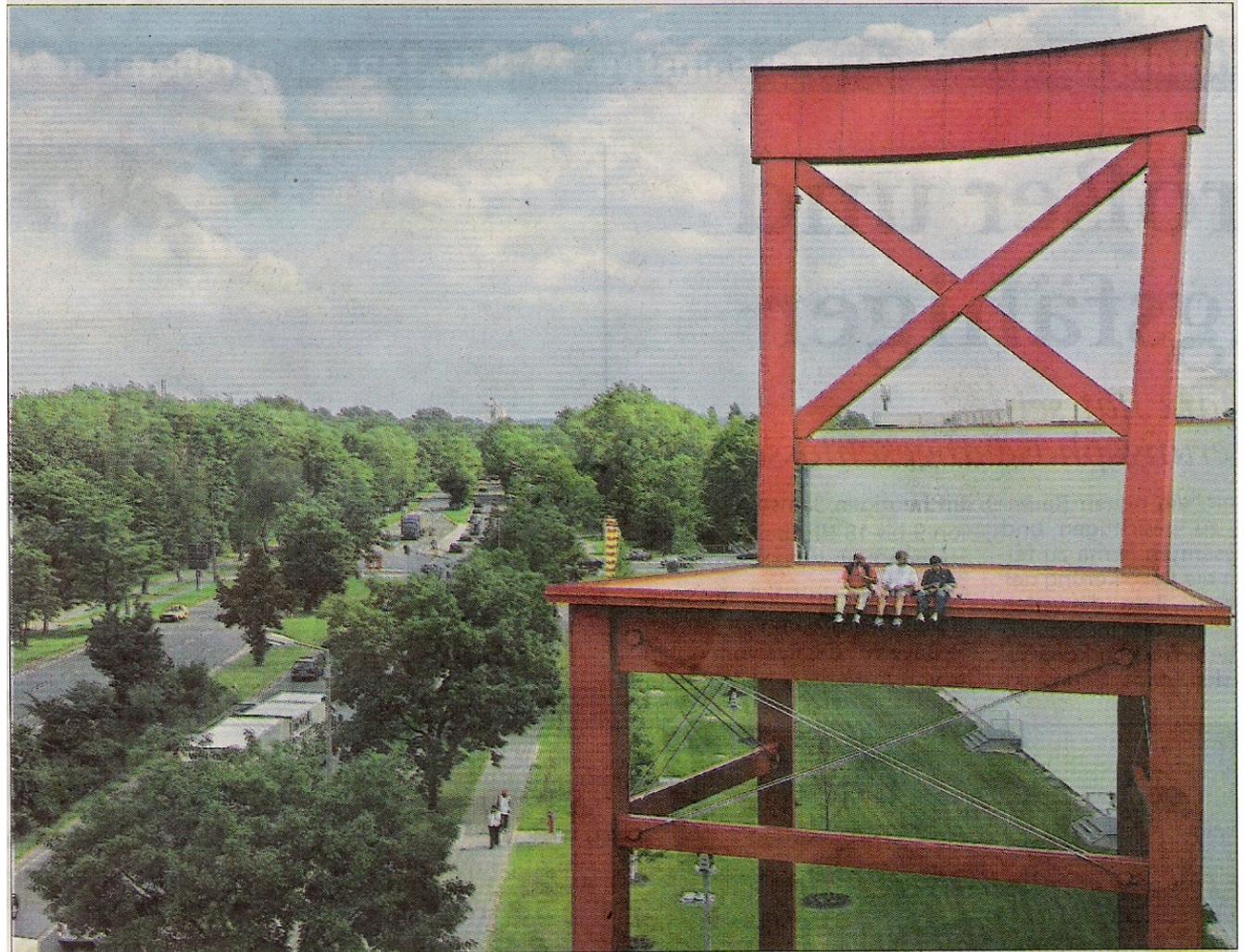
Cooperation with Partners

Presentation of Ideas

Summary of Results

Formulate questions

Formulate questions about the picture and try to answer them.



Größter Stuhl der Welt steht in Nürnberg

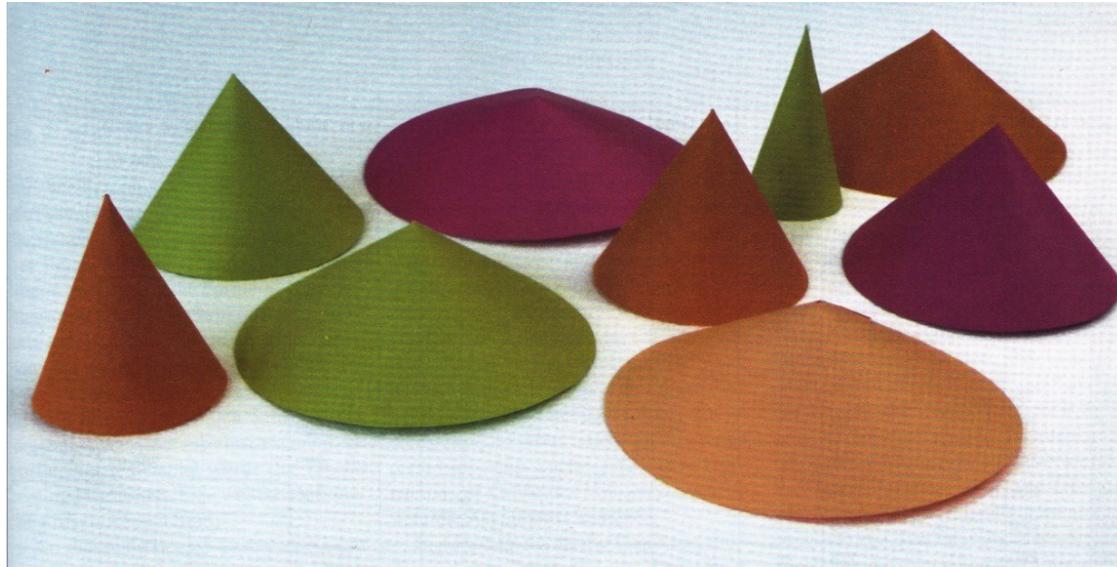
Explore mathematical objects

Try to find many properties of the function:

$$f(x) = \sqrt{x(6-x)}, \quad x \in D_{\max}$$

Explore mathematical objects

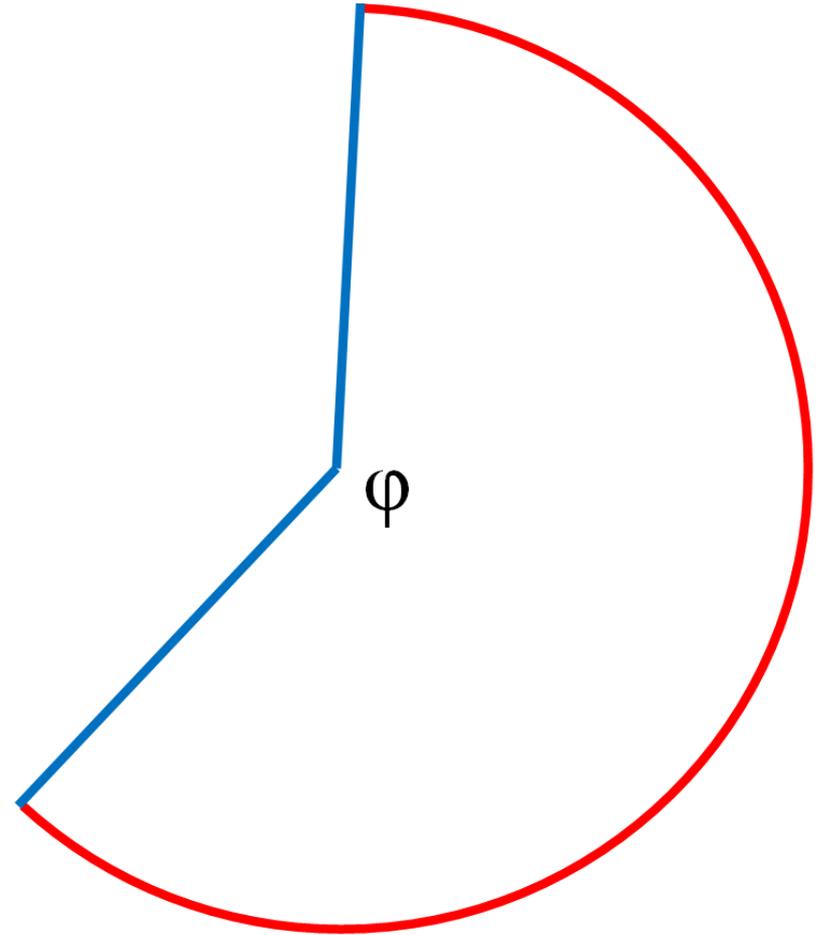
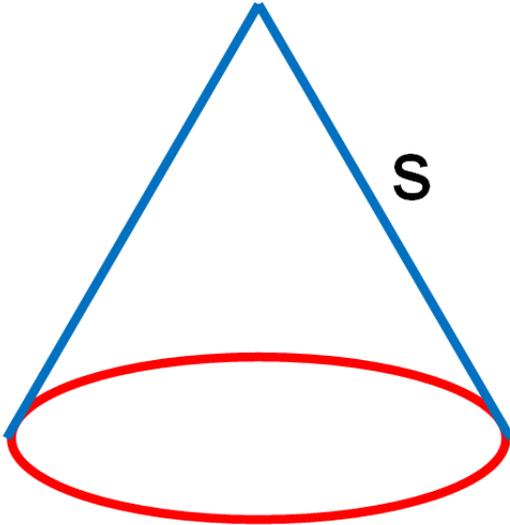
You would like to make a cone with paper.



Which form do you have to cut out of the paper?

Explore how the size of the cone (e.g. height, surface, volume) depends on the size of the cut out paper!

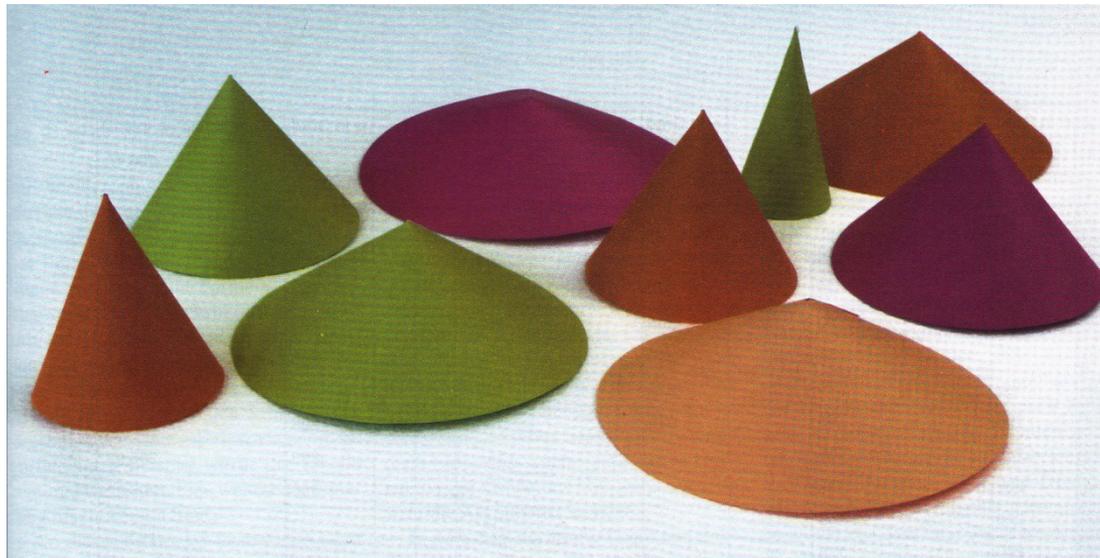
Geometric dependencies



Build and measure

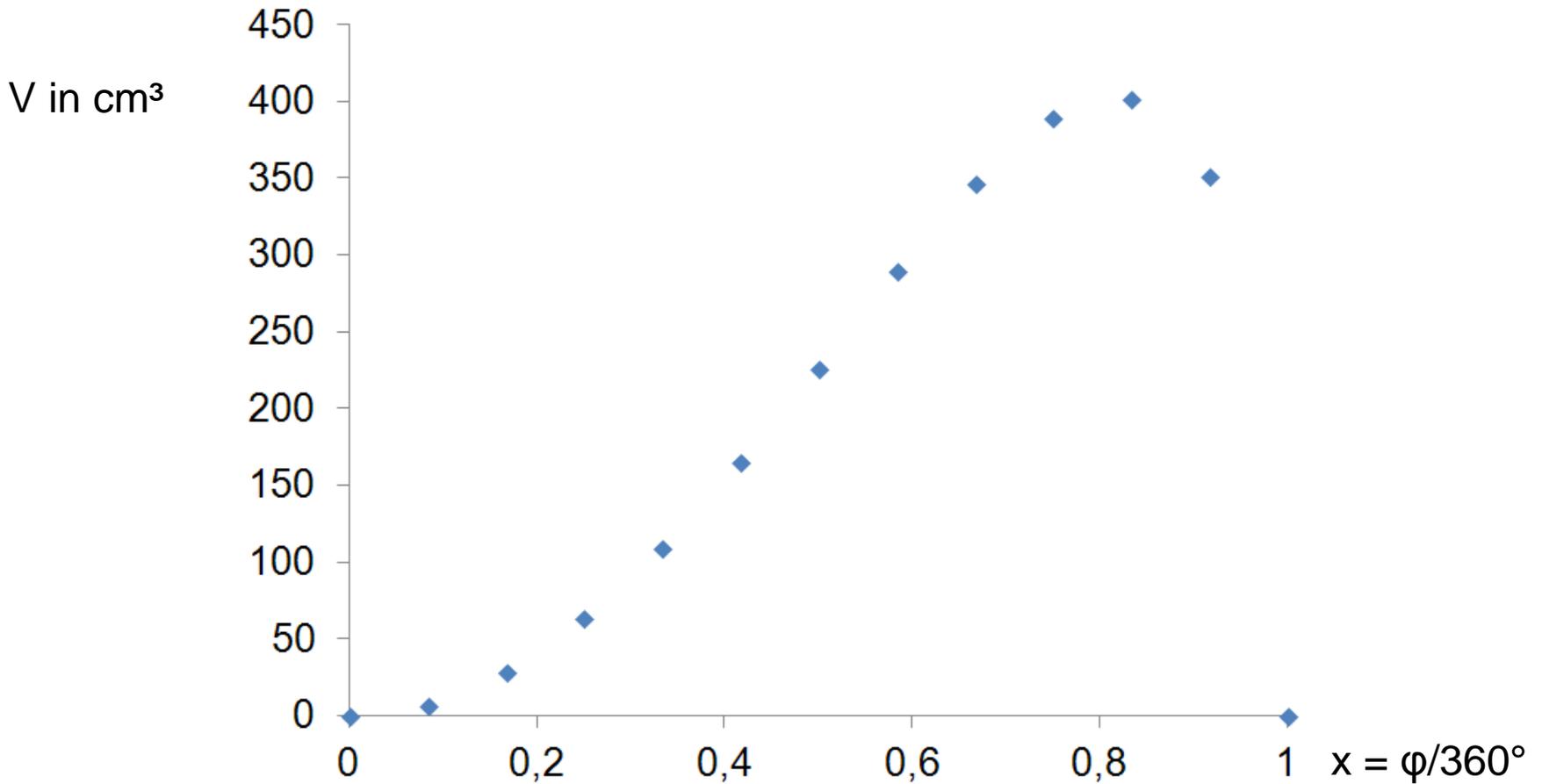
φ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
V in cm^3	0	7	29	63	110	165	227	289	347	390	402	352	0

s = 10 cm



Diagram

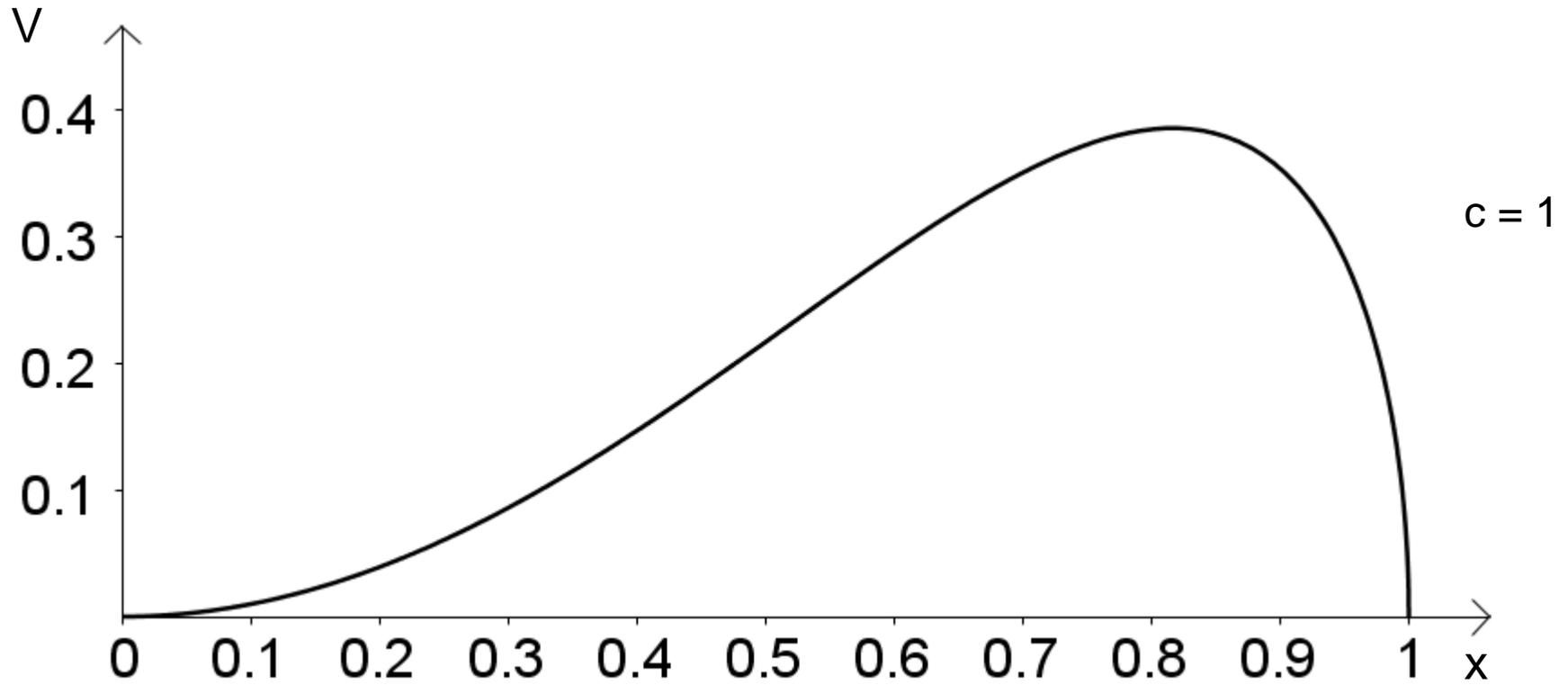
φ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
V in cm ³	0	7	29	63	110	165	227	289	347	390	402	352	0



Term and function

$$V(x) = c \cdot x^2 \cdot \sqrt{1 - x^2}$$

$$x = \frac{\varphi}{360^\circ}, \quad c = \frac{\pi}{3} s^3$$



Invent tasks

Convergent formulation	Open formulation
Calculate: 3^5 , 6^3 , 2^7 , 12^2	Calculate some powers! Find powers with a three-digit value.
Calculate: $24 \cdot (9 + 8 : 2)$	Create terms with the numbers 24, 9, 8 and 2 and calculate them. Create with these numbers terms which have a value as large (or small) as possible. Invent terms with numbers and brackets and calculate them.

Invent tasks

Convergent formulation	Open formulation
<p>Solve the equation $7x - 11 = 24$.</p>	<p>Create some equations with the solution $x = 5$.</p> <p>Create quadratic equations with the solutions 1 and 5. Describe all possibilities.</p> <p>Create exponential equations with the solution 5.</p> <p>Create a word problem corresponding to the equation $7x - 11 = 24$.</p>

Invent tasks

Convergent formulation	Open formulation
<p>Given is the function $f(x) = (x + \sin x)^2$.</p> <p>Calculate the first derivative, extreme values,</p>	<p>Invent a function which has „sin x“ in its term, and explore it.</p>

Estimate and argue

Many car drivers started in their holidays in traffic jams on the motorways.
Last Friday the traffic jams had a total length of 200 kilometres.

How many persons were involved in these traffic jams?
Explain your considerations.

Natural Differentiation

(1) All pupils in class work on the same topic.

(2) The tasks

- **are open, i.e. they support different ways of doing mathematics,**
- **are accessible to all pupils, such that all pupils have the chance to feel success,**
- **are rich with respect to mathematical content,**
- **support working on different levels.**

Natural Differentiation

(3) The method combines

Individual Working

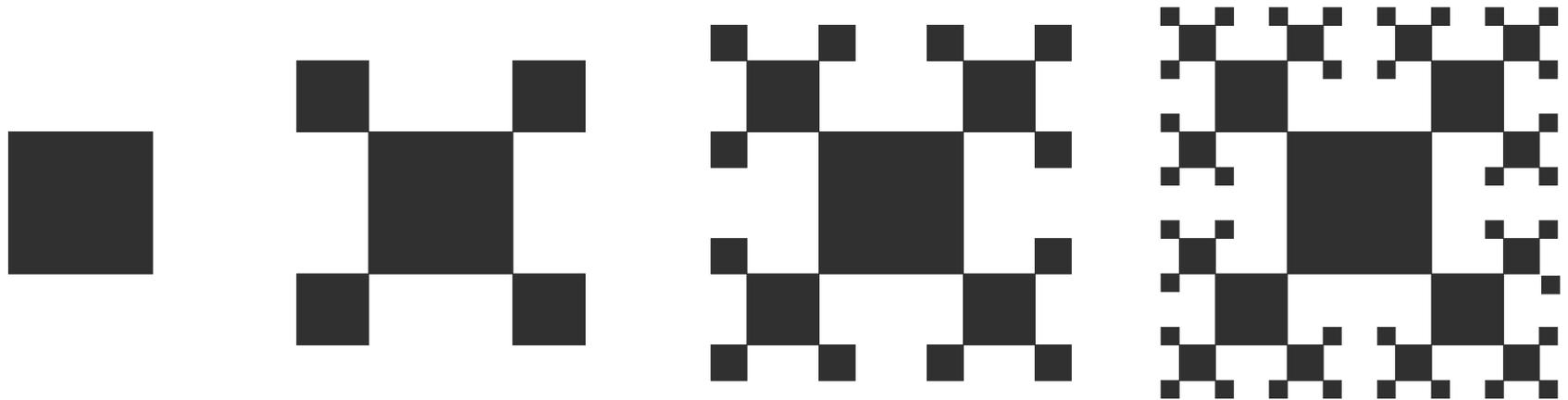
Cooperation with Partners

Presentation of Ideas

Summary of Results

6. Additional Offers in Regular Lessons

Explore mathematical objects



Explore this series of figures.

Formulate questions and try to answer them.

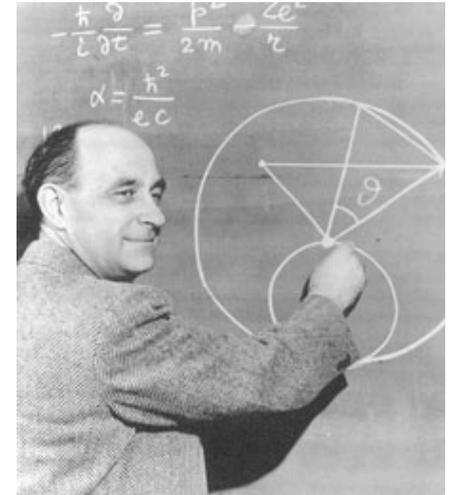
Additional tasks as brain-teasers

How many zeros are at the end of the number
with the value

$$1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot \dots \cdot 99 \cdot 100 ?$$

Fermi questions

- How long did you look TV in your life up to now?
- How many beats did your heart make up to now?
- How many dentists are there in the Czech Republic?
- How much drinking water does your family use per year?
How much drinking water is used in the Czech Republic?
- Which mass has the air in our classroom?
- How many pupils could go into our classroom?

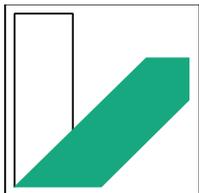


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